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Structural damage prediction based on Bayesian updating with Monte Carlo sampling

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Abstract. A series of engineering accidents in recent years have caused people to worry about the safety of engineering structures. As buildings continue to age, engineering accidents are increasing, and it is necessary to monitor their health. Bayesian updating can determine the probability of damage to a structure by collecting a posteriori information, and can more accurately estimate the location and extent of damage, thereby predicting the risk of structural damage and repairing the damage in time, significantly improving engineering efficiency. This paper attempts to update the information of the simulated structure based on the Bayesian updating method to obtain a more accurate posterior distribution of the potentially damaged components to demonstrate the feasibility and superiority of the technique. This paper finds that Bayesian updating can be used in conjunction with modeling software such as SMsolver. By constructing a structural model with SMsolver and processing the data with Bayesian updating, it is possible to calculate the damage problems of the structure accurately. At the same time, this paper also uses Monte Carlo sampling technology in combination with the Bayesian updating method to explore the prediction effect of structural damage. Using Monte Carlo sampling methods, the failure probability and damage location of the structure can be effectively estimated under the condition of known prior information, thereby improving production efficiency and the accuracy of structural inspection.

Keywords: Bayesian updating, Bayesian principle, structural damage, Monte Carlo sampling.

1. Introduction

With China's reform and opening up at the end of the last century, the economy developed rapidly, and domestic infrastructure construction also ushered in rapid development. Many bridges, roads and high-rise buildings were quickly built and put into use in a short period. However, with the passage of time and the increase in traffic volume and vehicle weight requirements, China's existing steel structures are aging, and the degree of fatigue deterioration is also increasing. Therefore, methods for evaluating and updating the actual fatigue life of such structures are urgently needed.

The randomness of fatigue growth and the uncertainty of the load history of steel structures require regular and repeated inspections, which have become an important part of the bridge evaluation and management process. In this case, Bayesian updating derived from the Bayesian theorem can be well applied to assess and update the actual fatigue of the structure. Bayesian theory can be used to more effectively update fatigue reliability models using the results of NDI technology, improving the estimation of the remaining fatigue life and safety of existing steel bridge structures, allowing them to continue to be used while minimizing unnecessary repairs or re-installation [1].

In the long-term operation of structural facilities, the aging and fatigue of structures become inevitable problems over time. In order to ensure the safety and reliability of these structures during their life cycle, and to improve the efficiency of damage detection and maintenance, more advanced detection and evaluation methods are needed.

In 1998, Beck [2] and others first applied Bayesian theory to model updating in the field of civil engineering, proposing a basic framework for Bayesian updating. To solve the problem of integrating the posterior distribution of parameters, Beck and others introduced the Metropolis-Hastings (MH)-based Markov Chain Monte Carlo (MCMC) algorithm into Bayesian updating and used the shear model to prove the reliability of the method [3]. In this study, two structural models will be established using SM Solver, and the structure's posterior information and the structure's failure probability will be updated using the Bayesian update method and the Monte Carlo sampling method, respectively, to propose a more efficient structural damage prediction method.

This paper aims to predict and assess structural damage based on Bayesian updating and Monte Carlo sampling methods. This paper aims to explore the feasibility and superiority of these two methods in structural damage prediction and verify their effectiveness through specific experiments. To achieve this goal, this paper introduces the basic principles of Bayesian updating and Monte Carlo sampling and their applications in structural damage prediction, including specific calculation formulas and processes. Second, this paper verifies the application of Bayesian updating and Monte Carlo sampling in structural damage prediction through two specific experiments, and finally, this paper proposes future research directions and application prospects.

2. Basic methods

2.1. Bayesian updating method

Based on the principle of Bayesian, the new information about an object obtained through factual evidence or experiments is used as the likelihood (i.e., the probability of the event actually occurring and the probability of the event not occurring), and the original prior probability of the object is calculated through the Bayesian formula to obtain the posterior distribution of the object, thereby obtaining more reliable posterior information about the object [4]. This study established a model of a plane truss with possible damage using SM Solver. The lengths and stiffnesses of some members and supports were known, while the stiffnesses of the members with possible damage were unknown and assigned a possible prior distribution. The author used the actual deformation of the truss structure as new information to perform Bayesian updating on the model to obtain a more accurate stiffness distribution of the damaged members (see Experiment 1 for details).

In this study, the unknown stiffness of the damaged member is regarded as a random variable, and the true value of the structural deformation is used to update the prior probability density function of the parameters to obtain the posterior probability density function [5]. The basic formula is as follows:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = cP(A|B)P(B) \tag{1}$$

Where B is the vector of parameters to be updated, i.e. the result vector obtained after Monte Carlo sampling; A is the observed data, i.e. the true displacement of the structure; P(B) is the prior distribution of the damaged member, which is often regarded as a generalized unbiased uniform distribution in the absence of historical experience; the likelihood function P(A|B) is the conditional probability of

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displacement A when B occurs; c is a normalization constant to ensure that the posterior probability density function is integrated to 1 [6]. P(B|A) is the posterior distribution probability of the stiffness of the damaged member that is desired to be obtained from the test results.

2.2. Monte Carlo sampling

Monte Carlo sampling is a computational method based on probability, statistical theory, and methods. It simulates the behavior of a random process or system through a large number of random samples to obtain the statistical characteristics or approximate solution of the process or system [7]. PMBOK uses Monte Carlo simulation as a technical means of quantitative risk analysis in project risk management. In engineering, Monte Carlo simulation is an effective risk analysis method, often used in engineering investment risk analysis and industrial system reliability and risk analysis [8]. With the advent of the information age, Monte Carlo sampling analysis of structural failure probability has gradually become the mainstream structural damage prediction method in engineering [9,10].

In order to demonstrate the efficiency and convenience of combining the Monte Carlo sampling method with the Bayesian updating method to calculate the failure probability of structures, the author established a statically determinate two-force beam model in the study, and the member area, material strength, and load on the structure (hereinafter referred to as the relevant parameters) all obeyed the corresponding distribution. The author used Excel to randomly sample the relevant parameters 10,000 times to calculate the failure probability of the model, and the results can be used to determine the damage probability of the structure (see Experiment 2 for details).

2.3. Introduction to the tool

SMSolver was developed by the Structural Mechanics Solver Development Group of the Department of Civil Engineering at Tsinghua University. It is a computer-aided analysis and calculation software (course) for teachers, students and engineers. The solver includes a series of problems involved in the classic structural mechanics courses, such as the geometric composition, statics, super statics, displacement, internal forces, influence lines, free vibration, elastic stability, and ultimate load of two-dimensional planar structures (systems). All of these problems are solved using accurate algorithms.

3. Experimental results and discussion

3.1. Test 1: Bayesian updating of a planar truss model

Use SM Solver to build the structural model: define the length unit in the model as meters (m), the unit of force as kilonewtons (KN), and the unit of member stiffness as kilonewtons multiplied by the square of meters (KN×m^2); define the nodes of the structure, take the length of each cross-member as 15m; connect all nodes to form individual member units, and combine them into the final structure as shown in Figure 1. In this model, the researcher defines the prior information as follows: the stiffness data of the known intact members (as shown in Table 1) and the support locations; the prior information of the stiffness of the damaged members (members 3-4, i.e., element 3) obeys a uniform distribution U (0.90, 1.98).

Table 1. Related Rod Stiffness Tables

1-2	2-3	4-5	1-6	6-7	7-8
1.8	1.71	1.8	1.98	1.89	1.89
8-5	2-6	3-6	3-7	4-8	
1.98	1.62	1.53	1.539	1.62	

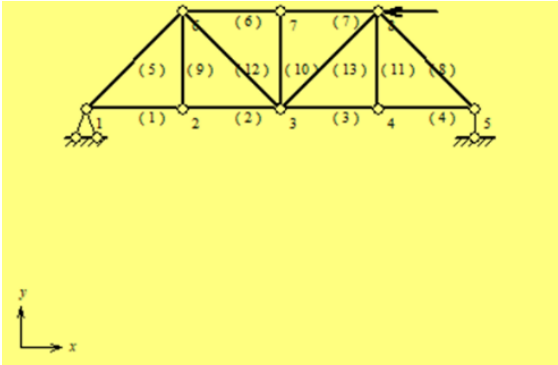

Figure 1. Diagram of the truss model

3.2. Test 1 process

After the truss model is built, the prior belief about the stiffness distribution of member 3 needs to be updated based on the actual deformation of the structure (in this experiment, the displacement at node 3, Δ, is used as the deformation parameter) when the structure is subjected to a load (the fact that occurs in Bayesian theory) The experimenter assumes that the upper limit of the single load applied is 100KN. The SMSolver was used to simulate the effects of various loading methods on the stress of each member of the structure and the overall stability of the structure. It was finally decided to apply a horizontal left lateral force of 100KN at node 8, as shown in Figure 2 (omitted here, does not affect the test results).

3.3. Calculate the deformation at node 3

SM Solver is used to simulate the displacement at node 3 when the load is applied according to uniform distribution under different stiffness values of element 3, as shown in Table 2. Table 2 shows partial results of uniformly distributed values. The tester gives the true displacement at node 3 as Δ_r=0.01287m, and the standard deviation of the test is σ=0.5.

Table 2. Values of element 3(partial)

EA	u -Horizontal displacement	v -Vertical displacement	θ-Corner
0.9	-0.01282895	0.00162577	-0.00042596
0.92	-0.01282895	0.00158048	-0.00042747
0.94	-0.01282895	0.00153712	-0.00042891
0.96	-0.01282895	0.00149556	-0.0004303
0.98	-0.01282895	0.0014557	-0.00043163
1.00	-0.01282895	0.00141744	-0.0004329
1.02	-0.01282895	0.00138067	-0.00043413
1.04	-0.01282895	0.00134532	-0.00043531
1.06	-0.01282895	0.0013113	-0.00043644
1.08	-0.01282895	0.00127855	-0.00043753
1.10	-0.01282895	0.00124698	-0.00043858
1.12	-0.01282895	0.00121654	-0.0004396
1.14	-0.01282895	0.00118717	-0.00044058

3.4. Calculation of displacement using the principle of virtual work

The axial force diagrams of the axial forces of each member when P = 1KN and when P = 100KN are shown in Table 3 and Table 4, respectively. The expression for calculating the displacement Δ at node 3 using the virtual work principle

$$\Delta_{ij}=\sum\int\frac{N\overline{N}}{EA}ds=\sum\frac{Nl\overline{N}}{EA} \tag{2}$$

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Table 3. Axial force diagram of each rod when P is 1KN

Rod end 1				Rod end 2		
Unitary code	Axial force	Shearing force	Bending moment	Axial force	Shearing force	Bending moment
1	-0.75000000	0	0	-0.75000000	0	0
2	-0.75000000	0	0	-0.75000000	0	0
3	-0.25000000	0	0	-0.25000000	0	0
4	-0.25000000	0	0	-0.25000000	0	0
5	-0.35355339	0	0	-0.35355339	0	0
6	-0.50000000	0	0	-0.50000000	0	0
7	-0.50000000	0	0	-0.50000000	0	0
8	0.35355339	0	0	0.35355339	0	0
9	0.00000000	0	0	0.00000000	0	0
10	0.00000000	0	0	0.00000000	0	0
11	-0.00000000	0	0	-0.00000000	0	0
12	0.35355339	0	0	0.35355339	0	0
13	-0.35355339	0	0	-0.35355339	0	0

Table 4. Axial force diagram of each rod when P is 100KN

Rod end 1				Rod end 2		
Unitary code	Axial force	Shearing force	Bending moment	Axial force	Shearing force	Bending moment
1	-75.0000000	0	0	-75.0000000	0	0
2	-75.0000000	0	0	-75.0000000	0	0
3	-25.0000000	0	0	-25.0000000	0	0
4	-25.0000000	0	0	-25.0000000	0	0
5	-35.3553390	0	0	-35.3553390	0	0
6	-50.0000000	0	0	-50.0000000	0	0
7	-50.0000000	0	0	-50.0000000	0	0
8	35.3553390	0	0	35.3553390	0	0
9	0.00000000	0	0	0.00000000	0	0
10	0.00000000	0	0	0.00000000	0	0
11	0.00000000	0	0	0.00000000	0	0
12	35.3553390	0	0	35.3553390	0	0
13	-35.3553390	0	0	-35.3553390	0	0

The above data is multiplied by the virtual work principle to obtain the nodal 3 processing theory displacement formula:

$$\Delta = 0.01214 + \frac{93.75}{EA} \tag{3}$$

3.5. Calculate the likelihood function and output the updated results

The researcher integrated the above-mentioned data in Excel and updated it using Bayesian principles.

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Table 5. Partial diagram of the update process

prior belief	Δ	Likelihood	Prior	likelihood*Prior	Post
0.9	0.0131817	0.6571716	0.925926	0.608492268	0.8328379
0.92	0.013159	0.6752955	0.925926	0.625273589	0.8558064
0.94	0.0131373	0.6917853	0.925926	0.640541954	0.876704
0.96	0.0131166	0.7067177	0.925926	0.654368269	0.895628
0.98	0.0130966	0.7201739	0.925926	0.666827712	0.9126811
1	0.0130775	0.7322375	0.925926	0.677997722	0.9279694
1.02	0.0130591	0.7429929	0.925926	0.687956397	0.9415998
1.04	0.0130414	0.7525237	0.925926	0.696781237	0.9536783
1.06	0.0130244	0.760912	0.925926	0.704548189	0.9643088

In Table 5, prior belief is the uniform distribution of the stiffness of element 3; Δ represents the theoretical displacement value of node 3 calculated by the principle of virtual work under the corresponding stiffness of element 3; likelihood is the likelihood function, that is P(A ∣ B), in the Bayesian formula, the difference between the theoretical displacement value and the true displacement value x=Δ-Δ_r, and the standard deviation σ=0.5 is substituted into the normal distribution error formula:

$$f_e(x) = \frac{1}{\sqrt{2\pi} * \sigma} e^{-\frac{1}{2}(\frac{x}{\sigma})^2} \tag{4}$$

The formula for calculating the likelihood function is:

$$L(Y|X) = \frac{1}{\sqrt{2\pi} \times 0.5} \times e^{\left(-\frac{1}{2}\times\left(\frac{12.14+\frac{93750}{X\times10^5}-12.87}{0.5}\right)^2\right)} \tag{5}$$

In the actual calculation, since the prior distribution is a uniform distribution ranging from 0.9 to 1.92, the probability of each value is:

$$P(X) = \frac{1}{1.92 - 1.91} \tag{6}$$

Normalize it and then calculate the posterior distribution.

Using the Bayesian formula, the above calculation results are used to calculate the posterior information under different stiffness distributions. The researchers plot the prior distribution and the updated results of the structure in the statistical charts shown in Figure 2 and Figure 3 , which can more intuitively show the effect of the Bayesian updating method on updating the structural damage information:

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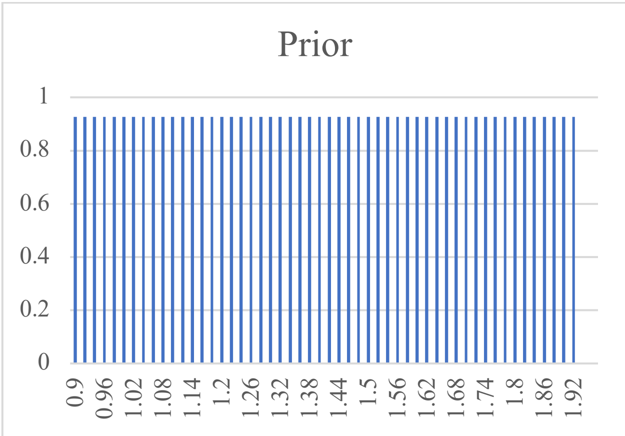

Figure 2. Prior probability distribution (uniform distribution) for static structural bars

Figure 2 shows the prior probability of a uniform distribution, which means that the probability of each possible value is equal when there is no observation data. The probability of each X value is:

$$P(X) = \frac{1}{1.92-0.91} \tag{7}$$

This means that the probability of each value is equal within the range of all possible initial stiffness values, and this uniform distribution assumption provides the basis for subsequent Bayesian updates.

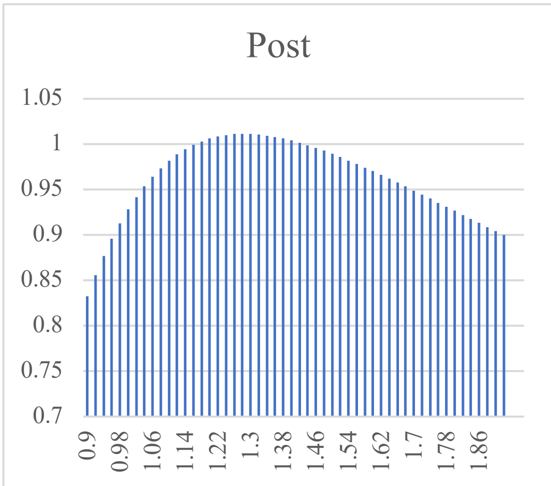

Figure 3. Posterior probability distributions for static structural bars

Figure 4 shows the posterior distribution (the likelihood function y = 0.5 mm has been calculated from the structural solver), which shows how the probability of each possible value changes under the influence of the observed data. The Bayesian update method dynamically adjusts the probability of each possible value by combining the prior distribution and the likelihood function of the observed data.

$$\text{Posterior} = \frac{P(X \mid Y)}{\sum P(X \mid Y)} \tag{8}$$

As can be seen from the figure, the posterior distribution has changed significantly from the prior distribution. In particular, the probability in the middle section (between 1.14 and 1.54) has increased significantly, while the probability at the two ends has decreased relatively. This shows that the initial stiffness value in the middle section is more likely to reflect the actual situation under the influence of

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the observed data. The Bayesian update makes the posterior distribution more reflective of the damage in the actual project by introducing the observed data into the calculation of the prior distribution.

In the figure on the right, the peak of the posterior probability density function appears around 1.34, indicating that the probability of an initial stiffness of 1.34 is highest after the observation data is incorporated. Through this redistribution of probability, the researchers can more accurately identify and assess possible damage in the truss structure.

The posterior distribution clearly shows the influence of the observation data on the probability distribution. The results show that the Bayesian updating method is highly effective and reliable in structural health monitoring. This not only verifies the theoretical feasibility of the method, but also provides a solid foundation for practical engineering applications.

4. Test 2: Monte Carlo sampling calculation of the failure probability of a statically determinate two-force beam

In order to explore the predictive effect of the Bayesian updating method on structural damage, this study established a static two-force beam model after clarifying the updating process in Experiment 1, and used the Monte Carlo sampling method to calculate its failure probability. To this end, the researchers set a series of conditions:

The load P (KN) conforms to the extreme value I-type distribution (α=0.3, u=20);
When the variables conform to the extreme value I-type distribution, the probability formula is satisfied:

$$F_X(x) = \exp\left[-e^{-\alpha(x-\mu)}\right] \tag{9}$$

$$f_X(x) = \alpha \exp\left[-\alpha(x-\mu) - e^{-\alpha(x-\mu)}\right] \tag{10}$$

The material strength (MPa) conforms to a lognormal distribution (λ=3.8, ξ=0.3);
When the variable conforms to a lognormal distribution, the probability formula is satisfied:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(lnx-\mu)^2}{2\sigma^2}} \tag{11}$$

The cross-sectional areas A1 (15 cm2 to 20 cm2) and A2 (10 cm2 to 15 cm2) are uniformly distributed.

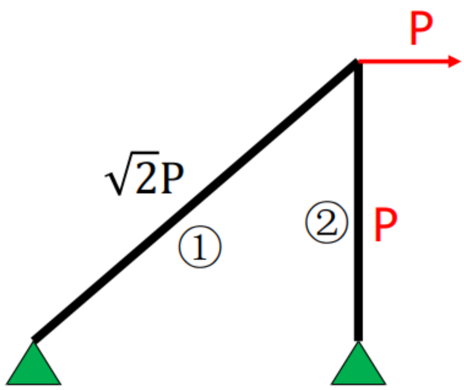

Figure 4. Static two-force rod model diagram

This model (Figure 4) has a strong randomness. The load, material strength, and cross-sectional area are randomly generated parameters based on known a priori information. As long as the stress on one member is greater than its material strength, the structure is considered to have failed. The approximate failure probability can be simulated by Monte Carlo sampling.

Using Excel, four columns of random numbers ranging from 0 to 1 are generated, with 11,000 rows each. According to the distribution types of load P, cross-sectional area A1 and A2, and material strength, the corresponding codes are written according to the formulas listed above, and applied to the four columns of random numbers to create 11,000 random values for the four indicators. It is determined whether the structure fails by comparing the relationship between the stress on the two members and the material strength. After 11,000 samples, the study gets the results shown in Table 6 and Table 7. Table 6 only selected a small portion of the 11000 data points, while Table 7 presents the calculation results of failure probability:

Table 6. Results of big data sampling (partial)

r1	P	r2	A1	r3	A2	r4		Fy	P/ √2/A1(Mpa)	P/A2(Mpa)	Fail1?	Fail2?
0.398	20.272	0.0926	15.463	0.245	11.224	0.148	-1.04	32.682	9.27	18.062	0	0
0.072	16.782	0.3647	16.823	0.962	14.81	0.026	-1.95	24.928	7.054	11.331	0	0
0.157	17.945	0.9772	19.886	0.302	11.51	0.94	1.553	71.223	6.381	15.59	0	0
0.2	18.41	0.1859	15.93	0.682	13.41	0.665	0.427	50.812	8.172	13.729	0	0
0.794	24.885	0.215	16.075	0.518	12.589	0.93	1.477	69.617	10.95	19.768	0	0
0.479	21.024	0.2237	16.119	0.572	12.862	0.447	-0.13	42.958	9.223	16.346	0	0
0.261	19.014	0.9532	19.766	0.843	14.214	0.022	-2.02	24.404	6.802	13.377	0	0
0.177	18.171	0.7023	18.511	0.881	14.403	0.938	1.535	70.84	6.941	12.616	0	0

Table 7. Results of big data sampling

total failure numbers	59
failure probability	0.005363636

According to the calculation results, we used the Monte Carlo sampling method to sample 11,000 times for this statically determinate two-force beam. The result was that the number of failures of member 1 was 1, the number of failures of member 2 was 58, and the total number of failures was 59. Therefore, the failure probability of this structure was 59/11,000, or 0.536%. This experiment shows that with certain prior information, the failure probability and damage location of the structure can be estimated using Monte Carlo sampling, thereby more accurately identifying and locating structural damage and greatly improving production efficiency.

5. Conclusion

This study explores the operational method of using Bayesian methods to update infrastructure information. During the study, SMsolver was used for modeling and Excel's probability statistics function was used to process the data. First, a truss model with possible damage was simulated, including the physical quantities of the material properties of the node elements and the stiffness of the elements, as well as the probability distribution of the stiffness of the damaged members. SM Solver then performs the loading simulation for the most unfavorable loads to explore the loading scenarios that are most likely to result in damage to the members. Then, the internal forces and displacements under the most unfavorable load are obtained using the virtual work principle, and the posterior probability of stiffness corresponding to all stiffness possibilities is calculated using the Bayesian theorem to complete the update of the damaged member information.

This study established a statically determinate two-force beam model and used the Monte Carlo sampling method to explore the effectiveness of the Bayesian updating method in structural damage prediction with known a priori information. The experimental results show that the structure's failure probability and damage location can be estimated more accurately by random sampling of the load, material strength and cross-sectional area. This result shows that the Monte Carlo sampling method can be effectively applied to the estimation of the failure probability of the structure, which has important application value in practical engineering. It can improve production efficiency and ensure the structure's safety and reliability through accurate damage identification and location. This approach provides an effective technical tool for structural health monitoring and maintenance.

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The research findings show that the Bayesian updating method can significantly improve the accuracy of structural damage assessment. Specifically, by dynamically correcting and improving the assessment results of the structural health status with the latest inspection data, the accuracy and reliability of the inspection can be improved, especially in dynamic systems. In addition, the Bayesian updating method can also be used to build risk assessment models to dynamically assess the risk level of structures and provide scientific decision support for managers. The study also shows that the use of Monte Carlo sampling in combination with the Bayesian updating method can more effectively calculate the probability of structural failure, providing a more efficient method for structural damage prediction.

In today's solid mechanics research, damage is mainly dealt with at the material point level, and in the constitutive law, it is formulated as a boundary value problem in continuum damage mechanics. On the other hand, damage to a specific structure is mainly observed at the structural level. Previously used detection methods for assessing the condition of structures or individual components have been widely used. In general, these methods are only used for qualitative state description rather than quantitative description. In summary, Bayesian updating combined with modeling software such as SMsolver can provide a solid theoretical and technical basis for developing the next generation of intelligent structural health monitoring systems, enabling real-time monitoring and dynamic assessment of infrastructure. By continuously adjusting and improving model parameters through Bayesian updating, the structural health status can be dynamically reflected, supporting scientific decision-making and providing a strong guarantee for the long-term safe operation of infrastructure. This research not only has important theoretical significance, but also provides practical solutions for engineering practice.

Authors Contribution

All the authors contributed equally and their names were listed in alphabetical order.

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