

Theoretical Design and Modeling of a Novel Composite Flexible Mechanism

Zifeng Lin

*Yanshan University, Qinhuangdao, China
linzifeng@stumail.ysu.edu.cn*

Abstract: To enhance the comprehensive performance of flexible motion platforms in terms of precise positioning and large displacement amplification, this paper designs and investigates a novel composite mechanism that combines a rhombic amplification mechanism with a parallelogram-based flexible structure. Based on the Euler-Bernoulli beam theory and Castigliano's second theorem, the displacement amplification ratio and stiffness analytical model of the rhombic flexible mechanism are derived, and the Lagrangian equation is employed to establish an analytical model for the natural frequency of the flexible mechanism. Additionally, an analytical stiffness model for a single parallelogram is developed. Finite element analysis is then used to simulate the dynamic performance of the mechanism, verifying the rhombic amplification ratio, overall stiffness, and natural frequency. The results indicate that the designed composite mechanism achieves a high displacement amplification ratio, with a fundamental resonance frequency reaching 652 Hz, while maintaining high positioning accuracy and stability in two-dimensional motion. This meets the demands for high-precision and long-travel applications in flexible motion platforms. The findings provide innovative design ideas for applications in micro-nano manipulation, precision positioning platforms, and flexible sensors.

Keywords: flexible mechanism, piezoelectric ceramics, rhombic structure, parallelogram mechanism.

1. Introduction

Flexible mechanisms [1] emerged in the 1980s and have since become a vital part of mechanism design, finding wide applications in everyday life and precision manufacturing [2]. Due to their advantages of minimal assembly requirements, frictionless and gap-free motion, no need for lubrication, and high motion accuracy, they are an ideal choice for micro- and nano-positioning technologies [3]. Flexible mechanisms play a crucial role in high-precision fields such as micro-positioning [4], micromanipulation, and biomedical applications [5].

Previous research has explored numerous displacement amplification mechanisms [6], including traditional rhombic [7], bridge, Moonie, and Cymbal-type mechanisms, which are widely used in precision positioning and micro-displacement control due to their compact structure, high amplification ratio, and good linearity. However, traditional displacement amplification mechanisms [8] often encounter challenges of insufficient stiffness and low natural frequency when achieving large displacement amplification. This limits their applicability in situations requiring additional load-

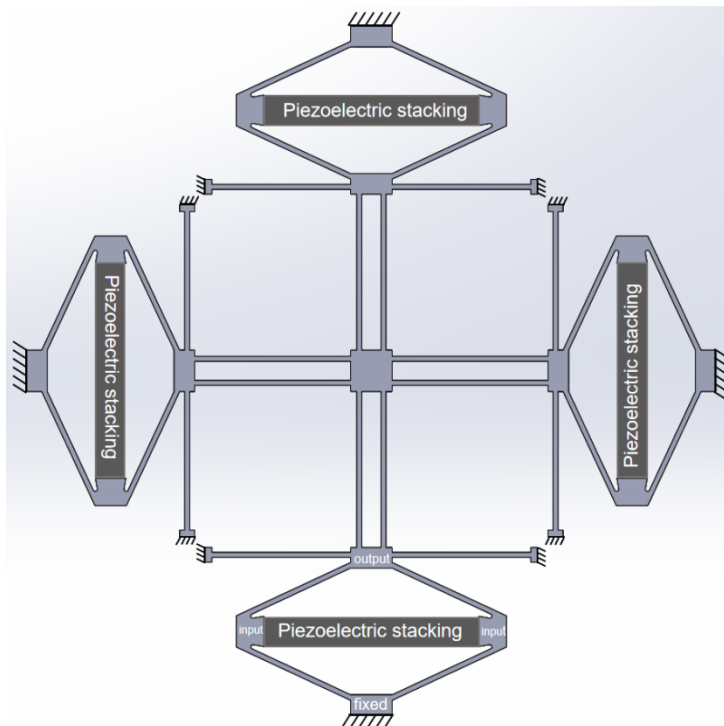
bearing capacity or high-frequency response. Consequently, many researchers have aimed to enhance output stiffness and dynamic response in their designs to better meet engineering demands.

The parallelogram flexible mechanism, another widely used type, exhibits high output stiffness and smooth multidimensional motion stability, especially excelling in flexible motion platform designs that require high load capacity and minimal distortion [9]. However, the displacement amplification capability of parallelogram structures is limited, making it difficult to meet the demands of precision positioning systems for large displacement amplification [9]. Therefore, designing a novel mechanism that combines the stiffness benefits of the parallelogram structure with the high amplification characteristics of the rhombic structure—achieving large displacement amplification, high stiffness, and superior dynamic response—remains an urgent challenge.

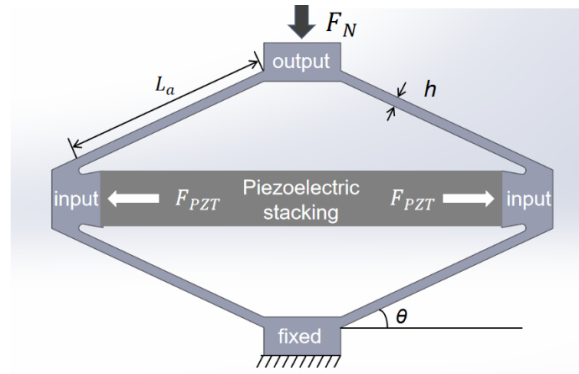
To address this, this paper proposes a composite structure that combines a parallelogram with a rhombic amplification mechanism. Through static and dynamic modeling, the study examines its performance in displacement amplification, stiffness, and resonance frequency. This structure is expected to yield a high-performance flexible amplification mechanism suitable for precision optical platforms, meeting the needs of modern precision positioning and high-frequency response systems.

2. A Novel Composite Flexible Mechanism

Combining the high stiffness of a parallelogram mechanism with the high displacement amplification capability of a rhombic amplification mechanism is a pressing challenge in the fields of precision positioning systems and high-frequency response applications. This paper proposes a composite structure that integrates a parallelogram structure with a rhombic amplification mechanism, designed using SolidWorks. The mechanism's performance in displacement amplification, system stiffness, and resonance frequency is analyzed. Through this innovative design, we aim to develop a high-performance flexible amplification mechanism suitable for precision optical platforms, meeting modern demands for large displacement, high stiffness, and excellent dynamic response. The specific structure is shown in Figure 1.



(a) Schematic of the Composite Mechanism Model



(b) Principle Diagram of the Rhombic Displacement Amplifier

Figure 1: Composite Flexible Mechanism

3. Analytical Model of the Composite Flexible Mechanism

3.1. Displacement Amplification Ratio Model

As shown in Figure 2, let the output force of the piezoelectric ceramic and the load be F_{PZT} and F_N , respectively, with other geometric parameters defined in Figure 1. When the piezoelectric ceramic is subjected to a positive voltage, the displacement amplification mechanism contracts horizontally by $2\Delta x$ with one end fixed, while the other end extends vertically by $2\Delta y$. Due to the symmetry of the mechanism, one-fourth of the flexible displacement amplification mechanism is selected for mechanical analysis, yielding the following relationships: $F_{PZT} = 2f_x$; $F_N = 2f_y$. As shown in Figure 2, to simplify the mechanical analysis, the flexible arm AB of the rhombic displacement amplification mechanism is used for force analysis. Considering the fixed boundary conditions of the flexible arm, points A and B rotate at equal angles, with point A moving horizontally and point B vertically. The flexible arm AB, constrained by points A and B, experiences a moment $2M_r$, leading to the following force equilibrium and moment equilibrium equations [10]:

$$f_{Ax} = f_{Bx} = f_x = F_{PZT}/2 \quad (1)$$

$$f_{Ay} = f_{By} = f_y = F_N/2 \quad (2)$$

$$2M_r = f_x l_a \sin\theta + f_y l_a \cos\theta \quad (3)$$

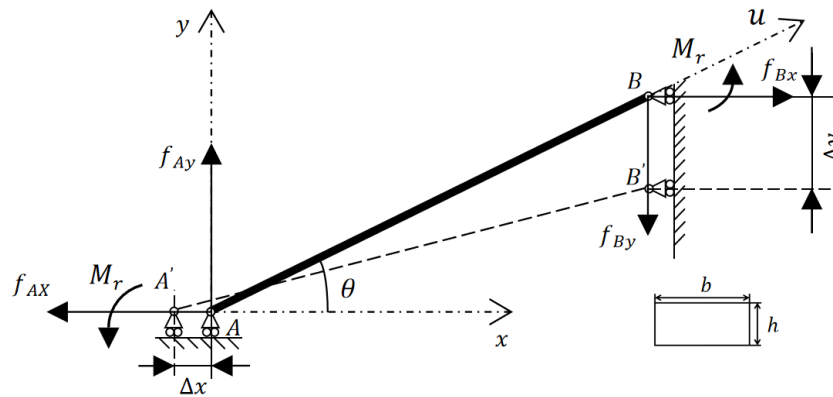


Figure 2: Force Analysis of the Rhombic Mechanism

For the simplified flexible arm AB, an analysis shows that AB is subjected to a tensile force along its length and a pair of moments perpendicular to it. Constraints at points A and B cause a moment

$2M_r$, which bends the flexible arm during motion, thereby amplifying displacement. Using force and moment equilibrium equations, the forces and moments along AB at a distance u from point A are:

$$\begin{cases} f_N = f_x \cos\theta - f_y \sin\theta \\ M = M_r - (f_x u \sin\theta + f_y u \cos\theta) \end{cases} \quad (4)$$

Based on the Euler-Bernoulli beam theory, the total strain energy of the flexible arm is the sum of tensile strain energy $V_{\varepsilon 1}$ and bending strain energy $V_{\varepsilon 2}$, given by:

$$V_{\varepsilon} = V_{\varepsilon 1} + V_{\varepsilon 2} \quad (5)$$

Using Castigliano's second theorem, the displacement of the mechanism in the x direction is:

$$\begin{aligned} \Delta x &= \frac{\partial V_{\varepsilon}}{\partial f_x} = \frac{f_N l_a}{EA} \frac{\partial f_N}{\partial f_x} + \int_0^{l_a} \frac{M}{EI} \frac{\partial M}{\partial f_x} du = \frac{l_a}{EA} (f_x \cos\theta - f_y \sin\theta) \cos\theta + \frac{l_a^3}{12EI} (f_x \sin\theta + \\ f_y \cos\theta) \sin\theta \\ \Delta x &= \frac{\partial V_{\varepsilon}}{\partial f_x} = \frac{f_N l_a}{EA} \frac{\partial f_N}{\partial f_x} + \int_0^{l_a} \frac{M}{EI} \frac{\partial M}{\partial f_x} du = \frac{l_a}{EA} (f_x \cos\theta - f_y \sin\theta) \cos\theta + \frac{l_a^3}{12EI} (f_x \sin\theta + \\ f_y \cos\theta) \sin\theta \end{aligned} \quad (6)$$

Similarly, the displacement in the y direction is:

$$\Delta y = \frac{\partial V_{\varepsilon}}{\partial f_y} = \frac{f_N l_a}{EA} \frac{\partial f_N}{\partial f_y} + \int_0^{l_a} \frac{M}{EI} \frac{\partial M}{\partial f_y} du = \frac{l_a}{EA} (f_x \cos\theta - f_y \sin\theta) \sin\theta + \frac{l_a^3}{12EI} (f_x \sin\theta + f_y \cos\theta) \cos\theta \quad (7)$$

where EA and EI are the tensile stiffness and bending stiffness of the flexible arm AB, respectively. Based on equations (6) and (7), the displacement amplification ratio under no load ($f_y=0$) is:

$$A_d = \frac{(l^2 + h^2) \sin\theta \cos\theta}{l^2 \sin^2\theta + h^2 \cos^2\theta} \quad (8)$$

3.2. Stiffness Model

To calculate the overall stiffness of the composite flexible mechanism, a two-step approach is used. Based on the derived amplification ratio formula in Section 2.1, the input stiffness of the displacement amplification mechanism is obtained as [10]:

$$K_{in} = \frac{2f_x}{\Delta x} = \frac{Edh^3}{l_a h^2 \cos^2\theta + l_a^3 \sin^2\theta} \quad (9)$$

and the output stiffness is:

$$K_{out} = \frac{K_{in}}{A_d} \quad (10)$$

The design principle of the dual-parallelgram mechanism is shown in Figure 3.

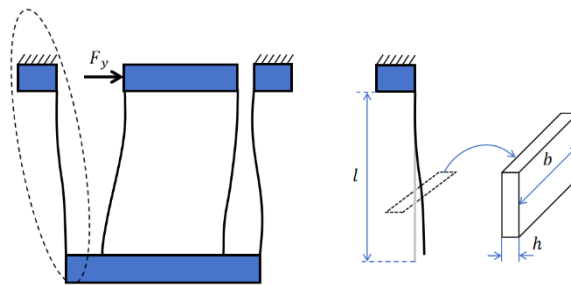


Figure 3: Principle Diagram of the Dual-Parallelgram Mechanism

From the deformation principle of the dual-parallelogram flexible unit, the main platform's stiffness K_y along the working direction can be expressed as [11]:

$$K_y \approx \left[12 - \frac{3}{100} \left(\frac{F_y l^2}{EI} \right)^2 \right] \cdot \frac{EI}{l^3} \quad (11)$$

where b and l represent the width and length of the straight beam, E is the Young's modulus of the material, and $I = bh^3/12$ with h being the thickness of the straight beam. F_y represents the load on the main platform along the working direction.

Ignoring the nonlinear terms in equation (2), the stiffness of the dual-parallelogram flexible unit in the working direction is:

$$K_y \approx \frac{12EI}{l^3} \quad (12)$$

For a single-parallelogram flexible mechanism, the stiffness in the working direction is double:

$$K_m = 2 \cdot K_y = \frac{2Ebh^3}{l^3} \quad (13)$$

From the operational state in Figure 4, the output stiffness in the y direction can be simplified as a combination of four single-parallelogram mechanisms and two rhombic amplifiers. The specific geometric parameters of the single-parallelogram mechanism are listed in Table 1.

Table 1: Calculation Parameters of the Parallelogram Mechanism

Parameter	Value	Parameter	Value
$b(\text{mm})$	10	$l(\text{mm})$	40
$h(\text{mm})$	1.5	$E(\text{GPa})$	78

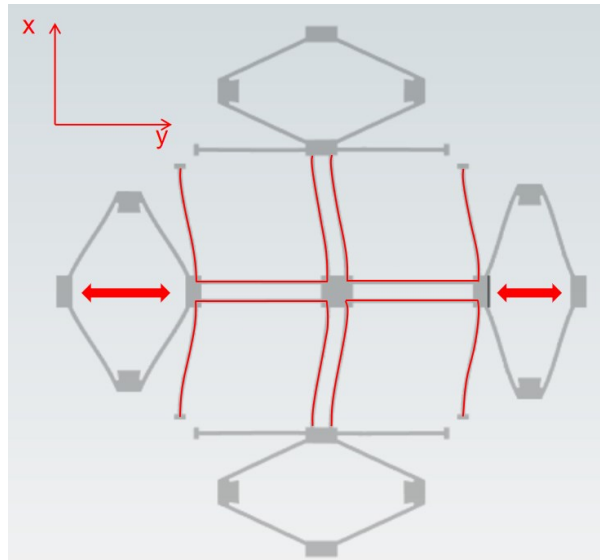


Figure 4: Working State of the Composite Mechanism

For the xy -axis flexible mechanism platform, the output stiffness is identical in the x and y directions, so calculating one direction's stiffness suffices. The stiffness of the composite flexible mechanism in a single direction is therefore:

$$K = 2K_{out} + 4K_m \quad (14)$$

3.3. Analytical Model of Natural Frequency

To better simulate real working conditions, an analytical model of natural frequency is established, where the base of the displacement amplification mechanism is fixed. Using aluminum as the material with a density of 2700 kg/m^3 , and calculated through SolidWorks, the mass is 0.013kg . During motion, it can be approximated as a spring with an effective mass m_d equal to one-third of its actual mass. The values $k_{out'}$ and $k_{in'}$ represent the output and input stiffnesses, respectively, derived from Ansys. The theoretically derived resonance frequency of the displacement amplification mechanism is [12]:

$$f = \frac{1}{2\pi} \sqrt{\frac{k_{out'}}{m_d}} = \frac{1}{2\pi} \sqrt{\frac{k_{in'}}{2A_d^2 m_d}} \quad (15)$$

4. Finite Element Analysis

4.1. Displacement Amplification Ratio Simulation

To validate the establishment of the analytical model and the accuracy of theoretical results in this study, a comparison is made between the analytical model's calculations and the results from finite element analysis (FEA) for the rhombus-shaped displacement amplification mechanism, using the FEA results as the benchmark. The specific geometric parameters for the displacement amplification mechanism are shown in Table 2, and the mesh division for the FEA model is illustrated in Figure 5.

In the FEA, the bottom of the model in Figure 5 is fully constrained, and opposing horizontal driving forces are applied symmetrically on either side of the flexible mechanism's long diagonal. The displacement amplification ratio for the flexible mechanism is calculated as the ratio of the vertical output displacement to the horizontal input displacement in the simulation results. Partial displacement amplification ratio results for the rhombus-shaped mechanism are presented in Table 3.

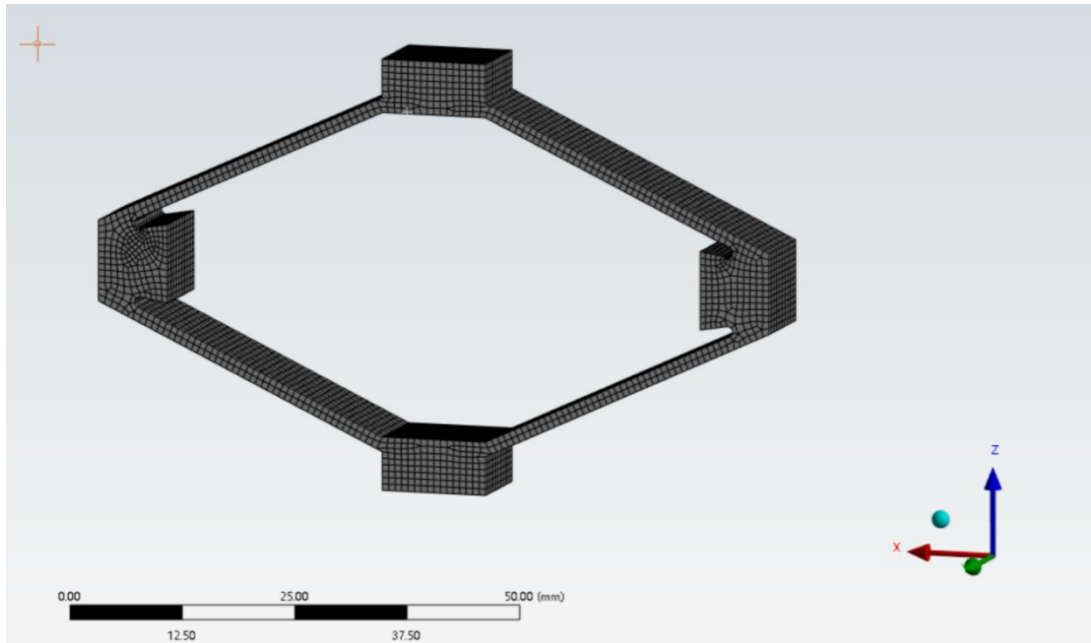


Figure 5: Finite Element Model

Table 2: Geometric Parameters of the Rhombus Mechanism

Parameter	Value	Parameter	Value
$l_a(\text{mm})$	31.4	$E(\text{GPa})$	78
$h(\text{mm})$	1.0	$\rho(\text{kg/m}^3)$	2700
$d(\text{mm})$	10		

Table 3: Displacement Amplification Ratio Results

Angle $\theta(^{\circ})$	Displacement Amplification Ratio		Error
	Analytical Model	FEA Result	
15	3.718	3.506	6.04%
20	2.705	2.663	1.57%
25	2.139	2.105	1.61%
30	1.728	1.715	0.75%

Table 2 shows that the analytical model based on Castigliano's second theorem closely aligns with the FEA results at larger angles θ , but exhibits a slight error at smaller angles due to the model not accounting for the influence of hollow regions when calculating bending moments. In practical engineering applications, an engineering factor should be applied to adjust these results, bringing the analytical model's calculations closer to the FEA results.

4.2. Stiffness Simulation

This study also verifies the overall stiffness of the theoretical model, using FEA results as a benchmark.

In the FEA, the basic constraints are as shown in Figure 1(a), while the stiffness in the y -direction is verified in Figure 4. A driving force is applied to one side of the short diagonal of the rhombus amplifier, and the y-direction displacement of the central platform is obtained. The stiffness in the y-direction for the composite mechanism is calculated as the ratio of the applied driving force to the y-direction displacement. The theoretical result calculated is 470N/mm, while the FEA result is 441N/mm, with an error of 6.5%.

4.3. Natural Frequency Simulation

Working frequency bandwidth and output displacement amplification ratio are critical parameters for evaluating the performance of displacement amplification mechanisms. Establishing an accurate analytical mechanics model between the natural frequency and geometric parameters is essential for structural optimization. To verify the accuracy of the analytical model for the natural frequency of the dual-arm rhombus-shaped displacement amplification mechanism, an FEA modal analysis of the same model was conducted using ANSYS software. The mesh division for the FEA model is illustrated in Figure 6, with specific structural parameters detailed in the table, and the angle θ of the flexible mechanism is set at 25°.

The first six modes of the rhombus-shaped displacement amplification mechanism are shown in Figure 5, with the third mode representing the deformation mode in the output displacement direction. The frequency for this mode is 652.9 Hz, while the theoretical calculation yields a natural frequency of 762 Hz, resulting in a relative error of 16%, which is within the acceptable range for engineering applications.

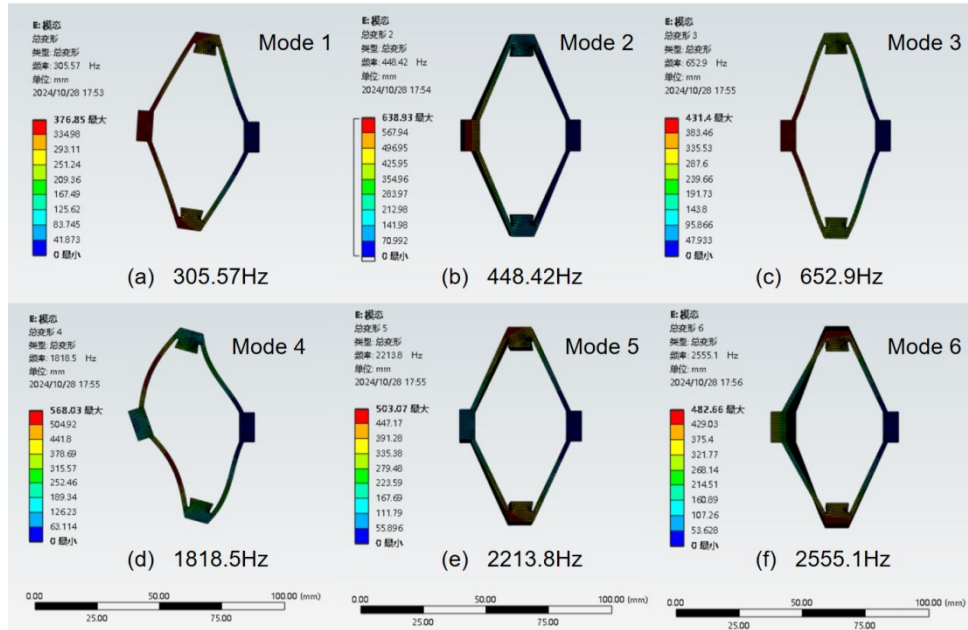


Figure 6: First Six Modes of the Rhombus-shaped Displacement Amplification Mechanism

5. Conclusion

This study investigates a composite structure combining a rhombus amplification mechanism and a parallelogram flexible mechanism to enhance the comprehensive performance of a flexible motion platform in precision positioning and large displacement amplification. Through theoretical modeling and FEA simulations, the performance of the composite mechanism in displacement amplification ratio, stiffness, and resonance frequency is validated. Experimental results indicate that this composite mechanism achieves high displacement amplification while maintaining substantial stiffness and low resonance frequency, meeting the requirements for high precision and large-stroke applications.

This research provides a new design approach for applications in micro-nano manipulation, precision positioning platforms, and flexible sensors. However, the performance of the mechanism was only validated within a specific load range; further research could explore dynamic performance optimization under complex load conditions. Additionally, material modification and structural optimization to further enhance the stability and applicability of the composite mechanism are directions worth exploring in future studies.

References

- [1] Abhijit, T., & Prasanna, G. (2022). Design and analysis of ultra-precise large range linear motion platforms using compliant mechanism. *IEEE Access*, 10, 94321-94336. <https://doi.org/10.1109/ACCESS.2022.3201343>
- [2] Gao, P., & Zhang, J. F. (2021). Development and prospects of flexible mechanisms. *China Equipment Engineering*, (5), 249-251.
- [3] Tian, Y., et al. (2020). A spatial deployable three-DOF compliant nano-positioner with a three-stage motion amplification mechanism. *IEEE/ASME Transactions on Mechatronics*, 25(3), 1322-1334. <https://doi.org/10.1109/TMECH.2020.2973175>
- [4] Lai, K. Z., Tran, K. S., Shirinzadeh, B., Ehrampoosh, A., Tian, Y., & Qin, Y. (2023). Design and analysis of a novel 3-DOF out-of-plane micropositioning mechanism. In *2023 26th International Conference on Mechatronics Technology (ICMT)* (pp. 1-3). Busan, Korea, Republic of. <https://doi.org/10.1109/ICMT59920.2023.10373668>
- [5] Song, Y., Wang, S., Luo, X., & Shi, C. (2022). Design and optimization of a 3D printed distal flexible joint for endoscopic surgery. *IEEE Transactions on Medical Robotics and Bionics*, 4(1), 38-49. <https://doi.org/10.1109/TMRB.2022.3142516>

- [6] Schlinquer, T., Mohand-Ousaid, A., & Rakotondrabe, M. (2018). *Displacement amplifier mechanism for piezoelectric actuators design using SIMP topology optimization approach*. In *2018 IEEE International Conference on Robotics and Automation (ICRA)* (pp. 4305-4311). Brisbane, QLD, Australia. <https://doi.org/10.1109/ICRA.2018.8460183>
- [7] Seong, H.-S., Zhakypov, Z., Okamura, A., & Ryu, J.-H. (2023). *A TSA-based haptic display with a rhombus structure for rendering linearized high force*. In *2023 23rd International Conference on Control, Automation and Systems (ICCAS)* (pp. 792-795). Yeosu, Korea, Republic of. <https://doi.org/10.23919/ICCAS59377.2023.10317056>
- [8] Chen, F., Zhang, Q., Gao, Y., & Dong, W. (2020). *A review on the flexure-based displacement amplification mechanisms*. *IEEE Access*, 8, 205919-205937. <https://doi.org/10.1109/ACCESS.2020.3037827>
- [9] Xu, Q. (2012). *New flexure parallel-kinematic micropositioning system with large workspace*. *IEEE Transactions on Robotics*, 28(2), 478-491. <https://doi.org/10.1109/TRO.2011.2173853>
- [10] Jiang, Z., Cao, J. Y., Ling, M. X., et al. (2017). *Theoretical design and modeling of a novel double-arm rhombic piezoelectric flexible mechanism*. *China Mechanical Engineering*, 28(21), 2557-2561+2566.
- [11] Xi, S. Z., & Lai, L. J. (2021). *Performance analysis of corrugated beam clamp-type double parallelogram flexible mechanism*. *Intelligent Computer and Applications*, 11(2), 190-194. Jiru Wang et al. 2021. *Smart Materials and Structures*, 30, 075021.
- [12] Fang, C., Guo, J., Xu, X. X., et al. (2016). *Design of rhombus micro-displacement amplification mechanism suitable for FSM systems*. *Infrared and Laser Engineering*, 45(10), 170-176.