

Numerical Modeling of Heat Pollution: An Advection-Diffusion Problem Analysis

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Abstract: Thermal pollution refers to the heating of water bodies caused by the release of factorial wastes, deforestation, solar radiation etc. This study presents a numerical model, specifically addressing the heat advection-diffusion equation, to analyze thermal pollution caused by discharges of industrial factories in river systems. The aim of this project is to present the pollution caused by factorial discharges visually as thermal pollution not only causes destabilization of aquatic ecosystems but may also negatively influence industries such as fisheries, tourism, and other water-based activities in the long term. Therefore, our study introduces and explores six distinct scenarios where this model can be applied. Our findings provide valuable insights into the dynamics of thermal pollution and its impacts on heating up river systems.

Keywords: Numerical model, Heat pollution, Advection and Diffusion

1. Introduction

Water is the driving force of nature, and all living things are heavily dependent on it. Therefore, unpolluted water sources have become particularly essential to all living organisms. The heat pollution, however, has had a harmful impact on the water resources for an extended amount of time. Because of water's high specific capacity (4.18J/g°C), water absorbs large amounts of heat but releases them slowly, and much energy is required to deal with this issue[1]. Nowadays, nearly 80% of the water used by industries worldwide is used for cooling[2].

Water temperature is the influencing factor of water quality, photosynthesis, dissolved oxygen, animals' respiration, and growth rate, and can vary geographically and longitudinally depending on the geomorphic features[3]. Measuring water temperature can help scientists determine the living conditions of aquatic living creatures and the health of the whole ecosystem, but it has had many obstacles due to the heat pollution issue[1]. Moreover, the average water temperature worldwide is increasing at the speed of 2-5°C annually[4]. The solubility of metal and toxicity will increase because of the rising temperature, which causes more species to die out[5-6].

Thermal pollution may have extreme impacts both in the short and long term on the ecosystem. In the short term, it can potentially lead to reduced sexual maturation, inhibition of critical stages of larval development, reduced feeding, reduced growth, increased susceptibility to predation, increased incidence of disease and parasitism, increased migration, and increased organism death[1]. These effects further layer up, eventually causing devastating long-term effects, including the loss of aquatic species whose survival and reproduction are temperature dependent, changes in the abundance and distribution of aquatic species, increased rate of surface water evaporation, and increased salinity and shrinkage of waterbodies and habitat loss[1]. On top of that, the variance in temperature can cause instabilities and disruptions in aquatic organism's life cycles. Those creatures base their migration patterns, habitat, and time to spawn in accordance with temperature cues. For example, Shellfish is a temperature-sensitive species that spawns within hours after reaching the threshold temperature[7]. In addition, trout can hatch their eggs within 165 days at 3°C, within 32 days at 12°C, or do not hatch when temperature is above 15°C[8]. Thus, it can be concluded that maintaining a stable water temperature is essential for protecting and preserving the health of aquatic life.

Previous researchers have been involved in several dimensions of dictating thermal activities in water areas. Issakhov, A. and Zhandaulet, Y. use two-dimensional formulation to simulate the influence of the heat source at different locations along the river [9], focusing particularly on water zones affected by power plants. They use shallow water equations to calculate the index in those zones. Alibek Issakhov, Yeldos Zhandaulet. C E Raptis and colleagues make a Global heat emission dataset and use salmon habitat as an indicator of water temperature [10]. In this way, they visualize the situation in a variety of places which becomes easier to monitor. In 2015, Marc A. Rosen and colleagues used wastewater thermal pollutant vector Plane Projection to monitor several power plants' wastewater outlet channels and present a simulation test [11], which provided valuable insights into the spatial distribution and intensity of thermal pollution in water bodies near industrial facilities. An Qiang and his researchers at the Institute of Hydrobiology of the Chinese Academy of Sciences (CAS) use gene expression models and system dynamics models to help researchers understand and explore the progress and direction of pollution, as well as the processes of change in aquatic ecosystems.[12] Their findings suggest that heat pollution may lead to ecological damage, water quality deterioration and instability of the whole ecosystem. Monika Barbara Kalinowska utilized RivMix models to emphasize the important role of water-air heat exchange in dealing with thermal pollution and, further, to help people simulate the impact of water-air heat exchange on water temperature.[13]

2. Methodology

Our study includes three variables, where T is the temperature, t is the amount of time steps, and x is the position. We start by defining parameters such as the length of the river (L), thermal diffusivity (α), water flow velocity (v), change in position (dx), change in time (dt), and the number of position and time points (nx and nt). On top of that, initial conditions are set with a uniform temperature distribution and boundary conditions at time=0. The main loop repeats over time and space, updating the temperature using our advection-diffusion model, with the diffusion term:

$$T_i^n + \alpha \frac{\Delta t}{(\Delta x)^2} (T_{i+1}^n - 2T_i^n + T_{i-1}^n)$$

and the advection term:

$$-v \frac{\Delta t}{\Delta x} (T_i^{n+1} - T_i^n)$$

which model heat conduction and heat transport by water flow respectively. Boundary conditions are considered separately, and some cases include additional features like heat sources or cold sources. Finally, the model visualizes the temperature distribution along the river and its effects over time

through plotting temperature with respect to space and time. This structure allows for detailed presentations of the spread of heat in a river, accounting for both diffusion and advection processes.

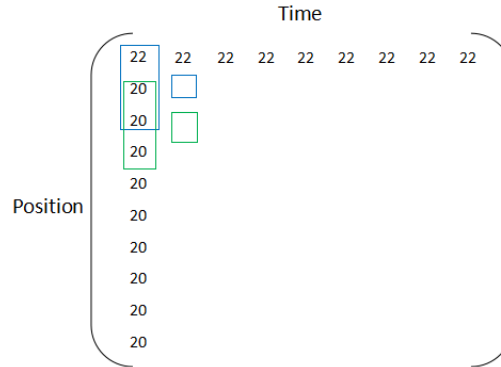


Figure 1: Time-position matrix for modeling the spread of heat

Our numerical model is based on the matrix in the figure 1. Originally, the first column of the river is set to 16 temperature to 22°C at the first position and 20°C at all other positions. Then, in the coming unit of time, the temperature at the first position is still 22°C due to the heat source being continuously turned on; the second position's temperature is now determined by the temperature in the formal unit of time using the temperature of the 3 positions circled by the blue box in the first column. Similarly, the third position's temperature is calculated by considering the temperature of the 3 positions circled by the green box in the first column. Through repeating this process, the whole matrix is obtained, further using it to plot our graphs.

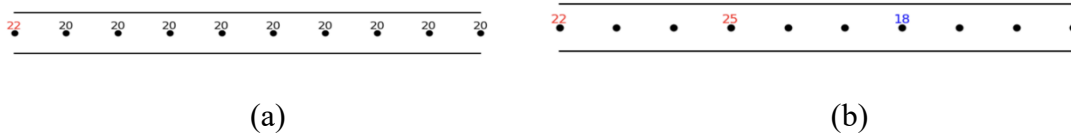


Figure 2: The initial condition of (a) one heat source and (b) additional heat and cold source

Figure 2(a) and 2(b) are the two original settings of our matrix, the first one represents only one heat source, and the second one represents two heat sources and one cold source. Both graphs are the thwartwise version of the first column in Figure 1.

2.1. Derivation of Model

1. Time Derivative:

$$\frac{\partial T}{\partial t} \approx \frac{T_i^{n+1} - T_i^n}{\Delta t}$$

2. Second Spatial Derivative (Diffusion Term):

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2}$$

3. First Spatial Derivative (Advection Term):

$$\frac{\partial T}{\partial x} \approx \frac{T_{i+1}^n - T_i^n}{\Delta x}$$

The analytical equation of advection and diffusion:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - v \frac{\partial T}{\partial x} [14]$$

Substitute these infinite difference approximations into the analytical equation:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2} - v \frac{T_i^{n+1} - T_i^n}{\Delta x}$$

Rearrange to solve for T_i^{n+1} , getting the numerical equation:

$$T_i^{n+1} = T_i^n + \alpha \frac{\Delta t}{(\Delta x)^2} (T_{i+1}^n - 2T_i^n + T_{i-1}^n) - v \frac{\Delta t}{\Delta x} (T_i^{n+1} - T_i^n) [15]$$

For better calculations, constant values are set for our parameters:

- α (thermal diffusivity) = 10km²/hr
- v (velocity of water flow) = 10 km/hr
- dt (change in time) = 0.01 hr
- dx (change in position) = 1 km

3. Results

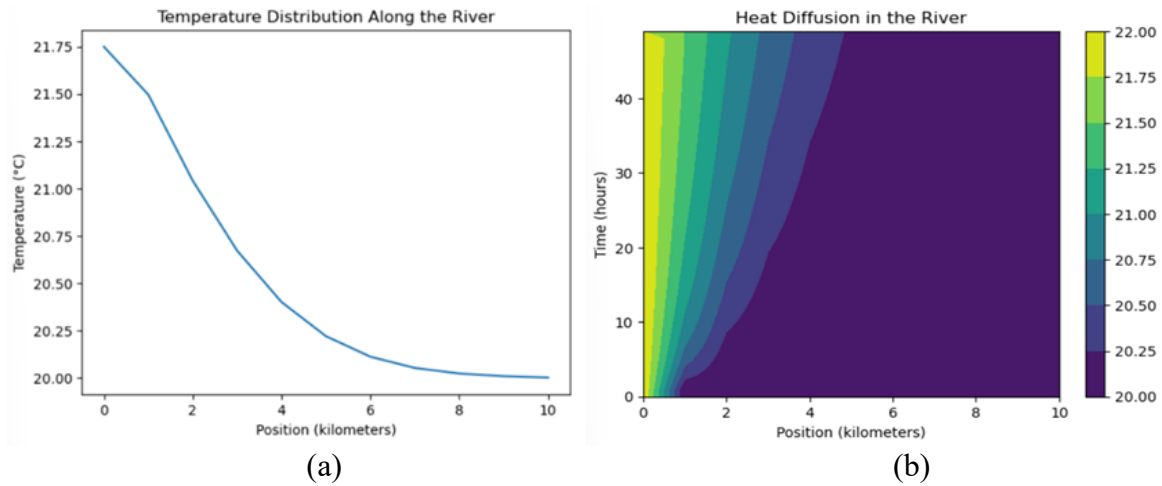


Figure 3: Diffusion-only model

First, we set the source of heat at the initial position with a temperature of 22°C. In this situation, no advection is being added and the result present is only a presentation of the diffusion of heat in water. Figure 3(a) is a temperature versus position graph, whereas temperature varies from 20°C to 22°C and position varies from 0 to 10 kilometers. In this graph, the curve that represents temperature is decreasing at a decreasing rate, indicating that the heat source is increasingly losing its influence the further away from the original heat source and that the diffusion of heat is nonlinear. Even without any advection factors, hotter water molecules lose energy in the process of collision while the cooler water molecules absorb the energy, thus leading to the diffusion of heat among their surrounding water bodies. In Figure 3(b), time is added as a third variable. Initially, at time 0, the area that heat influences is limited. Over time, heat spreads out both spatially and temporarily.

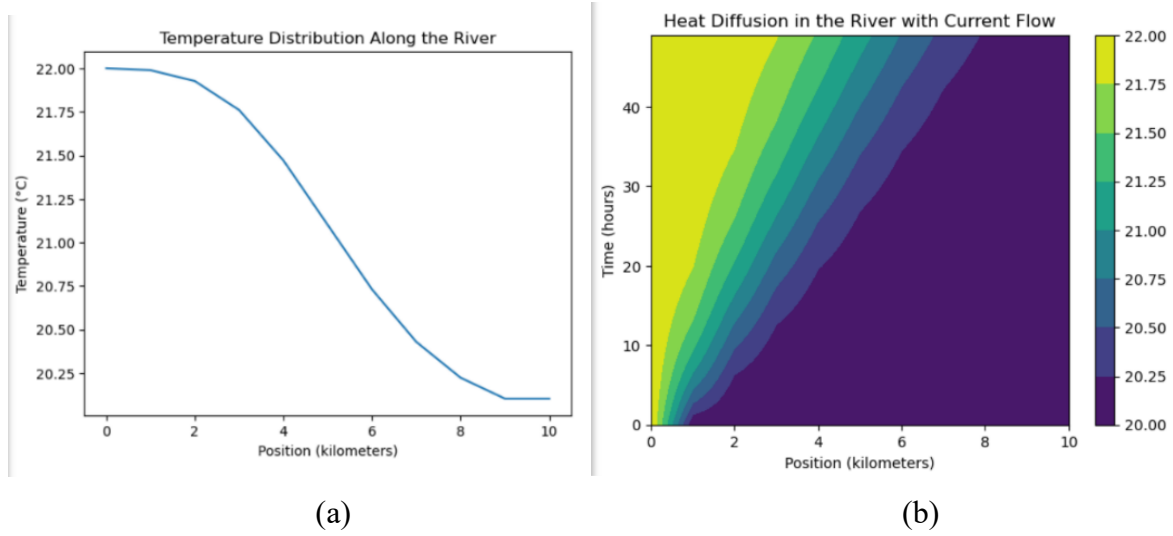


Figure 4: advection-only model

Figures 4(a) and 4(b) depict the effect of a 22°C heat source on the river, including only the advection factor. The first plot shows a trend in decreasing temperature similarly from the left end (22°C) to the right end (20°C). Since only advection is considered in this case, the distribution rate of heat increases because of the additional factor of water flow. This scenario highlights the absence of heat diffusivity, made possible by setting the constant coefficient α to zero. As presented in Figure 4(a), although the temperature curve is constantly decreasing, the curve is made up of two different parts. Initially at position range (0, 5) approximately, the curve is decreasing at an increasing rate (concave down) as water flow carries away the concentrated heat released in the river. But after five kilometers, the curve now decreases at a decreasing rate as heat becomes increasingly dispersed, and thus, its temperature will approach those of its surroundings.

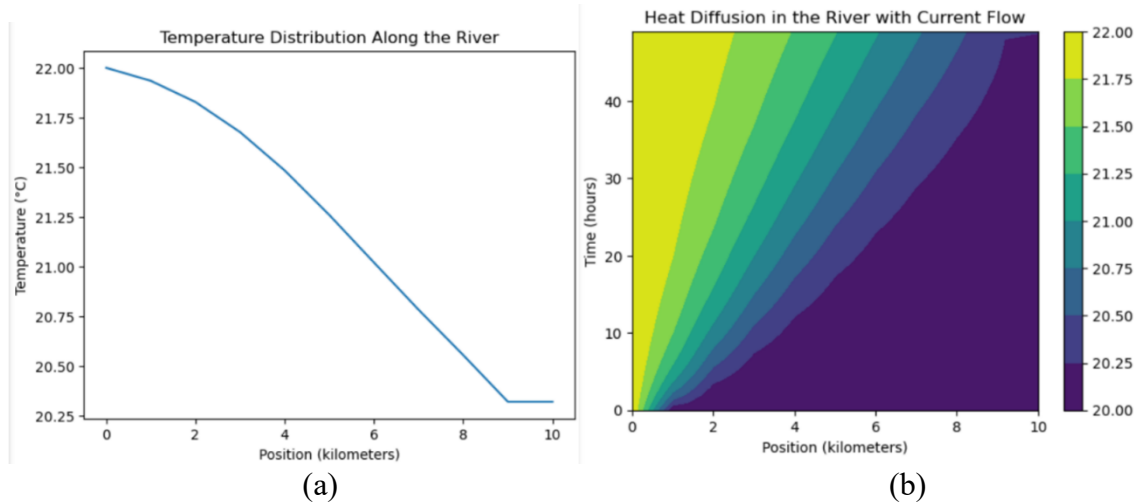


Figure 5: diffusion-advection model

In this case, our model considers both diffusion and advection, which matches more to real life than the first two cases, as neither of these factors is negligible. The initial condition remains unchanged. The energy stored in the river system decreases downstream. Since this case is a combination of the first two scenarios, the curve adds up the effects of the previous two, presenting a smoothed curve in Figure 5(a) and a wider spread of heat in Figure 5(b) compared to the previous

scenario. This distribution considers more factors that may influence the overall result of the spread of heat, thus taking a closer approach to the actual condition.

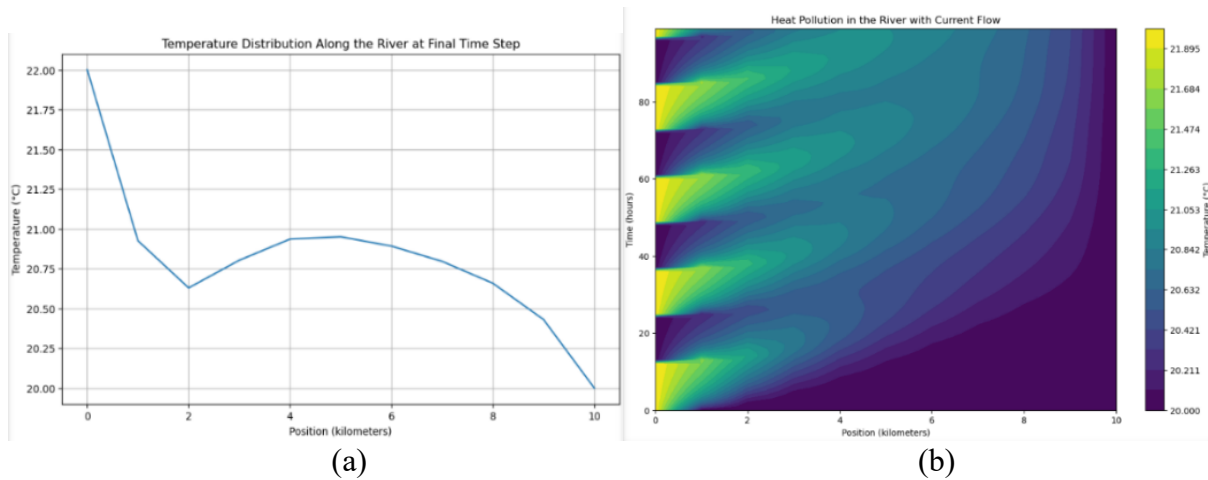


Figure 6: Diffusion-advection model with intermittent heat source

Figures 6(a) and 6(b) simulate the influence on the river caused by a 22°C heat source shifting its state every 12 hours. The graph shows the effect of both diffusion and advection. The temperature doesn't remain constant at 22°C downstream due to the cooling effect of the river flow and the condition of the state-shifting heat source. The transition from a downward to an upward temperature trend at two kilometers from the origin can be attributed to the influence of a heat wave originating from the formal warm current. The general heating progress is significantly slower than the formal cases.

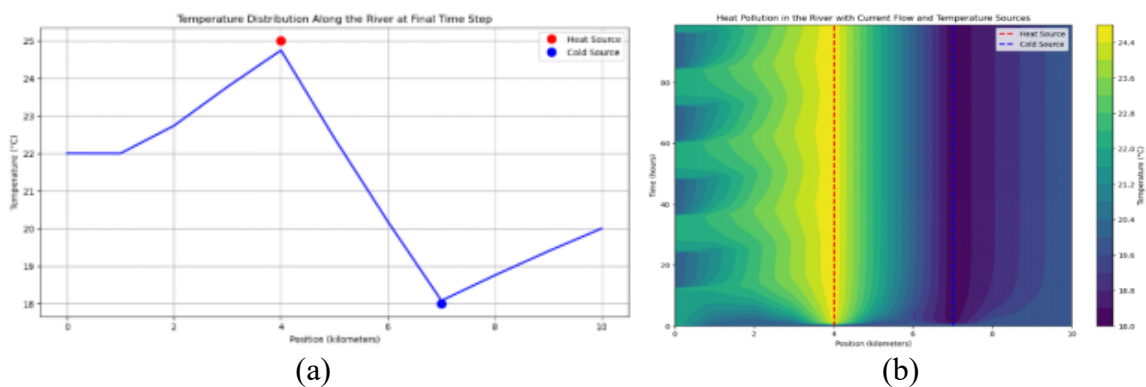


Figure 7: Diffusion-advection model with additional heat and cold source

Figures 7(a) and 7(b) show the extra heat source and cold source's influence on the river temperature. The heat source is located at 4km with the temperature of 25°C, and the cold source is located at 7km with the temperature of 18°C. The temperature state of the 22°C heat source changes every 12 hours and only diffuses to the right direction, while the 25°C heat source and the 18°C source diffuse in both two directions (left and right), leading to the maximum temperature at 4km and the minimum temperature at 7km, as shown in the figure.

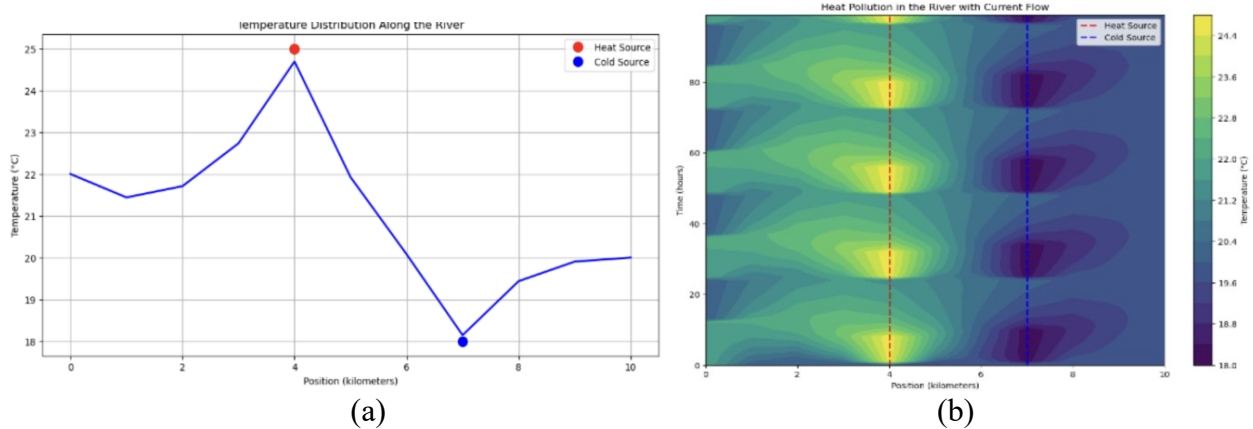


Figure 8: Diffusion-advection model with additional intermittent heat and cold source

Figures 8(a) and 8(b) represent the impact on the river system from periodic heat sources and cold sources. As shown, the heat source is located at 4km with a temperature of 25° C, while the cold source is located at 7km, with a temperature of 18° C. Factories do not operate all day long, so a cycle of 24 hours is set, which is equal to one day. The heat source is turned on for 8 hours a day and then on for the remainder of the cycle. The cold source is turned on for 10 hours a day and then also turned off for the rest of the cycle. The influence of heat and cold sources gradually spread to the upstream and downstream of the river, as shown in Figure 8(b).

4. Conclusion

Based on the calculation, the position, the opening time, and the number of the heat source all significantly influence the heating progress of the river, which are the aspects governments should monitor if they want to deal with river thermal pollution.

This paper mainly focuses on numerical modeling and presenting results in the form of graphs. However, there are more factors that can be added into consideration to improve the precision of our results. Further improvements can be made by using real parameters of the river that can be obtained from the database, which can make the calculation of T_i^{n+1} closer to reality.

Furthermore, other environmental interference cannot be ignored in real cases. Our paper only considers changes in temperature in the river systems caused by artificial actions, such as chemical wastewater released by factories in water bodies. However, there are an immense number of factors that can further influence the result of temperature distribution. For example, deforestation, urbanization, as well as natural factors like solar radiation and climate change can all significantly influence our results. Thus, our model can be further complicated and improved with consideration of the factors listed above.

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