

Model Reference Adaptive Attitude Control of a Quadrotor Unmanned Aerial Vehicle

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Abstract: This study explores the integration of the adaptive control approach with the Model Reference Adaptive Control (MRAC) method, with a view to examining the impact of this integration on the performance of dynamic systems. Adaptive control is highlighted as a versatile and robust methodology that enables systems to cope with uncertainties and adapt to rapidly changing conditions in their operating environments. As a branch of adaptive control, the MRAC method focuses on efficiently achieving desired performance objectives by adjusting system parameters based on a predefined reference model. The primary focus is on investigating how these methods can be applied to optimize the functionality of systems characterized by dynamic and variable nature, such as unmanned aerial vehicles. By addressing challenges related to uncertainties, disturbances, and adaptability, the integration of adaptive control with the MRAC method demonstrates significant potential in improving the robustness, responsiveness, and reliability of such systems. This research investigates the performance of adaptive techniques in enhancing the capabilities of modern control systems. The objective of the research is to assess the effectiveness of MRAC in terms of reference signal tracking. Simulation results demonstrate the ability of MRAC to maintain accurate tracking of the desired reference for key vehicle dynamics, including roll, pitch and yaw. The results demonstrate the robustness of the MRAC algorithm in attitude control, which may lead to support for sustainability issues such as energy consumption.

Keywords: Unmanned Aerial Vehicles, Adaptive Control, Model Reference Adaptive Controller.

1. Introduction

A quadrotor unmanned aerial vehicle (QUAV) owes its high maneuverability, stability, and controllability to its four motors and propeller structure. QUAVs are widely used in various applications, including research, military operations, mapping, agriculture, and aerial photography [1-3]. In recent studies, various methods are being studied for the attitude control of QUAVs, including Proportional-Integral-Derivative (PID) control [4,5], sliding mode control [6,7], and model predictive control [8]. Research also exists where Linear Quadratic Regulator (LQR) is designed to ensure attitude stability using nonlinear attitude equations of QUAVs (e.g., [9]). Classical control methods, particularly PID control, are commonly used in combination with different methods like

fuzzy controllers [10] for attitude control. In fact, studies can be found where five different control methods are applied within the same work [11]. These studies are based on Lyapunov theory, others PID, LQR, backstepping and sliding mode control. However, these methods often assume that the parameters in the system model remain constant. Classical control algorithms like PID struggle to provide ideal control under external disturbances or changing environments, making it challenging to adapt to balanced attitudes.

Adaptive control algorithms offer significant advantages due to their robust self-adaptability and capability to suppress slowly time-varying disturbances and uncertainties in the system [12]. The foundation of adaptive control was laid by Whitaker in which the error signal was utilized to adjust the controller parameters. This is recognized as one of the earliest studies on Model Reference Adaptive Control (MRAC) systems [13,14]. The contribution of Kalman filter to adaptive control systems is in providing a theoretical basis for the development of MRAC methods. The Kalman filter is crucial in MRAC approaches as it enables accurate prediction of changes in system dynamics and parameters over time. This feature plays a critical role in real-time monitoring and adaptation of system parameters [15].

Adaptive control approaches, particularly MRAC, offer an ideal solution for enhancing the robustness, precision, and performance of UAVs. Adaptive control algorithms can make QUAVs more reliable and efficient by improving their maneuverability and resilience against external influences [16]. These approaches allow systems to adapt to changing conditions, helping them cope with uncertainties effectively. Adaptive control aims to enable systems to handle uncertainties and adapt to time-varying conditions.

In this paper, MRAC control is used to control the attitude of the QUAV. The outline of the paper is as follows: the modelling of quadrotor is presented in Section 2, control methodology of quadrotor is given in Section 3, simulation and results are presented in Section 4, and the conclusion is given in Section 5.

2. Modeling of QUAV

In this section, the kinematic and dynamic model of the quadrotor is detailed. The “X” frame is used for the quadrotor motors in this paper. Through the force analysis of the X-type quadrotor, it should be known that it is mainly affected by gravity mg , the thrust force of the four propellers F_i , and the air resistance moment τ_i of the air to the four propellers. ω_i is the rotational speed of the i th propeller and τ_i is the drag torque produced by the i th propeller. The thrust F_i and the drag torque τ_i can be expressed by (1) and (2) respectively as

$$F_i = C_T \omega_i^2 \quad (1)$$

$$\tau_i = \pm C_D \omega_i^2 \quad (2)$$

where C_T is the thrust coefficient, which depends on the rotor blade diameter, air density, flight speed, and other parameters, and C_D is the drag coefficient, which depends on the air density, propeller size, and other aerodynamic parameters. The four control inputs of the four rotors are given below; F is the total drag force produced by the propeller. Here, τ_ϕ is the rolling torque, τ_θ is the pitch torque, and τ_ψ is the yaw torque. l is the distance between the center of the rotor rotation axes and the center of the quadrotor.

The nonlinear model of the quadrotor is described by (3). Through this formula, the resulting thrust and torque can be matched to the thrust of each propeller.

$$\begin{cases} F = C_T(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ \tau_\phi = \sqrt{2}C_T l(\omega_1^2 - \omega_2^2 - \omega_3^2 + \omega_4^2)/2 \\ \tau_\theta = \sqrt{2}C_T l(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2)/2 \\ \tau_\psi = C_D(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{cases} \quad (3)$$

To simplify the model, the QUAUV is assumed to be a rigid body and the dynamic characteristics of the drive motor are not taken into account. According to the kinematics and dynamics information, the dynamic model formula of the multirotor UAV (4) is,

$$\begin{cases} \dot{p} = v \\ \dot{v} = g e_3 - \frac{F}{m} R_b^w e_3 \\ \dot{R}_b^w = R_b^w \hat{\Omega} \\ J \dot{\Omega} = -\Omega x(J\Omega) + G_a + \tau \\ G_a = \sum_{i=1}^4 J_a(\Omega x e_3)(-1)^{i+1} \omega_i \end{cases} \quad (4)$$

where p represents the position and linear velocity of the vehicle expressed in the inertial frame $v \in R^3$. g is the standard acceleration due to gravity. $J \in R^{3 \times 3}$ represents the moment of inertia. G_a is the gyroscopic torque. J_a is the total moment of inertia of the entire engine rotor and propeller about the axis of rotation. Ω is the angular velocity of rotation of the object. $\tau \in R^3$ is the torque produced by the propeller on the fuselage shaft. $\hat{\Omega}$ is the skew symmetry matrix function of the vector. Then, use (x, y, z) to represent the linear displacement of the QUAUV in the world coordinate system, the angular velocity (p, q, r) with respect to the body frame, and the Euler (ϕ, θ, ψ) ratios in the world frame. The nonlinear mathematical model of the QUAUV according to Newton's second law and the Euler equation (5) is,

$$\begin{cases} \ddot{x} = -F(\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi)/m \\ \ddot{y} = F(\cos\psi \sin\phi - \sin\psi \sin\theta \cos\phi)/m \\ \ddot{z} = -F(\cos\theta \cos\phi)/m + g \\ \dot{\phi} = p + q \sin\phi \tan\theta + r \cos\phi \tan\theta \\ \dot{\theta} = q \cos\phi - r \sin\phi \\ \dot{\psi} = q \sin\phi / \cos\theta + r \cos\phi / \cos\theta \\ \ddot{\phi} = \tau_\phi / I_{xx} + \dot{\theta} \dot{\psi} (I_{yy} - I_{zz}) / I_{xx} \\ \ddot{\theta} = \tau_\theta / I_{yy} + \dot{\phi} \dot{\psi} (I_{zz} - I_{xx}) / I_{yy} \\ \ddot{\psi} = \tau_\psi / I_{zz} + \dot{\theta} \dot{\phi} (I_{xx} - I_{yy}) / I_{zz} \end{cases} \quad (5)$$

where I_{xx}, I_{yy}, I_{zz} , are the moments of inertia about the x, y and z axes, respectively. The QUAUV has four degrees of freedom input $(F, \tau_\phi, \tau_\theta, \tau_\psi)$ with thrust force F perpendicular to the body plane and three moments that create roll, pitch and yaw. What needs to be controlled is the UAV's six degrees of freedom (x, y, z) in displacement and (ϕ, θ, ψ) in rotation, so it is an underactuated system.

3. Control Methodology of QUAUV

QUAVs possess a complex control structure, particularly in terms of stability and maneuverability. These vehicles are challenging systems to control due to their nonlinear dynamics and the intensity

of external disturbances. In this study, the attitude control of a quadrotor UAV is aimed to be achieved using the MRAC method. MRAC is a control method designed to optimize system performance based on a reference model. By employing MRAC in the attitude control of the UAV, the roll, pitch, and yaw angles of the system are matched with the target reference values. In this method, the adaptive mechanism detects parameter changes and adjusts the control signals in real time. This guarantees the system achieves the desired stability and performance.

The reference model defines the desired behavior of the UAV, while the required values specify the concrete targets of the system. The actual values are compared with the reference to evaluate how close the system is to these targets, and the necessary adjustments are made accordingly. In the MRAC system depicted in Figure 1, the reference signal (r) represents the desired input signal, defining the target behavior the system should achieve. The adaptive parameters (a) are parameters utilized by the adaptive mechanism to adjust the controller, which are influenced by the reference model and system feedback. The control signal (u) is the input signal applied by the controller to the system to influence its behavior. The reference output (y_m) represents the output signal of the reference model, indicating the ideal response the system should follow. Finally, the system output (y) is the actual output signal of the system, showing how closely the system tracks the desired behavior. The primary goal of the control system is to minimize the error between y and y_m .

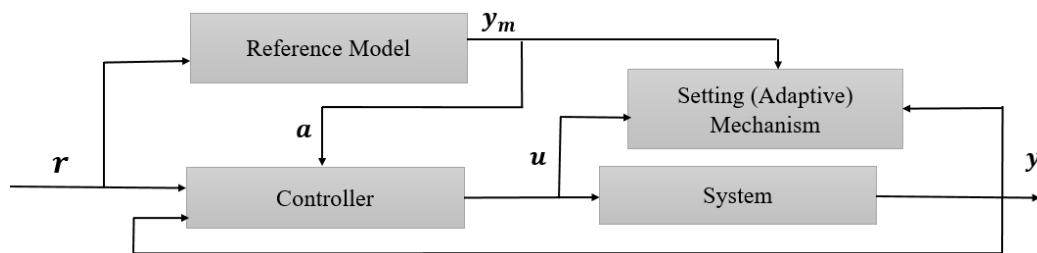


Figure 1: MRAC methodology diagram

4. Simulation and Results

In order to investigate the performance of MRAC in quadrotor UAV attitude control, it is applied in the control of a mini quadcopter model in the simulation environment. The parameters for the QUAV are $m = 0.8$ kg, $I_{xx} = I_{yy} = 0.009$, $I_{zz} = 0.015$, $C_T = 1$, $C_D = 0.08$, $l = 0.06$ m. Figure 2 depicts the QUAV's motion in three dimensions, where the red line represents the actual position of the QUAV (real position), and the blue line denotes the desired trajectory (desired position). The takeoff and landing points are clearly marked, providing a comprehensive visualization of the UAV's flight path. The primary objective of the MRAC algorithm is to minimize the deviation between the actual and desired positions, ensuring accurate trajectory tracking. The results demonstrate the algorithm's effectiveness, as the UAV closely follows the desired trajectory with only minor deviations during the transient phase, such as during takeoff and landing. These deviations are quickly mitigated, highlighting the system's rapid convergence capabilities. Throughout the flight, the UAV maintains a high level of precision in tracking the target trajectory, showcasing the adaptability of the MRAC algorithm to dynamic changes and disturbances. Overall, this figure underscores the effectiveness of MRAC in achieving precise and adaptive control in QUAVs. It validates the algorithm's capability to handle complex flight paths while ensuring stability and accuracy, making it a robust solution for position and attitude control tasks.

The performance of the Model Reference Adaptive Control (MRAC) algorithm in controlling the yaw, pitch and roll axes of a QUAV is illustrated in Figure 3. The pitch angle demonstrates the highest accuracy and stability, with the actual and reference signals almost perfectly tracking. It exhibits rapid adaptation to external disturbances, delivering exceptional performance. The yaw angle maintains

high tracking accuracy overall but shows a slight deviation during the disturbance around the 50th sec. The roll angle, despite initial oscillations and minor deviations during disturbances, quickly converges and aligns closely with the reference signal. Overall, the MRAC algorithm performs effectively across all axes, with the pitch axis emerging as the most accurate and consistent. These results highlight both the strengths of the algorithm.

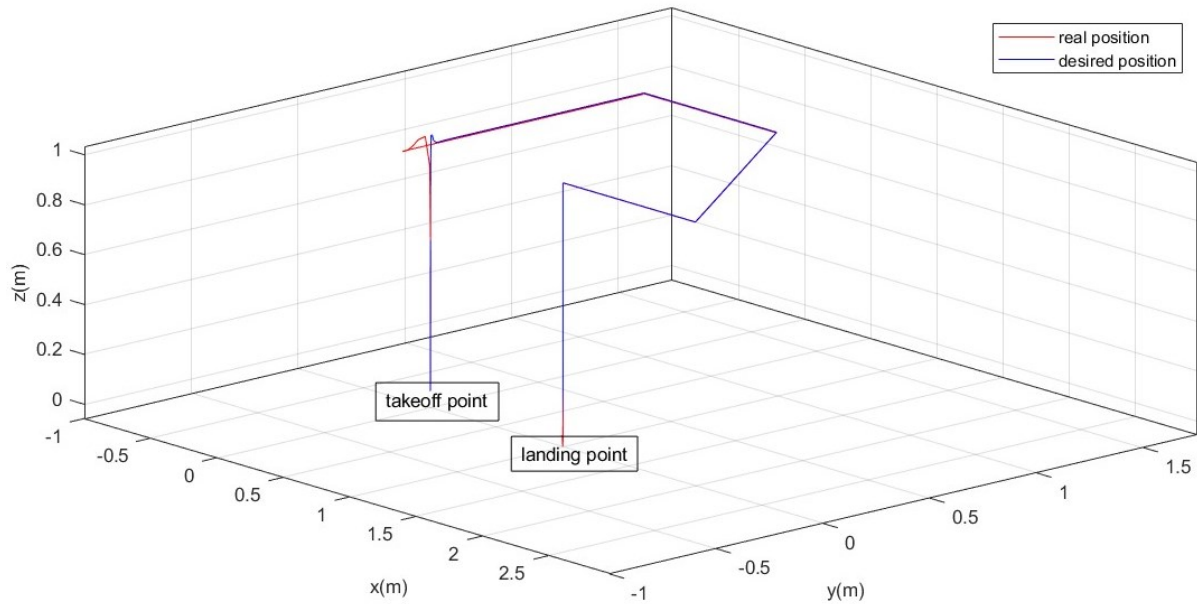


Figure 2: The tracked 3D trajectory

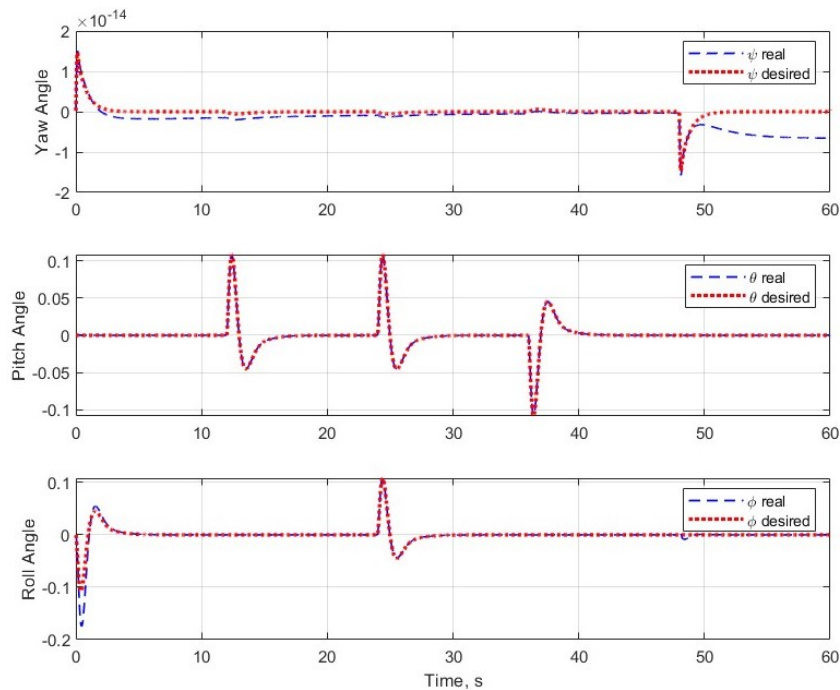


Figure 3: Performance of the MRAC for attitude control.

5. Conclusion

In conclusion, the performance evaluation of the MRAC algorithm based on the analyzed graphs demonstrates its effectiveness in providing robust and accurate control for a QUAV. The pitch angle exhibited the most consistent and precise tracking, with minimal deviations and rapid recovery from external disturbances, marking it as the best-performing axis. The yaw and roll angle also showed strong performance, effectively stabilizing after initial oscillations and disturbances, albeit with minor deviations compared to the pitch axis. These results confirm the algorithm's capability to maintain reliable control across multiple axes, while also highlighting areas for potential optimization, such as further enhancing the disturbance rejection and adaptation mechanisms in the yaw and roll axes to match the exemplary performance of the pitch axis. Overall, the MRAC algorithm proves to be a robust and adaptable control method suitable for complex dynamic systems.

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