

Study on the Sensitivity of Existing Truss Structures to Different Loading Modes

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Abstract: In actual engineering, it is necessary to frequently identify structural damage to ensure a clear understanding of the structure and to respond to accidents in advance. The methods for structural damage identification are currently mainly focused on Bayesian updating. Bayesian updating is a method that uses information obtained from experiments to update known conditions. However, implementing Bayesian updating in many actual engineering health detection, a method is needed to minimize the number of structural observations and tests and gain an understanding of the health status of engineering structures. This paper explores the maximum value of Bayesian updating sensitivity by applying different loads to the existing truss model. Specifically, applying pressure shear forces to the overall truss structure, Longitudinal comparison of the changes in update sensitivity under different loads, and summarized an optimal method to improve the sensitivity of partial truss structures through different loading methods to explore ways to reduce the cost of actual engineering tests. Finally, Bayesian inference is used to try to apply a larger load to the structure and record its displacement change value.

Keywords: Bayesian updating, Structural damage Identification, Structural health monitoring, Sensitivity, Truss

1. Introduction

There are many reasons that cause internal damage to structures in real life. Including local environmental erosion, structural damage, structural aging, loads exceeding the maximum design value that the structure can bear, etc. The reliability of engineering structures requires not only that the structures are consistent with the actual conditions and designs after they are built, but also that can be guaranteed to be able to be used normally even under corrosion and aging conditions within the specified time. Since the end of the last century, the work on structural damage identification has developed rapidly with the improvement of computer technology. In 1974, Collins et al. [1] proposed a method for structural statistical identification by modifying finite element models through experimental measurement data.

However, there are uncertainties in actual engineering. There is a lack of information on the impact of past buildings on the local environment, construction quality, etc. Currently, methods to improve structural damage identification are mainly focused on identification based on Bayesian updating method. In 1998, Beck and Katafygiotis [2] proposed a general Bayesian statistical framework for system identification for model updating and quantitatively assess this accuracy. Beck and Au [3] proposed an adaptive MCMC simulation method to evaluate the required integral. Since then, Markov

Chain Monte Carlo Sampling (MCMC) has become one of the most commonly used sample generation methods. Yi et al. [4] used structural modal parameters to monitor structural health based on Bayesian updating. Beck and Cheung [5] proposed a posterior distribution sampling algorithm based on MCMC, and test illustrative examples with simulated data from two shear walls. Since the posterior distribution of MCMC is unknown, a stochastic simulation method that is effective in almost all situations is needed, Ching and Chen [6] proposed the transition Markov chain Monte Carlo (TMCMC) method. Straub and Papaioannou [7] believe that MCMC is difficult to ensure the stability of the Markov chain, so the BUS algorithm is proposed, which can be regarded as an improvement on the simple rejection sampling algorithm of Bayesian updating. Xiao et al. [8] combined the concepts of group Monte Carlo and cross entropy and proposed an innovative adaptive importance sampling (AIS) algorithm. Ni et al. [9] proposed a new likelihood-free Bayesian inference method for structural parameter identification that can be used to estimate the probability of the posterior distribution of unknown structural parameters. In order to solve the computational cost problem of Bayesian updating, Cao et al. [10] proposed an adaptive information filtering (AIF) method based on Bayesian updating sensitivity analysis.

After the basic framework of structural damage identification and health monitoring methods based on Bayesian theory were established and improved, Scholars have made great contributions to improving the efficiency of Bayesian updating. However, in actual engineering, observation and testing of structures require a large amount of money. Therefore, a method is needed to minimize the number of structural observations and experiments, reduce the cost of overall health monitoring by gaining insights into structural health in as few tests as possible. Cao et al. [10] believe that some of the large number of different types of observations in actual engineering are related to the structural parameters themselves. As for the observation of structural response, low-value information not only does not help update the target but also increases the cost.

Based on the above problems, this paper uses the existing truss model, try to apply different load modes and load locations to the truss structure through simulation methods, and explore how to obtain the maximum Bayesian sensitivity of the upper and lower chords and webs of the truss at the lowest cost when detecting the health of the truss.

2. Bayesian evaluation method

2.1. Bayesian updating

The Bayesian evaluation method is based on Bayes' theorem, structural damage assessment using limited information. Compared with traditional methods, this method mainly combines the prior distribution with the results obtained after the experiment.,the new distribution after incorporating the outcome into the prior distribution is called the posterior distribution. Therefore, using this method, we can have a clearer understanding of the information of existing truss bridges. According to Bayes' theorem, the calculation expression of the posterior distribution is as follows:

$$f_x(X|U) = L(U|X) \cdot f_x(X) \quad (1)$$

In the formula: $f_x(X)$ and $f_x(X|U)$ respectively represent the prior distribution and posterior distribution of the truss stiffness in the evaluation method, $L(U|X)$ is the likelihood function. For the prior distribution in the evaluation method, X is a random variable, is summarized based on the data obtained in the past about the truss stiffness and the actual situation, which in this case is called the prior distribution. Therefore, in this evaluation method, the truss stiffness parameter will be regarded as a variable. Compared with traditional statistical methods for large amounts of data, Bayesian evaluation methods allow for smaller sample sizes.,each updated data is incorporated into the prior distribution, and new data is continuously added to the distribution.

2.2. Likelihood function

To obtain more accurate data on the stiffness of existing truss bridges, it is necessary to construct a likelihood function for a given structural model as shown in EQ. (2). In this experiment, the observed observation model value is $h_i(x)$, and the standard deviation is σ_ε .

$$L(U|X) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp \left\{ -\frac{1}{2} \frac{[h_i(x)-y_i]^2}{\sigma_\varepsilon^2} \right\} \quad (2)$$

In the formula: $h_i(x)$ is the modeling value, y_i is the observed value, and σ_ε is the observation error.

For the truss model, the formula can be simplified to EQ.(3):

$$f_x(X|Y) = \frac{1}{c} L(X) \cdot f_x(x) \quad (3)$$

In the formula, $L(X)$ is the likelihood function, $f_x(x)$ is the prior distribution, $f_x(X|Y)$ is the posterior distribution, and c is the coefficient.

In this test, there will be multiple sets of recorded displacements caused by the applied loads on the truss bridge. Therefore, under n observation values, assuming that these displacement observation groups are independent of each other, the likelihood function is defined:

$$L(x) = \prod_{i=1}^m L_i(x) \propto \prod_{i=1}^m \exp \left\{ -\frac{1}{2} \frac{[y_i-h_i(x,M)]^2}{\sigma_i^2} \right\} \quad (4)$$

Generally, when performing Bayesian updating and actual modeling, it is difficult to directly derive the relationship between the test results and the parameters to be updated.,or there is more than one parameter to be updated. In this experiment, five parameters to be updated are set. And it is difficult to directly calculate the integral to obtain the posterior distribution for modeling. Specifically, it is shown in EQ. (5):

$$c = \int_{-\infty}^{+\infty} \theta^x (1-\theta)^{1-x} \cdot \frac{\alpha}{\sqrt{2\pi}\sigma} e^{-\frac{(\theta-1/2)^2}{2\sigma^2}} d\theta \quad (5)$$

Therefore, in actual engineering, to obtain the posterior distribution, it is often necessary to use sampling calculations rather than theoretical calculations, and simple rejection sampling was used in this experiment.

2.3. Simple rejection sampling

In most experiments, it is difficult to write a specific analytical expression for the posterior distribution. Therefore, a sampling method that is effective for any distribution is needed. The simple rejection sampling method was used in this experiment. The specific flow chart is shown in Fig.1:

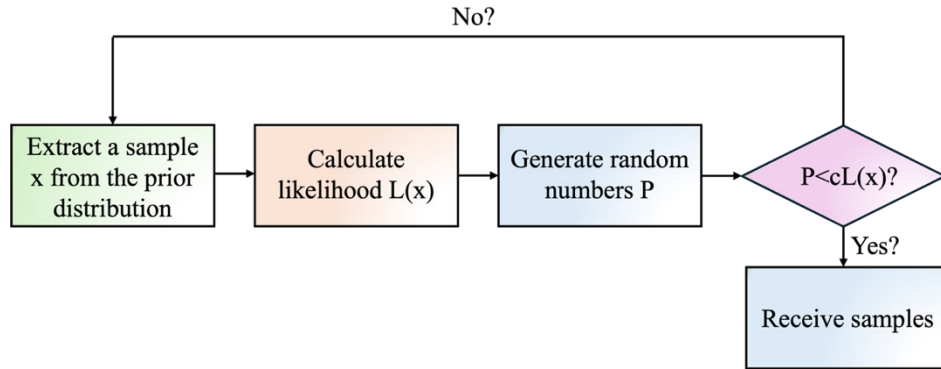


Figure 1: Simple rejection sampling flow chart

2.4. KL divergence

Kullback-Leibler Divergence (KL Divergence), it is used to describe the gap between the approximation of a probability distribution Q to the true distribution P , that is, the size of the information difference. The larger the KL divergence value, the worse the approximation of the probability distribution Q to the true distribution P . The definition of KL divergence is shown in EQ. (6):

$$D_{KL}(P||Q) = \sum_i P(i) \log \frac{P(i)}{Q(i)} \quad (6)$$

3. Project overview

The truss structure of this test was selected from the Songhua River Railway Bridge in Jilin City, Jilin Province, China. This railway bridge across the Songhua River in Jilin City was started in June 1926 and completed in August 1927. It has been in use for nearly a hundred years, so it is difficult to obtain the stiffness data of the original bridge trusses. This test uses the truss structure of the bridge truss, they are the average stiffness of the upper chord, the lower chord and the stiffness of the three special robs of the web. Five parameters are set for the corresponding stiffness, they are $EA_1=85000$, $EA_2=86000$, $EA_3=75000$, $EA_4=71000$, and $EA_5=81000$.

The health damage condition of the constructed model was evaluated, and the effect of applying different loads on the health damage assessment sensitivity of different robs in the truss was explored. The specific actual diagram and structure diagram are as follows (Fig.2, Fig.3):



Figure 2: Actual picture of the bridge

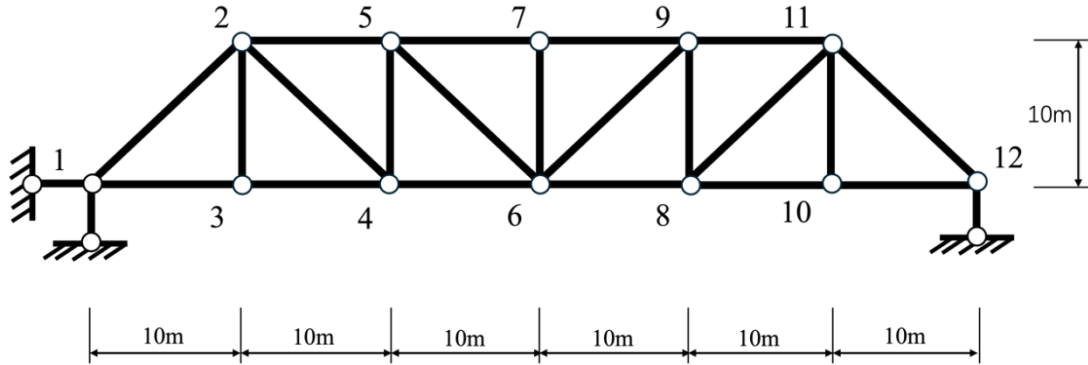


Figure 3: Truss structure diagram

Schematic diagram of the three special rods of the upper chord, lower chord and web member used in this test (Fig.4):

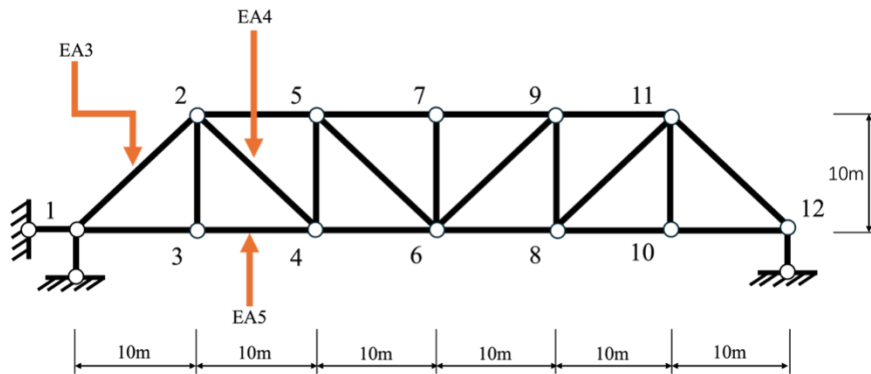


Figure 4: Schematic diagram of special rods

Based on the environment and specific data, the experiment sets a prior distribution for the assumed model. The prior distribution of EA3, EA4, and EA5 conforms to the normal distribution $X \sim N(70000, 7000)$, $X \sim N(70000, 7000)$, and $X \sim N(70000, 7000)$. The average stiffness of the upper and lower chords, except for special rods, is EA1 and conforms to the normal distribution $X \sim N(80000, 8000)$. The average stiffness of diagonal web rods and vertical rods except special rods is $X \sim N(80000, 8000)$.

In the test, sandbags were stacked, and cranes were used to provide forces in different directions. use a deflection meter to measure the displacement of the truss under static loading. Based on the displacement data, the analytical formula of the displacement measurement point is obtained. The posterior distribution is obtained through Bayesian evaluation methods.

4. Sensitivity analysis of loading mode on truss structure

The first consideration in the experiment was the way in which the force was applied to the truss, one is to apply bending force to the whole truss, and the other is to apply shear force to the whole truss. During the test, we also tried to apply tensile force to the upper and lower chords. However, there is no improvement in the update sensitivity of the overall truss.

4.1. Impact of pressure

First, apply pressure $P=10\text{kN}$ to the entire truss, which is the Press Group (PG) (Fig. 5):

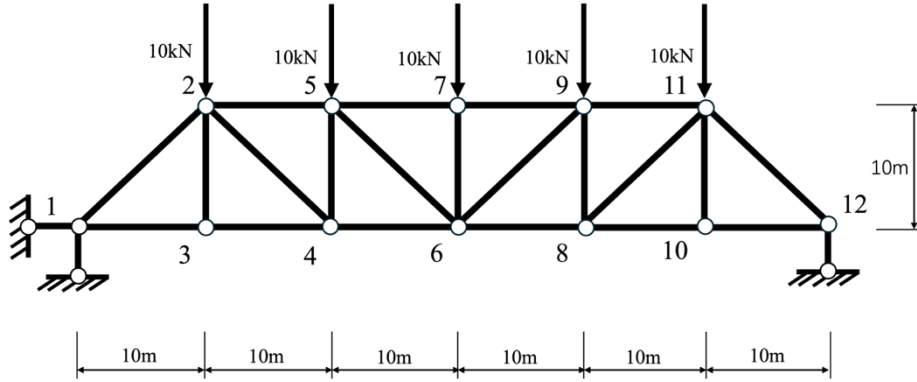


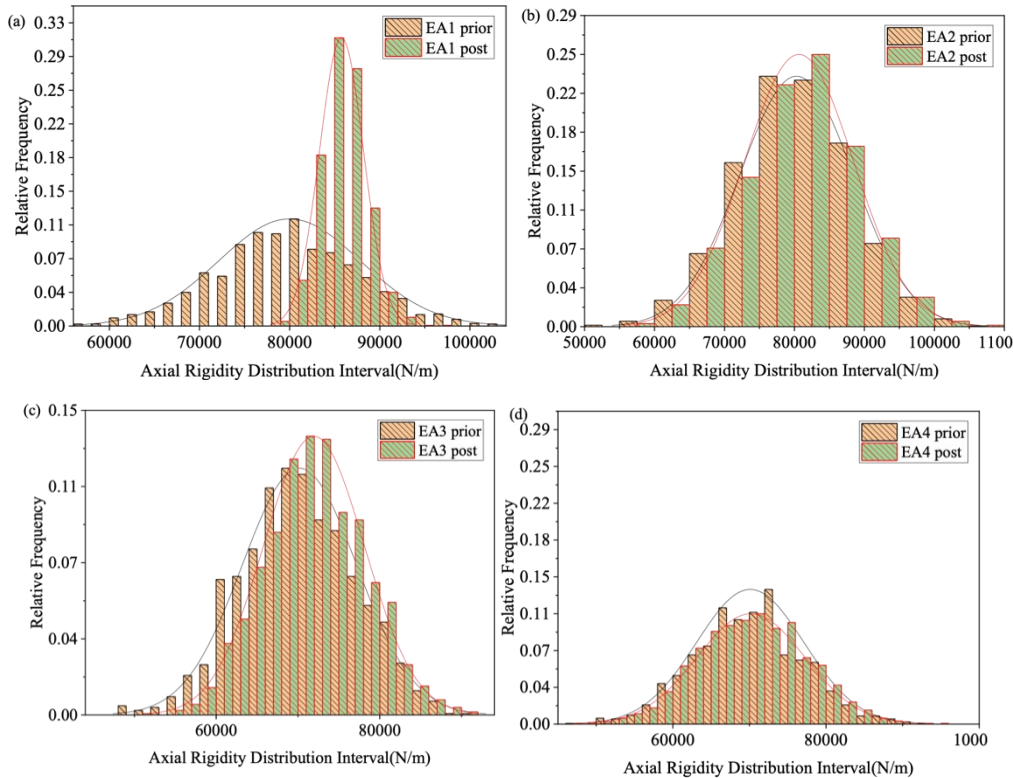
Figure 5: Schematic diagram of applied load in PG

Deflectometers were arranged at nodes 3 and 6, and the measured displacement values were 58mm and 33mm respectively. According to the preset a priori stiffness of each member and the virtual work theorem, the analytical expressions of the six-point displacement and the three-point displacement are obtained as shown in EQ. (7) and EQ. (8):

$$\Delta 6 = \frac{3325+200\sqrt{2}}{EA_1} + \frac{150+250\sqrt{2}}{EA_2} + \frac{250\sqrt{2}}{EA_3} + \frac{150\sqrt{2}}{EA_4} + \frac{125}{EA_5} \quad (7)$$

$$\Delta 3 = \frac{1659.026}{EA_1} + \frac{167.83}{EA_2} + \frac{589.427}{EA_3} + \frac{70.697}{EA_4} + \frac{208.3}{EA_5} \quad (8)$$

The likelihood function (error = 1mm) is obtained through the analytical formula, and then through the simple rejection sampling method, get the posterior distribution of Bayesian updating and organize the data (Fig.6):



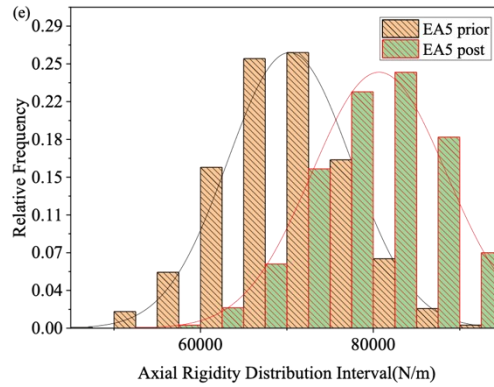


Figure 6: Prior and posterior comparison chart under the PG
(a)EA1 (b)EA2 (c)EA3 (d)EA4 (e)EA5

It can be seen intuitively from the above figure 6 that the pressure has a very high update efficiency for the stiffness EA1 of the upper and lower chords. The greater the update efficiency, the higher the sensitivity, the lower the sensitivity of the web and special rods. Therefore, various loading methods are tried next to improve the sensitivity of the web robs and special robs.

4.2. Impact of shear

In this test, four methods of applying shear force were tried on the truss. The first group of Single Shear force group 1 (SS1) (Fig.7):

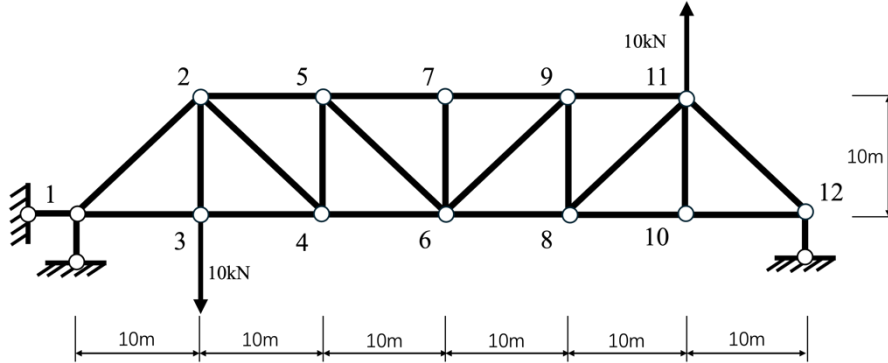


Figure 7: Schematic diagram of applied load in SS1

The deflectometers were placed at nodes 3, 6, and 11 to measure the displacement of three points. The displacement values obtained were: 77mm, 78mm, 56mm, and the displacement analytical formula of the three points is obtained by the same method:

$$\Delta_3 = \frac{198.15}{EA_1} + \frac{202.64}{EA_2} + \frac{157.2}{EA_3} + \frac{15.71}{EA_4} + \frac{55.54}{EA_5} \quad (9)$$

$$\Delta_6 = \frac{327.58}{EA_1} + \frac{174.74}{EA_2} + \frac{94.27}{EA_3} + \frac{47.14}{EA_4} + \frac{33.33}{EA_5} \quad (10)$$

$$\Delta_{11} = \frac{355.65}{EA_1} + \frac{58.31}{EA_2} + \frac{31.42}{EA_3} + \frac{15.71}{EA_4} + \frac{1.39}{EA_5} \quad (11)$$

The same sampling was done by simple rejection sampling (error = 1 mm). However, the sample acceptance rate (number of samples divided by the total number of samples) in this test group was as

high as 76.5%. A high acceptance rate will lead to a reduced Bayesian updating effect, so the test accuracy needs to be improved. The test increased the load P applied to each node from 10kN to 100kN. The Bayesian evaluation method with $P=100\text{kN}$ reduces the sample acceptance rate to 3.13%. Sort out the data after different load sampling (Fig.8, Fig.9):

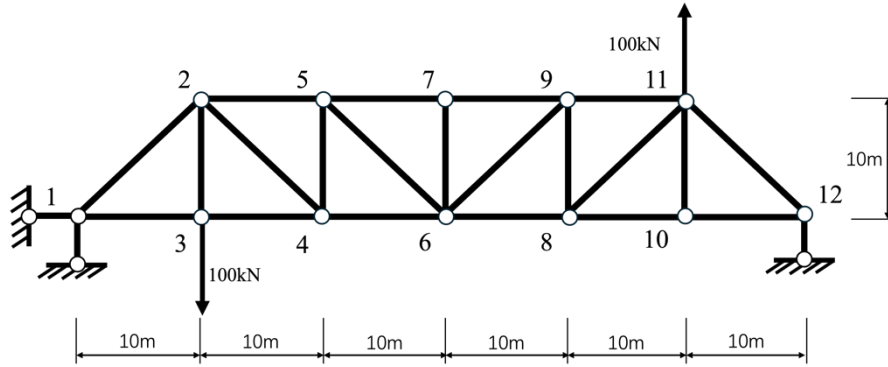


Figure 8: Schematic diagram of applied load $P=100\text{kN}$ for SS1

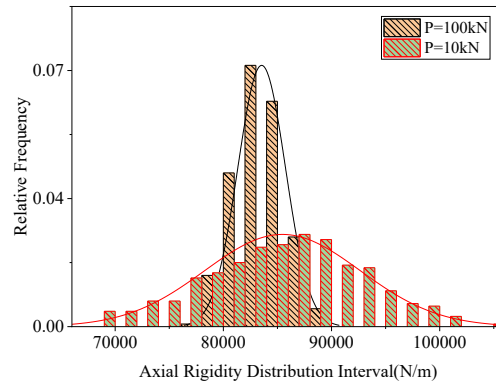
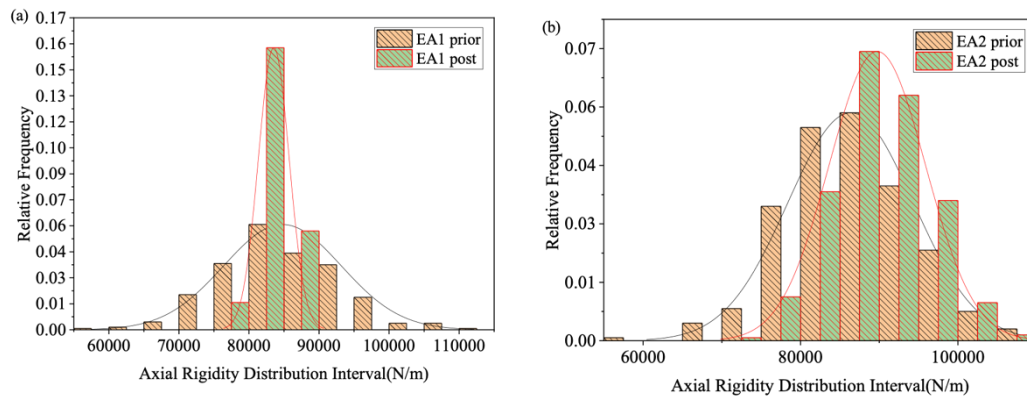


Figure 9: Comparison of SS1 after sampling under different loads

It can be seen intuitively in the figure 9 that the effect of Bayesian updating of the truss is improved to a certain extent by increasing the test accuracy by changing the load. The test data of load $P=100\text{kN}$ are sorted out (Fig.10):



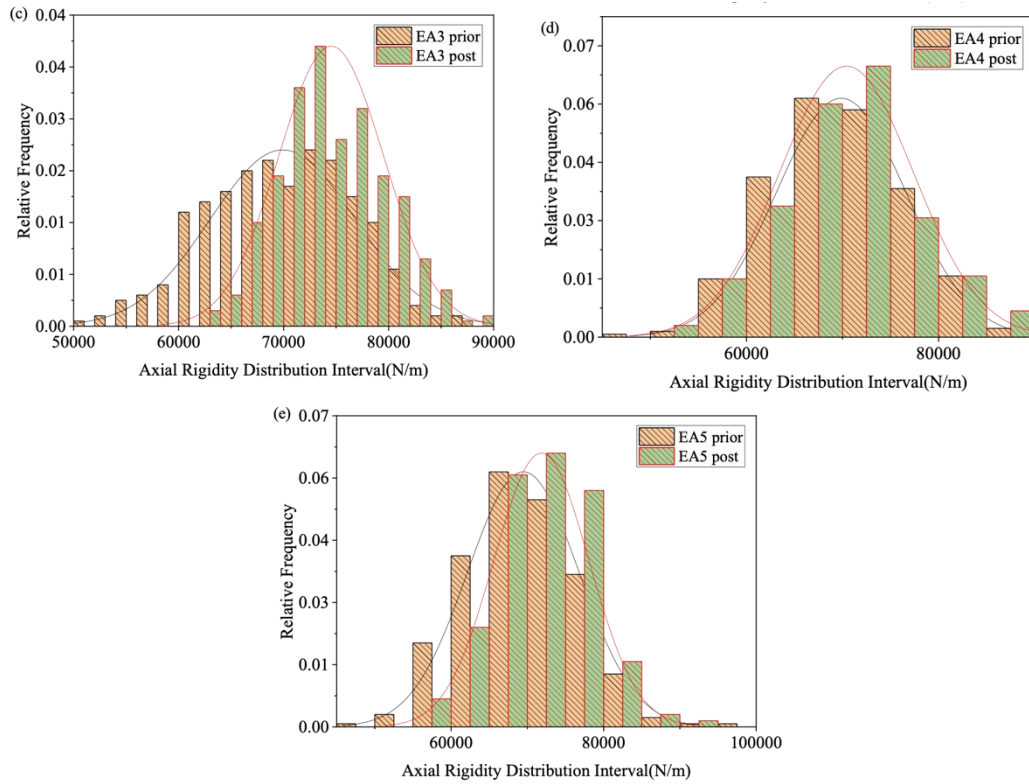


Figure 10: Prior and posterior comparison chart under the SS1
(a)EA1 (b)EA2 (c)EA3 (d)EA4 (e)EA5

As shown in the figure 10, the SS1 makes the upper and lower chord members stiffness EA1 updated more significantly than the pressure group. In addition, special members and web members, except EA4 members, have been updated to a certain extent.

The second way of applying shear force: Single Shear group 2 (SS2), structure and applied load P (P=100kN) (Fig.11):

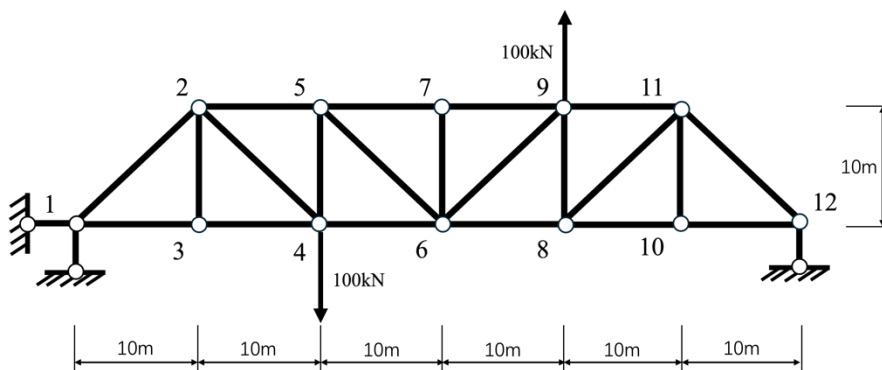


Figure 11: Schematic diagram of applied load in SS2

The deflectometers were placed at nodes 4 and 9, and the displacement values were measured to be 81mm and 65mm. The analytical formula was obtained using the same method:

$$\Delta 4 = \frac{3425.54}{EA_1} + \frac{1571.2}{EA_2} + \frac{628.5}{EA_3} + \frac{221.7}{EA_4} + \frac{222.3}{EA_5} \quad (12)$$

$$\Delta 9 = \frac{2084.49}{EA_1} + \frac{2552.9}{EA_2} + \frac{314.2}{EA_3} + \frac{314.2}{EA_4} + \frac{111.1}{EA_5} \quad (13)$$

The same simple rejection sampling method is used, and the data (Fig.12) are sorted:

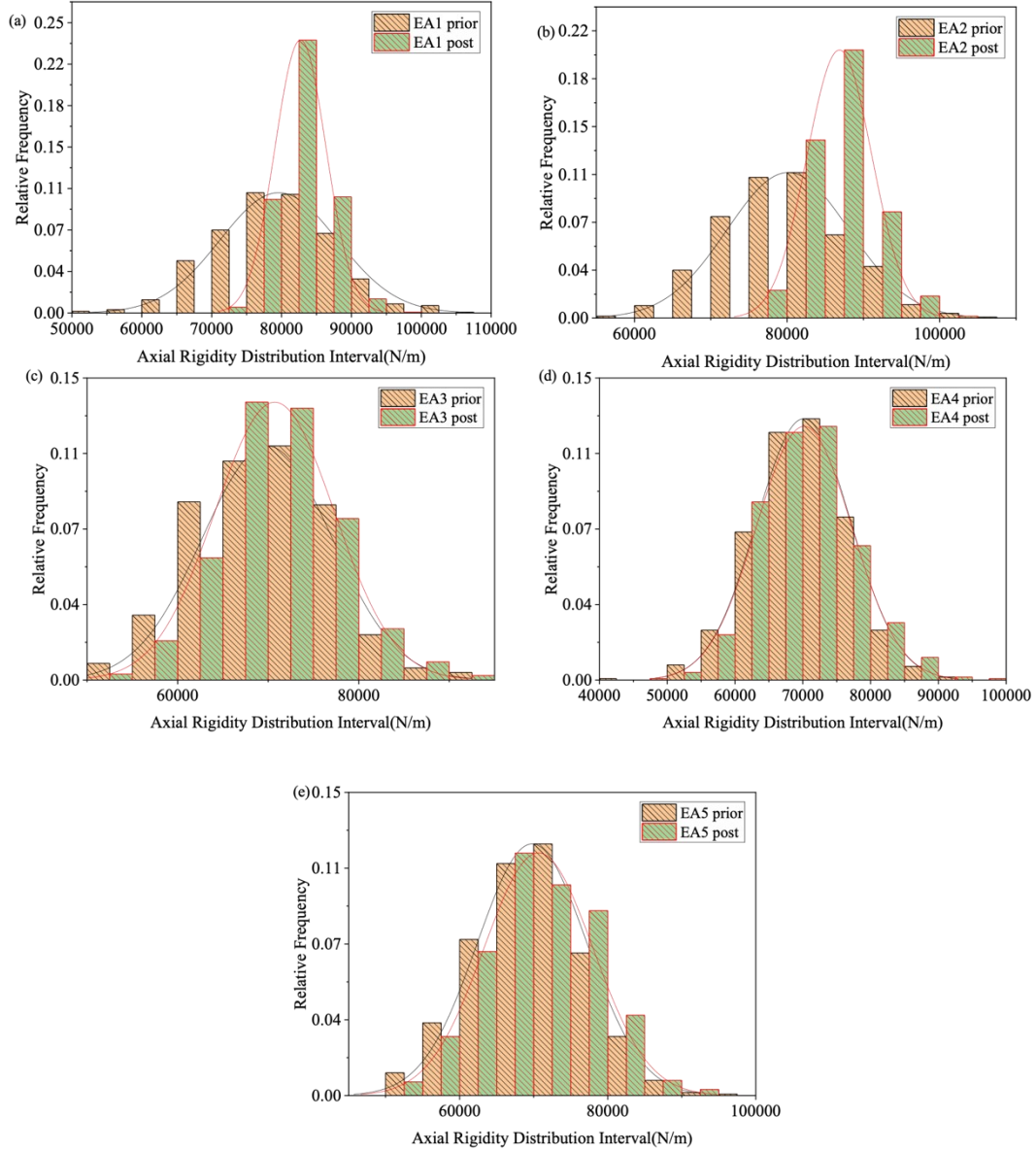


Figure 12: Prior and posterior comparison chart under the SS2
(a)EA1 (b)EA2 (c)EA3 (d)EA4 (e)EA5

As shown in the figure 12, this test has low sensitivity to the selection of special robs but has increased sensitivity to the web stiffness EA2.

The third loading method, Double Shear group (DS) (Fig.13):

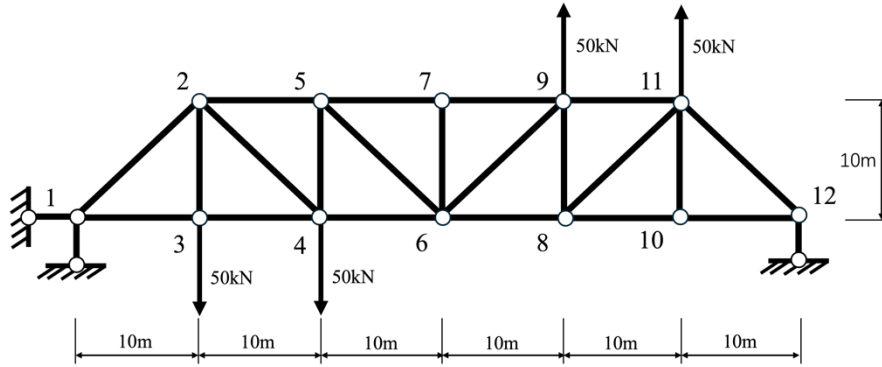


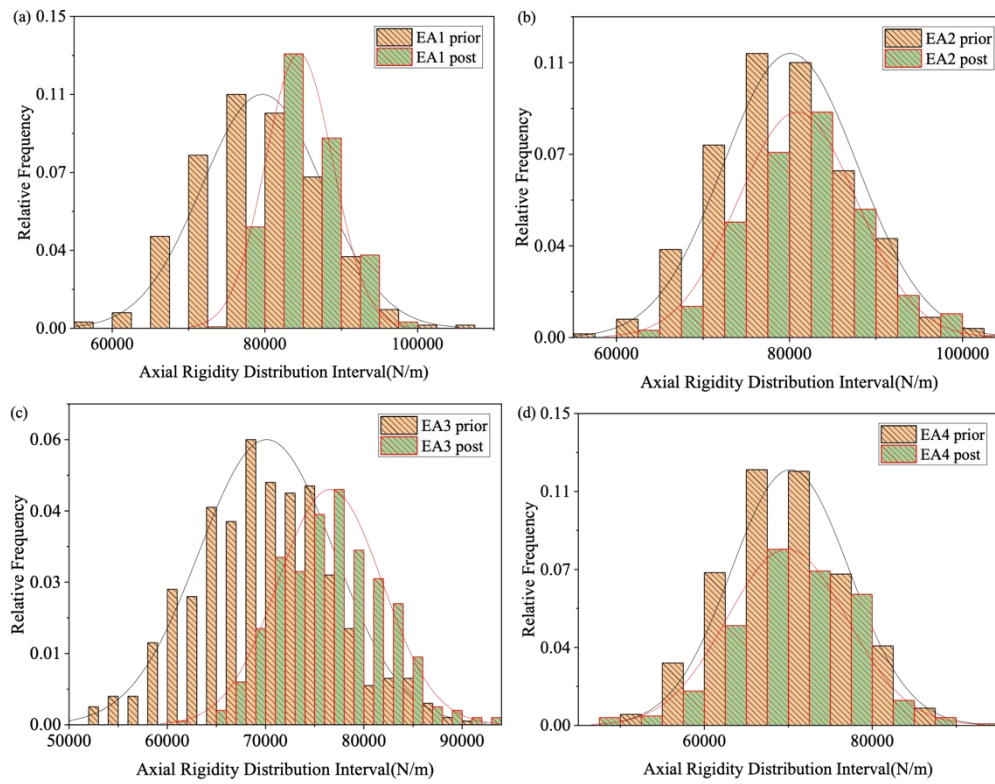
Figure 13: Schematic diagram of applied load in DS

Deflectometers were arranged at points 3 and 6 of the structure, and the displacements were measured to be 61mm and 55mm. A load of $P = 50\text{kN}$ was applied, and the analytical formula for the displacement was obtained as follows:

$$\Delta 3 = \frac{3456.95}{EA_1} + \frac{1663.95}{EA_2} + \frac{706.95}{EA_3} + \frac{0}{EA_4} + \frac{250}{EA_5} \quad (14)$$

$$\Delta 6 = \frac{2402.4}{EA_1} + \frac{971.35}{EA_2} + \frac{1178.75}{EA_3} + \frac{0}{EA_4} + \frac{417.5}{EA_5} \quad (15)$$

The data were also sorted using simple rejection sampling (Fig.14):



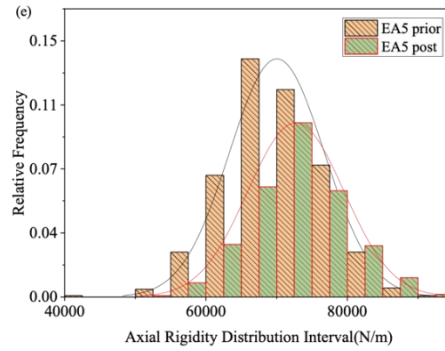


Figure 14: Prior and posterior comparison chart under the DS
(a)EA1 (b)EA2 (c)EA3 (d)EA4 (e)EA5

By comparing the figure 14 with the analytical expression, since the stiffness of the EA4 special member in the analytical expression is 0, Therefore, there was no update to EA4 in this test group. Except for this rod, the rest of the rods have been updated to some extent.

The last loading method: Pull Shear group (PS) (P=200kN, at 7 nodes, P=100kN, at 4 and 8 nodes) (Fig.15):

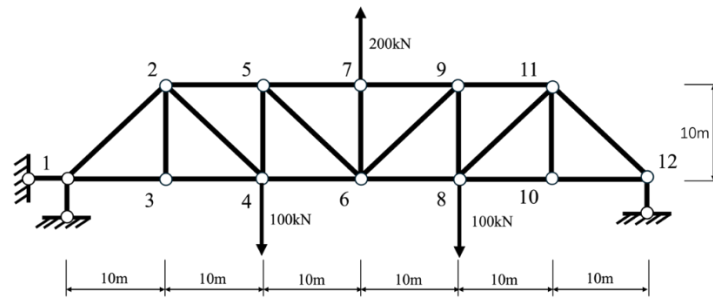


Figure 15: Schematic diagram of applied load in PS

The deflectometers were placed at nodes 4 and 6, and the displacement values were measured to be 58mm and 89mm. The analytical formula for the displacement is as follows:

$$\Delta 4 = \frac{3000}{EA_1} + \frac{3827.9}{EA_2} \quad (16)$$

$$\Delta 6 = \frac{2000}{EA_1} + \frac{2356.8}{EA_2} \quad (17)$$

By simple rejection sampling and tidying up the data (Fig.16):

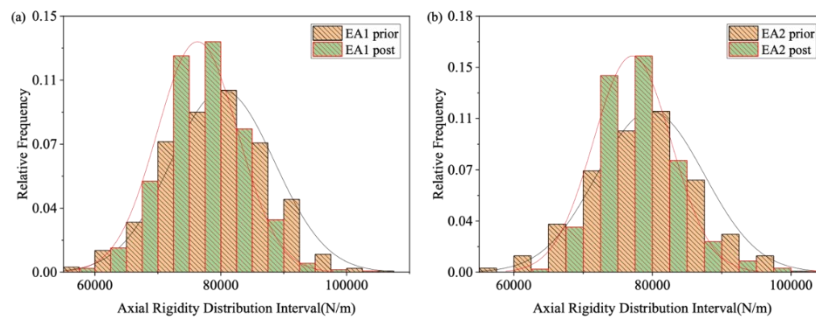


Figure 16: Prior and posterior comparison chart under the PS
(a)EA1 (b)EA2

Due to the load application method in this group of tests, the internal force at the selected special rod is 0. In the analytical formula, the coefficients of EA3, EA4, and EA5 are 0. Therefore, this group of experiments only updated EA1 and EA2. As shown in the figure 16, the sensitivity of the belly bar has been improved.

4.3. Comprehensive analysis

Sampling data of five experimental groups (Fig.17):

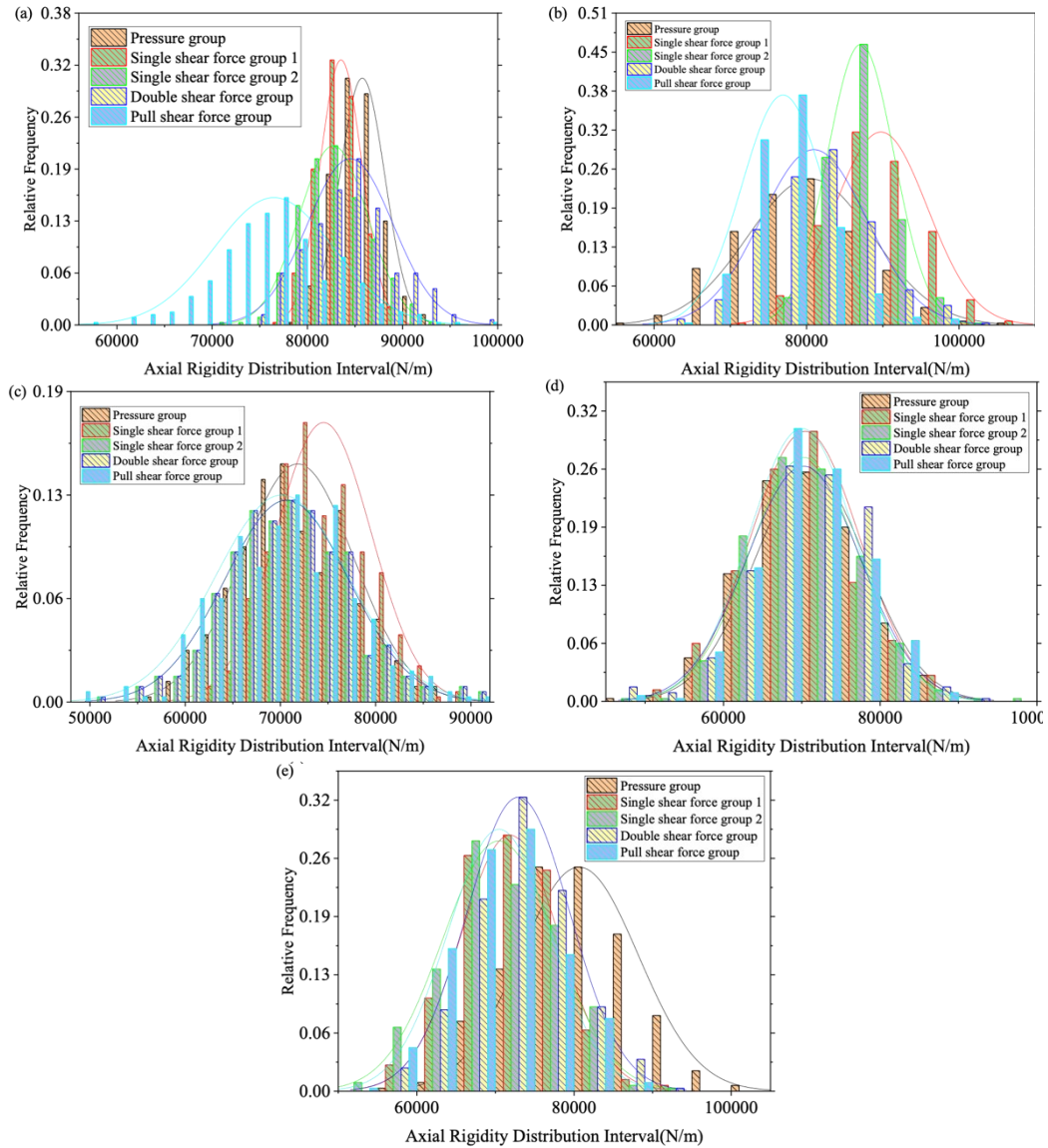


Figure 17: Posterior comparison of each group
(a)EA1 (b)EA2 (c)EA3 (d)EA4 (e)EA5

As shown in the figure 17, when comparing the stiffness of the upper and lower chords, the SS1 has the highest sensitivity, followed by the PS. When comparing the webs, the sensitivity of the two groups of SS2 and the PS is roughly similar, but the results are different. Through KL divergence calculation and actual observation of the model, the effect of SS2 is better. For the three special rods, the SS1 has the highest sensitivity. It can be concluded through experiments that increasing the force will increase the accuracy of the test, that is, the sensitivity of the update. Applying compression to

the truss increases the sensitivity of the upper and lower chords, while applying shear increases the sensitivity of the webs. That is to say, the overall truss is regarded as a simply supported beam, when the shear force of the overall simply supported beam is 0, the upper and lower chords have the greatest sensitivity, and when the bending moment is 0, the web member has the highest sensitivity.

5. Bayesian inference

In the SS1, the test group improved the test accuracy by increasing the magnitude of the applied load. So try to infer on the PG: If the load is applied in the same way as the pressure group, the applied load $P=10\text{kN}$ is increased to $P=30\text{kN}$. The 6-point displacement change value of the truss structure is obtained. The analytical formula of the 6-point displacement of the pressure group is as follows:

$$\Delta 6 = \frac{332.5P+20\sqrt{2P}}{EA_1} + \frac{15P+25\sqrt{2P}}{EA_2} + \frac{25\sqrt{2P}}{EA_3} + \frac{15\sqrt{2P}}{EA_4} + \frac{12.5P}{EA_5} \quad (18)$$

The average stiffness of the sample after sampling gave a 6-point displacement value of 173.9 mm.

6. Conclusion

In order to improve the efficiency of structural health detection of truss structures using Bayesian updating, that is, the sensitivity of Bayesian updating, this paper conducted multiple groups of tests and data comparisons by applying different loads. The optimal load application method was obtained. Bayesian inference is used to try to estimate the displacement change after applying a larger load to the structure. The main conclusions are as follows:

(1) Through the posterior comparison diagrams of each group, it can be concluded that, assuming that the truss structure is regarded as a simply supported beam, when the overall shear force of a simply supported beam is 0, the upper and lower chords have the highest sensitivity, the overall bending moment is 0 and the web member has the highest sensitivity.

(2) By comparing the SS1 with the SS2, it can be concluded that the same load is applied, and the force applied to the entire truss is the same, the location of the nodes where the loads are applied also affects the sensitivity of the Bayesian updating.

(3) In actual tests, the more loads are applied at other locations, the better it is for exploring the effect of the location of applied loads on the sensitivity of Bayesian updating. Next, the truss structure should be simplified into a simply supported beam and an attempt should be made to explore the effect of the load application position on the Bayesian structure of the overall structure.

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