

Review on Consensus Control of Multi-Agent Systems

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Abstract: The Consensus Control of Multi-Agent Systems is an essential and vibrant field of study. In recent times, spurred by the exponential growth of information technology and network communication technology, Multi-Agent Systems (MASs) have ascended to a prominent position as a major research direction across modern automation, artificial intelligence, and distributed control domains. This paper's theoretical research segment is dedicated to delving into the fundamental theoretical tenets underlying multi-agent consensus. It will meticulously comb through and systematize the extant literature and models, thereby crystallizing the research accomplishments in the realm of multi-agent consensus control. This comprehensive analysis will not only elucidate the complex theoretical landscape but also identify the gaps and opportunities for further innovation. In conclusion, the consensus control of MASs is a fertile ground for research, fraught with both daunting challenges and exciting prospects. My intention is to furnish readers with a profound and nuanced comprehension of the current research trajectories. As a result, I aim to catalyze the more profound and extensive deployment of MASs in diverse societal arenas and spotlight the crucial role and prospective beneficial effects of multi-agent consensus control in the evolution of modern society and the days to come.

Keywords: MASs, Consensus control, Distributed control

1. Introduction

With the rapid development of information technology and network communication, MASs has gradually become a significant research direction in modern automation, artificial intelligence, and distributed control. The MASs consists of multiple agents that are typically autonomous, interactive, and required to accomplish specific tasks in uncertain environments. The consensus control problem aims to ensures all agents can reach an agreement on a particular state (such as position or velocity), and it finds wide applications in fields such as UAV formations, robotic team collaboration, and intelligent transportation systems[1].

The research on consensus control can be traced back to the 1990s, with the primary goal of ensuring that multiple agents maintain a certain level of coordination in complex environments. In contemporary research, attention is no longer solely fixed on traditional centralized control methods. Instead, there is a growing emphasis on distributed control strategies, which are designed to make use of local information for decision-making without the need for central coordination. This approach enhances the robustness and flexibility of the system[2]. For example, Olfati-Saber et al. proposed a control strategy based on adjacency graphs, such that all the agents update their states through

information from neighboring agents, and reach an agreement[3]. And Chang, Z., & Other Authors. put forward noise-free and noisy data-driven consensus protocols. Through the state and input data collected locally, local control gains are calculated to achieve leader-follower consensus[4].

Currently, the research hotspots of consensus control for MASs mainly focus on the following aspects:

Robustness and Adaptability: In practical applications, agents may face various uncertainties and disturbances, prompting researchers to focus on control strategies that exhibit robustness and adaptability. For instance, Liu et al. introduced an improved control method that enhances the system's resistance to external disturbances through the incorporation of anti-interference mechanisms[5]. Nowadays, in order to address the challenges faced in the distributed adaptive consensus control problem of nonlinear MASs with unknown parameters and communication delays, Zhang M ,Wang G ,Wang C put forward a new adaptive state feedback control protocol based on the backstepping method by utilizing the reference dynamic system[6].

Dynamic Topology: In the realm of MASs, a fixed topology means that the connections among agents remain unchanged throughout the operation. For instance, Cao,etc. found that in the formation flight of UAV where the relative positions and communication links among them are predefined and remain static[7], it can be regarded as a fixed topology. However, in certain application scenarios, like in a mobile sensor network where agents are likely to move around and may encounter obstacles or changes in the environment, the connectivity among agents may vary over time. This then brings about the problem of consensus control under dynamic topologies, making the study of consensus control under dynamic topologies particularly important. Deng and Yuan conducted an in-depth exploration of this issue, proposing an adaptive control scheme that ensures the system maintains consensus even in dynamically changing network environments[8].

Delay and Time-Varying Systems: In practical applications, information transmission delays are unavoidable, and researchers have explored this extensively to find suitable consensus control solutions under time-varying network conditions. Zhang et al. investigated MASs with transmission delays and proposed a synchronization control method based on gain adjustment[9]. In response to Time-Varying Systems, Li, S., and Wang, Q. put forward a novel adaptive control algorithm, which can estimate and compensate for the time-varying uncertainties in the system in real time. By introducing the adaptive law, agents can dynamically adjust the control parameters according to their own and their neighbors' state information to adapt to the time-varying characteristics of the system[10]. Compared with traditional control algorithms, this adaptive control algorithm has better robustness and adaptability and can ensure that the multi-agent system asymptotically achieves consensus in the presence of time-varying uncertainties.

Nonlinear Systems: Numerous practical systems display nonlinear characteristics, which has thus made the consensus control of nonlinear MASs a research focus. Studies have shown that nonlinear control methods can effectively enhance the performance of the system. In 2020, Vázquez et al. proposed a nonlinear control strategy that successfully achieved state synchronization among multiple agents[11].

In summary, consensus control in MASs is a research field full of challenges and opportunities. The theoretical research section of this paper will explore the theoretical foundations relevant to multi-agent consensus, reviewing existing literature and models to establish a framework for the research findings. The subsequent section addressing existing problems will detail the specific issues currently faced in multi-agent control, using charts from existing studies to visualize these problems and enhance the persuasiveness of the arguments. Following this, the problem-solving section will propose solutions to these issues, combining theoretical analysis with simulation experiments, and will reference relevant charts and theoretical knowledge from the literature to support my viewpoints. Finally, the conclusion will summarize the content studied in this paper, highlighting the significance

of consensus control in MASs for modern societal development and its potential positive impacts in the future.

2. Mathematical Tools and Theoretical Foundations

2.1. Graph Theory

Definition of graph: In a multi-agent system, a single agent is seen as a point, and the information transfer between two agents is seen as an edge. $G = (V, E, A)$ is a weighted graph where $V = \{1, 2, \dots, n\}$ represents the set of n points; $E \subseteq V \times V$ represents a collection of edges; $A = [a_{ij}] \in \mathbb{R}_{n \times n}$ denotes the adjacency matrix of all elements. If there is a path for any two nodes, then graph G is connected, i.e., graph G is structurally balanced.

Adjacency matrix: $a_{ij} > 0$ if $(i, j) \in E$; $a_{ij} = 0$ otherwise.

Degree matrix: The degree matrix D is a diagonal matrix whose diagonal element d_{ii} represents the degree of node i , i.e. $d_{ii} = \sum_{j=1}^n a_{ij}$.

Laplace matrix: The Laplace matrix L is defined as $L = D - A$. The Laplace matrix has the following properties:

1. L is symmetrical.
2. The sum of the rows of L is zero, i.e. $\sum_{j=1}^n a_{ij} = 0$.
3. The minimum eigenvalue of L is 0, and the corresponding eigenvector is $\mathbf{1}_n$ (all 1 vectors).

2.2. Matrix Theory

Eigenvalues and eigenvectors: For the Laplace matrix L , if 0 is a eigenroot of L , then $e_0 = \mathbf{1}_n$ is also a eigenroot of $e - L$, and their corresponding eigenvectors are all $\mathbf{1}_n$.

3. Mathematical Models

3.1. Single Integrator Model

For a single-integrator dynamic system, the consensus algorithm is typically designed as follows:

$$\dot{x}_i = -\sum_{j \in N_i} a_{ij} (x_i - x_j), i \in V \quad (1)$$

where $a_{ij} > 0$ means that agent i can make use of agent j 's information, and N_i is the set of neighbors of agent i .

3.2. Multi-integrator Model

In a multi-integrator model, the system is usually composed of multiple agents, each of which can consider its own state and that of its neighbors at the same time, achieving more complex coordination and consistency. The design of the multi-integrator model can be done as follows:

For a multi-integrator dynamic system, assuming that the state of each agent i consists of position p_i and velocity v_i the dynamic equation of the system can be expressed as follows:

$$\dot{p}_i = v_i, \dot{v}_i = u_i \quad (2)$$

where u_i is the control input of agent i , which usually relies on the state information of its neighbors. The control input u_i can be designed with the following consensus algorithms:

$$u_i = -k \sum_{j \in N_i} (p_i - p_j) \quad (3)$$

where N_i is the set of neighbors of agent i , and k is a non-negative gain parameter that is used to adjust the convergence speed of the system.

4. Fractional multi-agent model

Dynamics of Leader:

$$D_\mu x_0(t) = Ax_0(t) + g(t, x_0(t)) \quad (4)$$

Dynamics of Follower:

$$D_\mu x_i(t) = Ax_i(t) + g(t, x_i(t)) + u_i(t), i \in \mathfrak{F} \quad (5)$$

where D_μ is the μ Caputo derivative, A is a known constant matrix, and g is a nonlinear function.

Definition of Caputo derivative:

$$D_\mu f(t) = (1/\Gamma(n - \mu)) \int_0^t (t - s)^{\mu-n+1} f^{(n)}(s) ds \quad (6)$$

$$n - 1 < \mu < n, n \in \mathbb{Z} + \quad (7)$$

where Γ is a Gamma function.

4.1. Control Protocol

$$u_i(t) = -\sum_{j \in \mathfrak{N}} a_{ij} (x_i(t) - x_j(t)), i \in \mathfrak{F} \quad (8)$$

4.2. System Dynamics

$$D_\mu x_i(t) = Ax_i(t) + g(t, x_i(t)) - \sum_{j \in \mathfrak{N}} a_{ij} (x_i(t) - x_j(t)) \quad (9)$$

$$i \in \mathfrak{F} \quad (10)$$

5. Conclusion

Future research in the domain of MASs, with a particular emphasis on multi-agent consensus control, will unquestionably progress further towards attaining elevated levels of robustness, adaptability, and real-time performance. The unwavering dedication to augmenting these essential attributes will be spurred by the perpetually evolving requisites of intricate and dynamic real-world circumstances.

By ceaselessly endeavoring to foster the seamless amalgamation of theoretical research outcomes and practical implementations, a more profound and extensive exploitation of MASs across a diverse range of societal sectors can be anticipated. In the realm of intelligent transportation, MASs could leverage consensus control to optimize traffic flow in real-time. Through the coordination of various agents such as traffic lights, vehicles, and roadside sensors, they could ensure that traffic signals are adjusted in a synchronized manner, minimizing waiting times and reducing congestion, thereby enhancing overall travel efficiency.

In the field of environmental monitoring, MASs could apply consensus control to enable more accurate and comprehensive data collection and analysis. Drones, ground sensors, and satellite-based agents could work in unison to gather data about air quality, temperature, and pollution levels. By achieving consensus on the data transmission and fusion, they could provide a more holistic view of the environment, facilitating prompt and effective responses to ecological changes.

In the industrial manufacturing sector, MASs might revolutionize production processes by employing consensus control to enable more flexible and efficient coordination among diverse machines and workstations. Agents representing different manufacturing units could communicate

and synchronize their operations, ensuring that production lines run smoothly and adaptively, minimizing production delays and maximizing output quality.

All in all, such advancements in multi-agent consensus control are expected to herald a new era of convenience and innovation in our daily lives, radically transforming the manner in which we engage with and administer complex systems.

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