

Bayesian Parameter Inversion and Analysis of Planar Truss Structures Based on OpenSees

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Abstract: The safety performance of engineering structures is inherently influenced by diverse uncertain factors, including material property variability, geometric imperfection, and stochastic external loads. Quantifying these uncertainties' effects on structural responses forms a critical foundation for reliability assessment and risk mitigation. Effective structural performance evaluation not only prevents catastrophic engineering failures but also supports decision-making in maintenance strategies. This study proposes a probabilistic reliability assessment framework for planar steel truss structures through systematic integration of finite element modeling using OpenSees with Bayesian updating theory. The methodology establishes a bidirectional relationship between numerical simulations and observed structural behaviors through probabilistic parameter calibration. Case studies reveal two principal outcomes: (1) The Bayesian-based approach significantly reduces model parameter uncertainties, leading to enhanced accuracy in predicting structural displacement responses; (2) The implementation of combined horizontal-vertical loading conditions strengthens the nonlinear interaction between structural parameters and displacement characteristics, thereby improving the completeness of parameter identification. These findings demonstrate the method's capability of addressing multi-source uncertainties and complex loading patterns, offering valuable insights for structural health monitoring and performance optimization in practical engineering applications.

Keywords: Bayesian updating, Model updating, Parameter inversion, Reliability assessment, Rejection sampling

1. Introduction

As critical hubs in modern transportation networks, bridges directly impact socioeconomic activities and public safety. Research on the reliability of civil engineering structures focuses on quantifying uncertainties in structural failure events [1-3]. Uncertainties arise from incomplete knowledge of material properties, geometric parameters of members, and computational models [4]. Truss bridges are widely used in long-span engineering due to their lightweight design, high load-bearing capacity, and ease of construction. However, long-term service under environmental erosion, fatigue loads, and accidental hazards leads to material degradation and member damage. For instance, the 2022 Pittsburgh bridge collapse in the United States was caused by undetected fatigue damage in steel structures. Therefore, developing efficient and accurate parameter inversion methods for truss structures is essential for real-time monitoring and early warning. Bayesian theory integrates prior

knowledge with observational data to update the understanding of random variables, reducing uncertainties and enabling reliability updates based on measured data [5-10].

Traditional parameter identification methods, such as least squares and frequency domain analysis [11], rely on deterministic assumptions and struggle to quantify model errors and measure noise. Bayesian methods explicitly express posterior parameter distributions by fusing prior knowledge and observational data [12]. However, existing studies primarily focus on continuum structures (e.g., beams and plates), leaving a gap in Bayesian inversion for truss systems.

Recent advancements in Bayesian inversion have demonstrated its unique advantages in uncertainty quantification and model updating for structural health monitoring. In civil engineering, scholars have conducted innovative research on various damage modes. Wang developed a Bayesian dynamic updating model for corrosion damage in reinforced concrete structures, enabling time-dependent probabilistic assessment through multi-source data fusion [13]. Zhou proposed an improved particle swarm optimization-Bayesian model updating method for simultaneous damage localization, quantification, and uncertainty analysis [14]. In energy exploration, Xu established a fracture weakness inversion framework by combining Bayesian theory with azimuthal seismic responses in HTI media [15]. Wang introduced a nonparametric sparse Bayesian inversion method for real-time damage detection in structural health monitoring [16]. These studies expand the theoretical dimensions of Bayesian inversion and provide domain-specific solutions, marking its transition from generic algorithms to specialized engineering applications.

Despite these advancements, few studies address two-dimensional planar steel truss structures. This paper innovatively combines deterministic finite element analysis with probabilistic methods to propose a Bayesian parameter inversion framework for planar trusses using OpenSees. First, a parameterized truss finite element model is constructed in OpenSees. Second, Bayesian updating theory is introduced to establish a posterior probability model with member cross-sectional areas as inversion parameters, sampled via rejection sampling. Finally, posterior distributions are derived through statistical analysis. This method overcomes the limitations of traditional deterministic inversion by applying Bayesian uncertainty quantification to planar trusses, offering new theoretical insights for addressing model uncertainties and measurement noise in structural damage diagnosis. The proposed reliability assessment model provides probabilistic decision-making support for truss bridge health monitoring.

2. Bayesian updating method for planar truss damage

The Bayesian updating method employed in this study is applicable to planar steel truss members under idealized linear deformation states. When damage occurs in these members, the method enables the evaluation of structural damage under specific loading conditions, thereby providing guidance for assessing structural failure probabilities. However, constrained by the complexity of engineering structures—including uncertainties in actual structural parameters, inherent model biases, and collective errors arising from field displacement monitoring instruments and operational practices—discrepancies between predicted and real-world data are inevitable. Consequently, when applying the Bayesian updating method proposed herein for damage parameter inversion, critical parameters must be recalibrated according to practical conditions. This study first conducts Monte Carlo sampling to generate a large number of parameter samples from prior distributions. Subsequently, forward model computations based on these sampled parameters are performed to determine displacement response data at target structural nodes. Finally, rejection sampling is implemented to filter samples according to likelihood function values, thereby approximating the posterior distribution.

2.1. Bayesian theoretical

Given the numerous uncertainties associated with the extent of damage in engineering members, this study employs a Bayesian approach to update and invert damage parameters. In accordance with Bayesian theory, all unknown parameters are treated as random variables θ , with the standard deviation of observation errors denoted as σ_ε . The Bayesian method effectively integrates prior engineering knowledge with actual observational data to jointly drive the inversion and updating of parameters. Within the Bayesian framework, the posterior probability distribution $P(\theta, \sigma_\varepsilon|Y)$ represents the outcome of parameter updating. According to Bayes' theorem (Ang and Tang, 2007), the posterior probability distribution $P(\theta, \sigma_\varepsilon|Y)$ is formulated as:

$$P(\theta, \sigma_\varepsilon|Y) = \eta P(\theta, \sigma_\varepsilon) L(Y|\theta, \sigma_\varepsilon) \quad (1)$$

Here, $P(\theta, \sigma_\varepsilon|Y)$ incorporates the latest estimates derived from engineering experience and observational data, representing the final updated results of the parameters. In this study, it specifically denotes the probability distribution of the cross-sectional area coefficient A of the target member under the given observational displacement data. The term $P(\theta, \sigma_\varepsilon)$ reflects the prior distribution of the random variable θ (representing displacement information at the target measurement nodes) and the standard deviation of observation errors σ_ε , primarily embodying engineering assumptions about the initial distributions of unknown parameters. The likelihood function $L(Y|\theta, \sigma_\varepsilon)$ quantifies the consistency between the observed data and model predictions under the given parameters θ and σ_ε . The normalization constant η ensures that the posterior distribution $P(\theta, \sigma_\varepsilon|Y)$ adheres to the definition of a probability density function. As indicated by Equation (1), the prior distribution $P(\theta, \sigma_\varepsilon)$ and the likelihood function $L(Y|\theta, \sigma_\varepsilon)$ are two critical components in Bayesian updating and parameter inversion. The subsequent sections will elaborate on the formulation of the prior probability distribution and the construction of the likelihood function.

2.2. Prior probability distribution

Based on previous research findings, the cross-sectional area coefficient A of planar steel truss bridges is generally assumed to follow a uniform distribution over the interval $[0.2, 1.1]$. Consequently, the prior distribution $P(\theta)$ can be formulated as:

$$P(\theta) = \frac{1}{(1.1-0.2)^2} \cdot I_{[0.2, 1.1]}(\theta) \quad (2)$$

Here, $I_{[a,b]}(\theta)$ denotes the indicator function for the interval $[a, b]$, which equals 1 if $\theta \in [a, b]$ and 0 otherwise. In this study, the prior distribution represents the probabilistic characterization of the cross-sectional area coefficient for damaged members. The adoption of the uniform distribution $P(\theta)$ over $[0.2, 1.1]$, consistent with prior research, provides parameter samples for Monte Carlo sampling. Furthermore, while this work employs a uniform distribution for prior probability modeling, readers or engineers may select alternative distribution forms based on specific structural characteristics and engineering expertise to enhance the efficiency of Monte Carlo sampling.

2.3. Likelihood function

The construction of the likelihood function constitutes a pivotal component in the Bayesian updating and inversion of target parameters. In the methodology presented herein, the formulation of the likelihood function directly governs both the efficiency of rejection sampling and the accuracy of inversion outcomes. The likelihood function fundamentally quantifies the degree of agreement between observed data and model predictions under the given parameters θ and the standard deviation of observational errors σ_ε . In this study, the likelihood function $L(Y|\theta, \sigma_\varepsilon)$ represents the

probability of obtaining displacement data corresponding to samples of the cross-sectional area coefficient A under the measured displacement responses at structural nodes. Observational errors σ_ε are typically assumed to follow a normal distribution. The proposed method adopts a Gaussian error model, where discrepancies between observed data and model predictions conform to a normal distribution. Consequently, the likelihood function $L(Y|\theta, \sigma_\varepsilon)$ is explicitly expressed as:

$$L(Y|\theta, \sigma_\varepsilon) = \frac{1}{\sqrt{2\pi} \cdot 0.2} \prod_{i=1}^N \exp \left\{ -\frac{1}{2} * \frac{(s_i - x_i)^2}{(0.05 \cdot x_i)^2} \right\} \quad (3)$$

In the constructed likelihood function $L(Y|\theta, \sigma_\varepsilon)$, s_i represents the model-predicted values, which correspond to either nodal displacement data or Monte Carlo samples of the cross-sectional area coefficient in the proposed methodology. Here, x_i denotes the true observed values, specifically referring to the measured displacements or cross-sectional area coefficients in this study, and N signifies the total number of observational data points. The likelihood function $L(Y|\theta, \sigma_\varepsilon)$ is designed with a mean centered at the true observed values x_i , aiming to ensure that model predictions align closely with actual measurements. The standard deviation $0.05 \cdot x_i$ reflects a permissible error tolerance of 5% relative to the true values. Within this framework, both displacements and cross-sectional area coefficients are uniformly treated as observational data. Despite their differing dimensionalities, consistency in units is achieved through prior error normalization (5% rule), which standardizes discrepancies across heterogeneous physical quantities.

3. Engineering case study

In industrial zones and secondary freight rail lines across North America and Europe, standardized steel Warren truss bridges are commonly utilized to span rivers, ravines, or other railway tracks. These structures are typically designed as modified Warren trusses with verticals, where triangular diagonal members are interspersed with vertical support. Constructed entirely of steel using welded and riveted I-beams, such bridges span 30 to 50 meters, making them suitable for medium-span applications, and are predominantly deployed in dedicated freight rail lines, industrial logistics corridors, or rural highway bridges. This study adopts an industrial freight railway bridge of this type as the case study.

3.1. Engineering context

The Ruhr Industrial Region in western Germany serves as a vital industrial hub, historically renowned for coal mining, steel production, and heavy industries from the late 19th to early 20th century. To meet the demands for raw material transportation, the region developed an extensive railway network, with freight bridges acting as critical infrastructure linking mines, steel mills, and ports. Most of these railway bridges were constructed between 1900 and 1930, many of which remain operational for coal transport routes. The Emscher Railway Bridge, spanning the Emscher River and connecting the Dortmund and Essen mining areas, is a prominent example. Due to prolonged exposure to harsh environmental conditions and insufficient maintenance, partial degradation of material properties and structural damage have been observed in its lower members, including a decline in the elastic modulus of the steel components. To quantify and inversely assess the damage extent in the bridge members, this study involves applying fixed loads to the structure while monitoring nodal displacements and member strain data.

3.2. Model development

To facilitate the study, the structural configuration of the Emscher Bridge is simplified into a two-dimensional planar truss model, as illustrated in Fig 1.

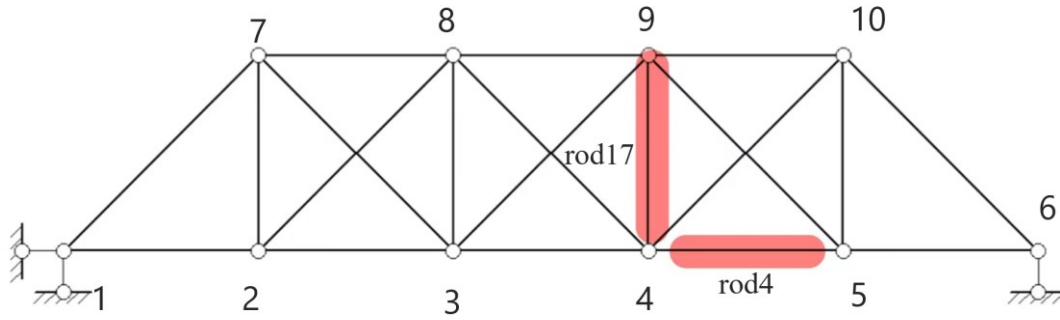


Figure 1: Schematic diagram of simplified 2D plane truss model of Emscher Railway Bridge

As the bridge is a statically indeterminate structure, a steel truss model was established using the finite element analysis software OpenSees for uncertainty analysis. The bridge model features uniform nodal spacings of 2 meters in both horizontal and vertical directions, with the two-dimensional planar truss configuration measuring 2 meters in height and 10 meters in width. Node 1, located at the bridge end, is designed as a fixed support providing full constraints in both horizontal and vertical directions. Node 6, serving as the critical support point, incorporates a sliding support that imposes vertical constraint while permitting horizontal displacement along the bridge axis. All remaining nodes in the bridge model employ pinned connections that allow free rotational movement of structural members at the joints. A finite element analysis was performed on the simplified two-dimensional planar truss model using OpenSees to obtain displacement response data at various nodes under fixed loading patterns.

3.3. Parameter settings of bridge model

To perform Bayesian updating and inversion for member damage identification in the bridge and validate the feasibility of the proposed methodology, this study assumes a priori that the damaged members are specifically designated as Rod 4 and Rod 17. The damage severity is characterized by a cross-sectional area reduction coefficient, where structural deterioration manifests as a decrease in this coefficient. The relationship between the current cross-sectional area and the reduction coefficient is defined as:

$$Area = A \cdot Area_0 \quad (4)$$

where $Area$ represents the current cross-sectional area of the member, A denotes the cross-sectional area reduction coefficient, and $Area_0$ corresponds to the designed cross-sectional area. Based on field measurements, the reduction coefficients are determined as $A_4=0.6$ for Rod 4 and $A_{17}=0.7$ for Rod 17. Displacement response data at all nodes under fixed loading patterns are acquired through displacement sensors. For analytical simplicity, only the linear elastic strain phase of members is considered, with a uniform elastic modulus $E = 10^5$ Pa and material yield strength $\sigma_y = 7 \times 10^9$ pa assigned to all members.

4. Result analysis

This study employs two distinct fixed loading patterns for Bayesian updating and inversion of bridge model parameters.

4.1. Inversion results and analysis under loading pattern 1

The first loading pattern involves applying a vertically downward concentrated force of 100KN at Node 9 of the bridge model, as illustrated in Fig. 2. The vertical displacement response data of Nodes 3, 4, 5, 8, 9, and 10, measured via displacement sensors, are summarized in Table 1.

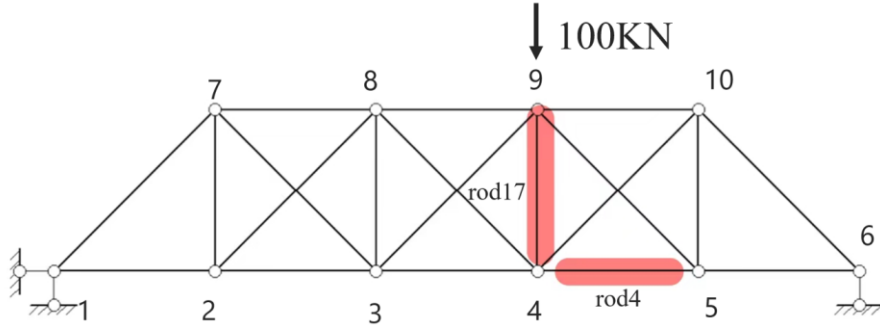


Figure 2: Diagram of the first loading mode

Table 1: Node vertical displacement data under the first loading mode

Node	Vertical displacement(m)
3	0.01364044673567225
4	0.01483558563597397
5	0.01075701563757633
8	0.01353227551986932
9	0.01622147769861779
10	0.01015542369332102

Under Loading Pattern 1, a Monte Carlo random sampling of 1,000,000 datasets was performed based on the prior distribution $P(\theta)$ of the cross-sectional area reduction coefficients. Through rejection sampling utilizing the established likelihood function $L(Y|\theta, \sigma_\epsilon)$, 6,543 accepted samples were obtained, yielding an acceptance rate of 0.6543%.

The cross-sectional area reduction coefficients of Rod 4 and 17 in the accepted samples were statistically analyzed. The inversion results for the cross-sectional area reduction coefficient of Rod 4 are presented in Fig. 3, while those for Rod 17 are shown in Fig. 4.

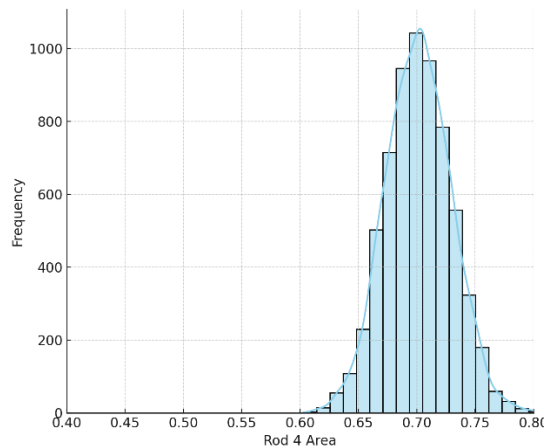


Figure 3: Rod 4 cross-sectional area coefficient frequency histogram

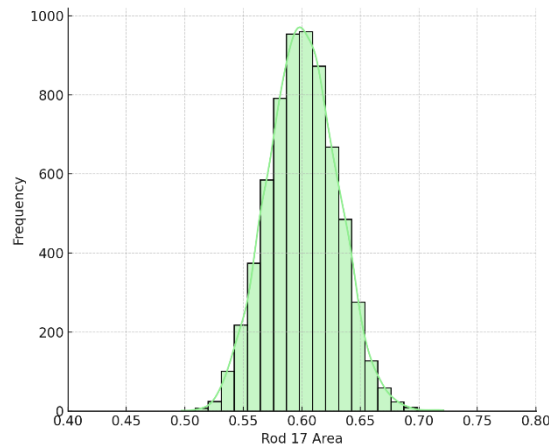


Figure 4: Rod 17 cross-sectional area coefficient frequency histogram

Statistical analysis of posterior samples revealed that the cross-sectional area reduction coefficient for Rod 4 exhibited a mean value of 0.7015 with a standard deviation of 0.0284, while Rod 17 demonstrated a mean value of 0.6005 and a standard deviation of 0.0293. The posterior mean values closely approximate the empirically measured cross-sectional area reduction coefficients, with standard deviations remaining below 5% of the measured values, consistent with theoretical expectations.

4.2. Inversion results and analysis under loading pattern 2

The study further implemented a second loading pattern to replicate the aforementioned procedures. In this configuration, a combined force of 100 kN in both vertical and horizontal directions was applied simultaneously to Node 9 of the bridge model, as depicted in Fig. 5. Subsequently, horizontal and vertical displacement response data for Nodes 3, 4, 5, 8, 9, and 10 were measured using displacement sensors and are tabulated in Table 2.

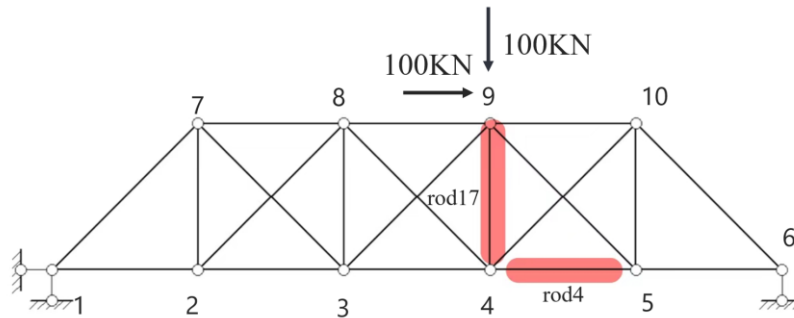


Figure 5: Diagram of the second loading mode

Table 2: Horizontal and vertical displacement data of nodes under the second loading mode

Node	Horizontal displacement(m)	Vertical displacement(m)
3	0.00477301851773672	0.01599245925997595
4	0.00776494560928651	0.01743731607813324
5	0.01117379331343185	0.01312112064293080
8	0.00838622897854506	0.01622751365068937
9	0.00737815607009490	0.01914044860404729
10	0.00496434946299666	0.01233492725002905

Under Loading Pattern 2, a Monte Carlo random sampling of 1,000,000 datasets was conducted based on the prior distribution $P(\theta)$ of the cross-sectional area reduction coefficients. Rejection sampling using the established likelihood function $L(Y|\theta, \sigma_\epsilon)$ yielded 7,367 accepted samples, corresponding to an acceptance rate of 0.7367%.

Statistical analysis of the cross-sectional area reduction coefficients for Rod 4 and 17 within the accepted samples was performed. The inversion results for the cross-sectional area reduction coefficient of Rod 4 are illustrated in Fig. 6, while those for Rod 17 are presented in Fig. 7.

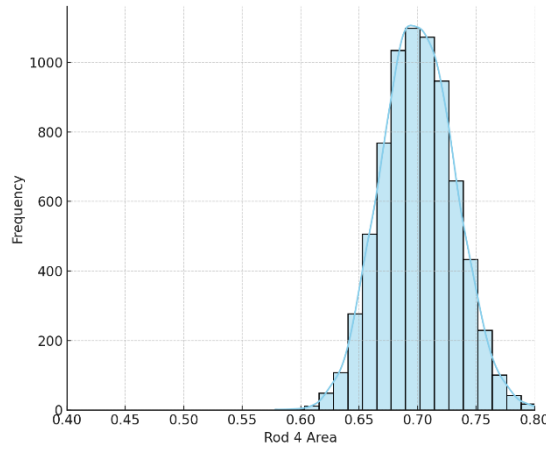


Figure 6: Rod 4 cross-sectional area coefficient frequency histogram

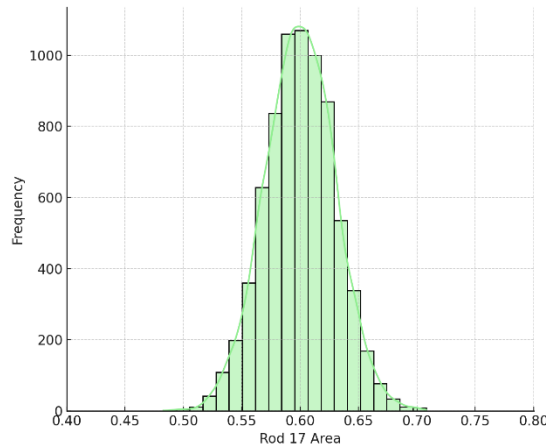


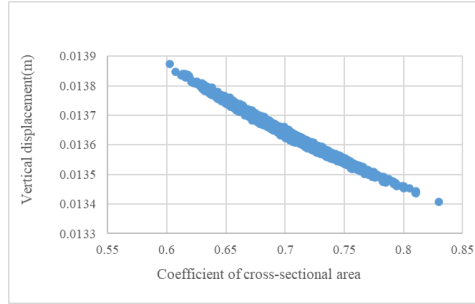
Figure 7: Rod 17 cross-sectional area coefficient frequency histogram

Statistical analysis of posterior samples demonstrated that the cross-sectional area reduction coefficient for Rod 4 exhibited a mean value of 0.7003 with a standard deviation of 0.0313, while Rod 17 showed a mean value of 0.5999 and a standard deviation of 0.0299. The posterior mean values align closely with the empirically measured cross-sectional area reduction coefficients, and the standard deviations remain below 5% of the measured values, consistent with theoretical expectations.

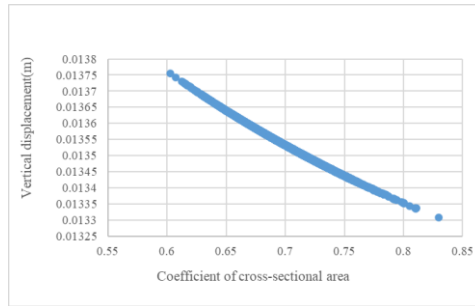
Compared to Loading Pattern 1, the combined loading configuration resulted in an increased acceptance rate during rejection sampling, with posterior mean values exhibiting enhanced proximity to the true coefficients. However, a slight increase in the standard deviations of posterior samples was observed.

4.3. Sensitivity of loading patterns to nodal displacements

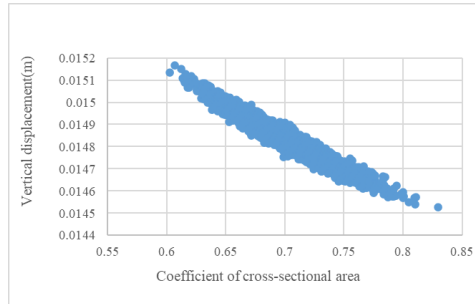
Under Loading Pattern 1, a correlation analysis was conducted between the posterior cross-sectional area reduction coefficients and the vertical displacements of all nodes. Scatter plots illustrating the correlation distribution for Rod 4 are presented in Fig. 8, while those for Rod 17 are shown in Fig. 9.



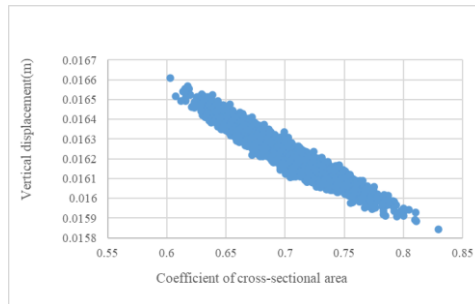
(a) Scatter plot of vertical displacement at Node 3 versus cross-sectional area reduction coefficient for Rod 4



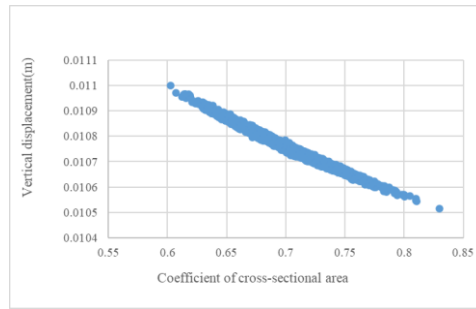
(b) Scatter plot of vertical displacement at Node 8 versus cross-sectional area reduction coefficient for Rod 4



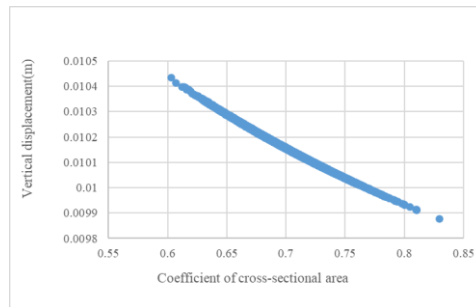
(c) Scatter plot of vertical displacement at Node 4 versus cross-sectional area reduction coefficient for Rod 4



(d) Scatter plot of vertical displacement at Node 9 versus cross-sectional area reduction coefficient for Rod 4

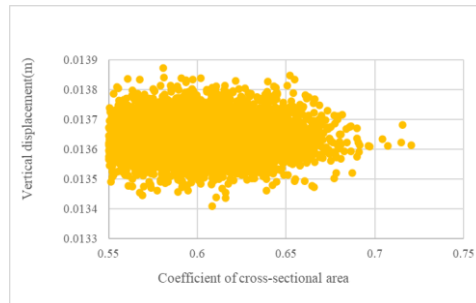


(e) Scatter plot of vertical displacement at Node 5 versus cross-sectional area reduction coefficient for Rod 4

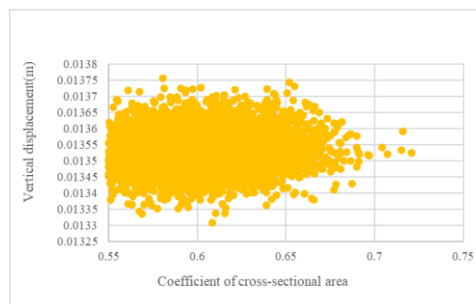


(f) Scatter plot of vertical displacement at Node 10 versus cross-sectional area reduction coefficient for Rod 4

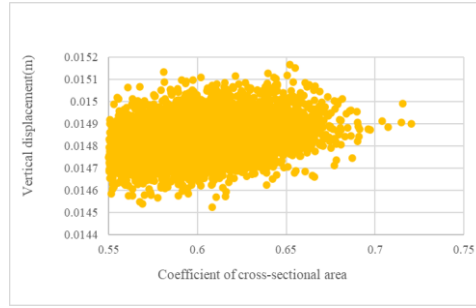
Figure 8: Scatter diagram of correlation between section coefficient of rod 4 and vertical displacement of each node



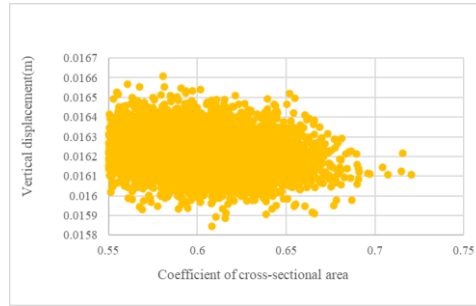
(a) Scatter plot of vertical displacement at Node 3 versus cross-sectional area reduction coefficient for Rod 17



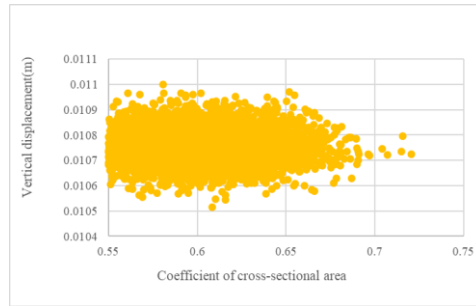
(b) Scatter plot of vertical displacement at Node 8 versus cross-sectional area reduction coefficient for Rod 17



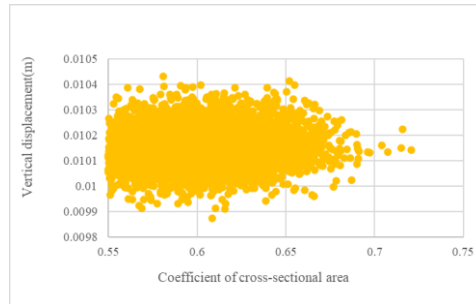
(c) Scatter plot of vertical displacement at Node 4 versus cross-sectional area reduction coefficient for Rod 17



(d) Scatter plot of vertical displacement at Node 9 versus cross-sectional area reduction coefficient for Rod 17



(e) Scatter plot of vertical displacement at Node 5 versus cross-sectional area reduction coefficient for Rod 17



(f) Scatter plot of vertical displacement at Node 10 versus cross-sectional area reduction coefficient for Rod 17

Figure 9: Scatter diagram of correlation between section coefficient of rod 17 and vertical displacement of each node

Rod 4 demonstrated significant correlations between its posterior cross-sectional area reduction coefficients and vertical displacements at all nodes, exhibiting linear distributions. In contrast, Rod 17 exhibited weak correlations between its cross-sectional area coefficients and nodal vertical

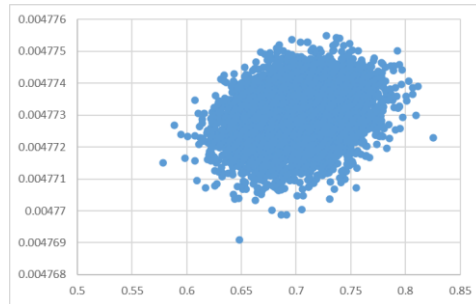
displacements. The correlation coefficients between structural members and vertical nodal displacements are summarized in Table 3.

Table 3: Correlation coefficient between section coefficient and vertical displacement of each node

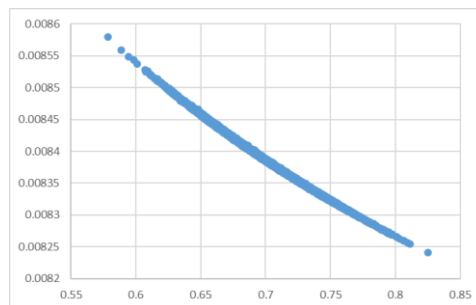
Parameter	Node3	Node4	Node5	Node8	Node9	Node10
A4	-0.9952	-0.968	-0.9931	-0.9987	-0.9544	-0.9986
A17	0.0127	0.3382	-0.0098	0.1005	-0.2009	0.0813

When the cross-sectional area reduction coefficient of Rod 4 increases, the actual cross-sectional area enlarges, leading to a reduction in nodal displacements. A significant negative correlation is observed between the reduction coefficient and vertical displacements at all measured nodes. This indicates that Rod 4 provides critical load-bearing support, and enhancing its strength effectively mitigates nodal displacements. As shown in Fig. 8, Rod 4 acts as the primary load-carrying component under Loading Pattern 1, suggesting that optimizing its dimensions could improve structural stiffness. However, adjusting its dimensions yields no significant improvement in target responses, implying lower priority in design modifications. In contrast, under the same loading pattern, Rod 17 exhibits negligible correlation between its cross-sectional area coefficient and vertical nodal displacements, demonstrating minimal influence on displacement responses.

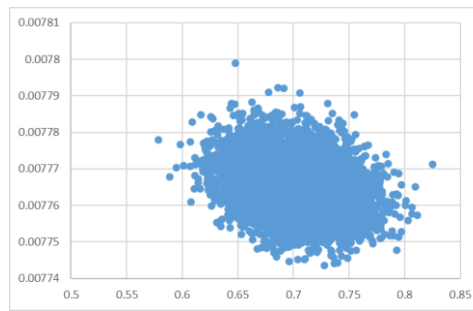
Subsequent analysis under the second combined loading pattern involved correlation statistics between posterior cross-sectional area reduction coefficients of Rod 4 and 17 and both vertical and horizontal displacements of all nodes. Analytical investigation revealed the scatter plot distributions for Rod 4 in Fig. 10 and Rod 17 in Fig. 11.



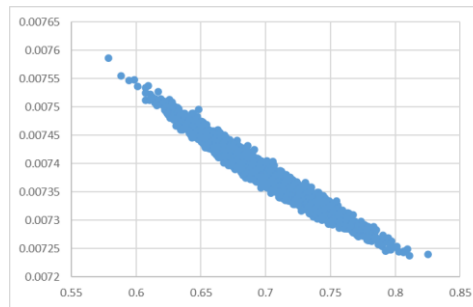
(a) Scatter plot of horizontal displacement at Node 3 versus cross-sectional area reduction coefficient for Rod 4



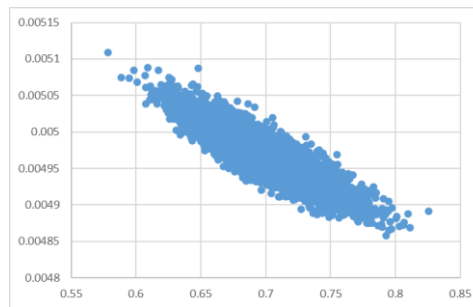
(b) Scatter plot of horizontal displacement at Node 8 versus cross-sectional area reduction coefficient for Rod 4



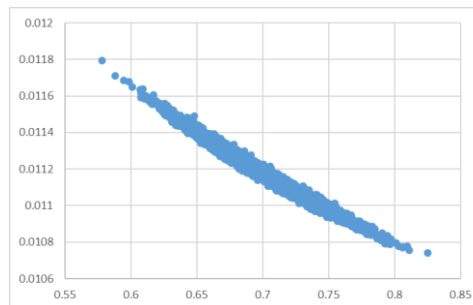
(c) Scatter plot of horizontal displacement at Node 4 versus cross-sectional area reduction coefficient for Rod 4



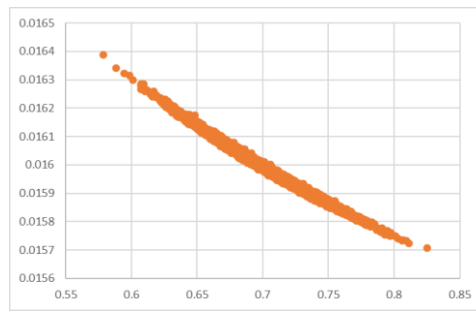
(d) Scatter plot of horizontal displacement at Node 9 versus cross-sectional area reduction coefficient for Rod 4



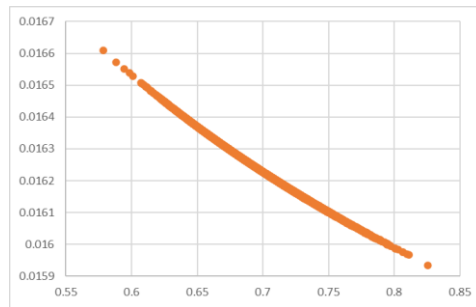
(e) Scatter plot of horizontal displacement at Node 5 versus cross-sectional area reduction coefficient for Rod 4



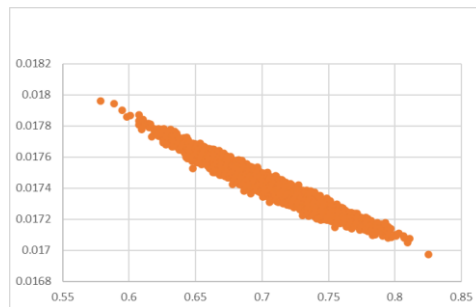
(f) Scatter plot of horizontal displacement at Node 10 versus cross-sectional area reduction coefficient for Rod 4



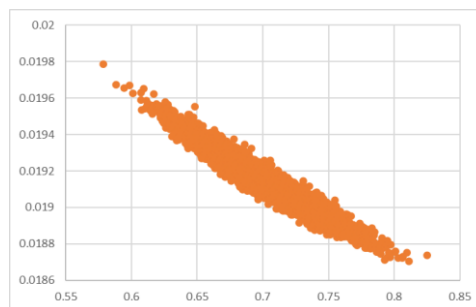
(g) Scatter plot of vertical displacement at Node 3 versus cross-sectional area reduction coefficient for Rod 4



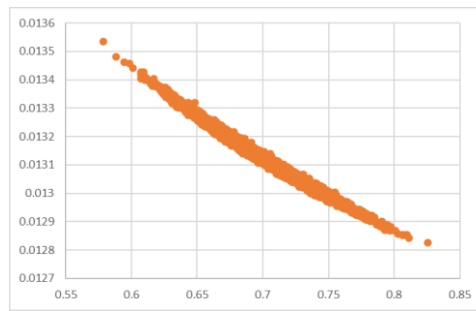
(h) Scatter plot of vertical displacement at Node 8 versus cross-sectional area reduction coefficient for Rod 4



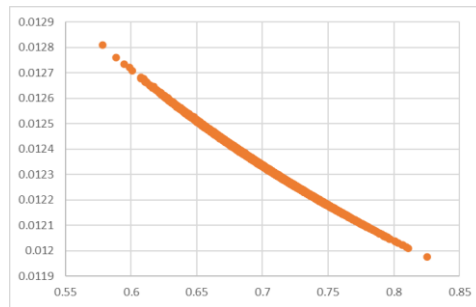
(i) Scatter plot of vertical displacement at Node 4 versus cross-sectional area reduction coefficient for Rod 4



(j) Scatter plot of vertical displacement at Node 9 versus cross-sectional area reduction coefficient for Rod 4

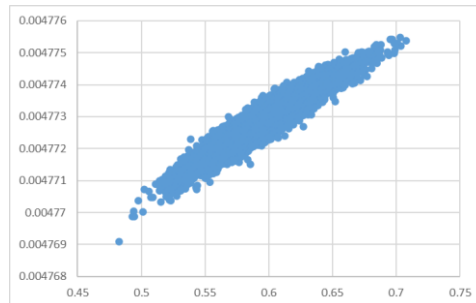


(k) Scatter plot of vertical displacement at Node 5 versus cross-sectional area reduction coefficient for Rod 4

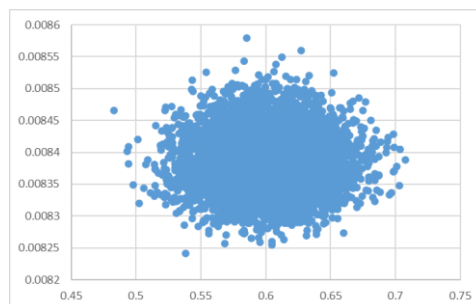


(l) Scatter plot of vertical displacement at Node 10 versus cross-sectional area reduction coefficient for Rod 4

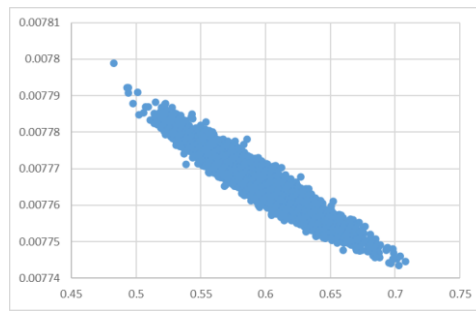
Figure 10: Scatter diagram of correlation between section coefficient of rod 4 and displacement of each node



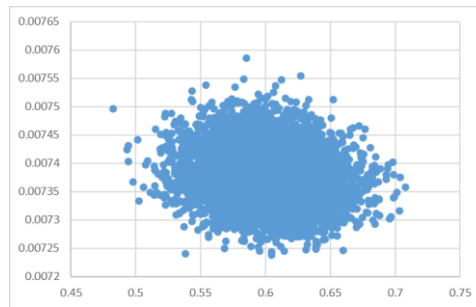
(a) Scatter plot of horizontal displacement at Node 3 versus cross-sectional area reduction coefficient for Rod 17



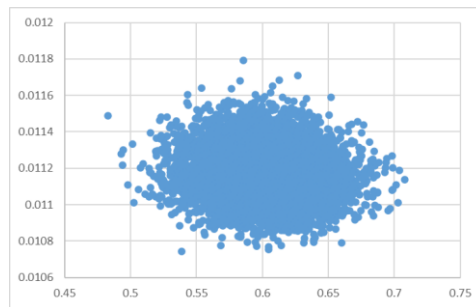
(b) Scatter plot of horizontal displacement at Node 8 versus cross-sectional area reduction coefficient for Rod 17



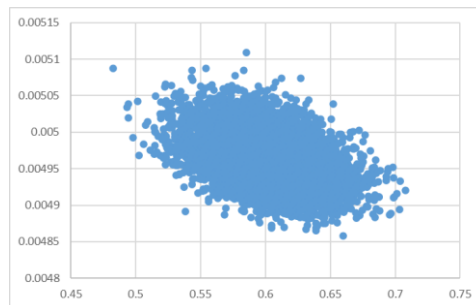
(c) Scatter plot of horizontal displacement at Node 4 versus cross-sectional area reduction coefficient for Rod 17



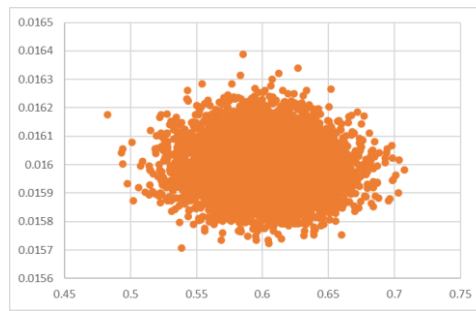
(d) Scatter plot of horizontal displacement at Node 9 versus cross-sectional area reduction coefficient for Rod 17



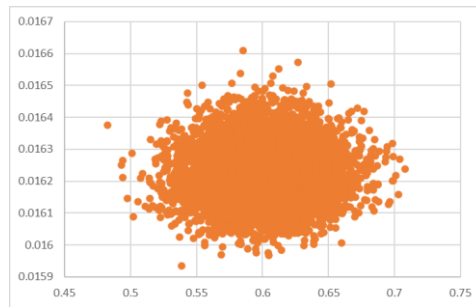
(e) Scatter plot of horizontal displacement at Node 5 versus cross-sectional area reduction coefficient for Rod 17



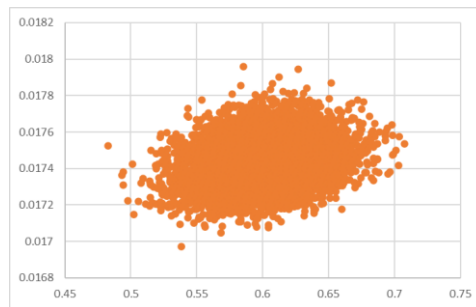
(f) Scatter plot of horizontal displacement at Node 10 versus cross-sectional area reduction coefficient for Rod 17



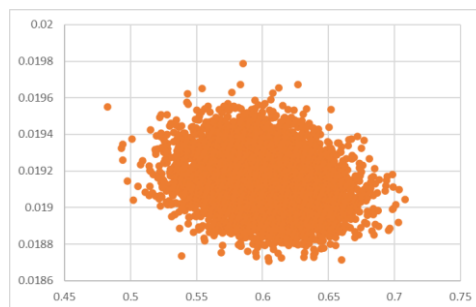
(g) Scatter plot of vertical displacement at Node 3 versus cross-sectional area reduction coefficient for Rod 17



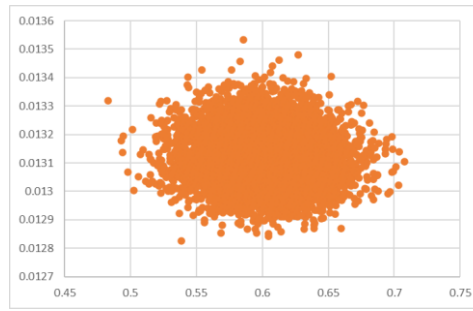
(h) Scatter plot of vertical displacement at Node 8 versus cross-sectional area reduction coefficient for Rod 17



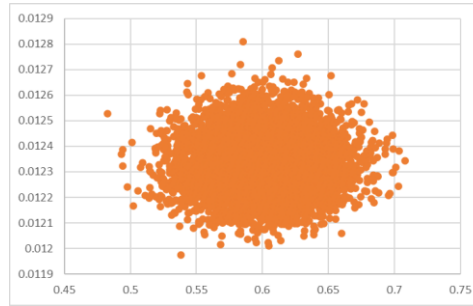
(i) Scatter plot of vertical displacement at Node 4 versus cross-sectional area reduction coefficient for Rod 17



(j) Scatter plot of vertical displacement at Node 9 versus cross-sectional area reduction coefficient for Rod 17



(k) Scatter plot of vertical displacement at Node 5 versus cross-sectional area reduction coefficient for Rod 17



(l) Scatter plot of vertical displacement at Node 10 versus cross-sectional area reduction coefficient for Rod 17

Figure 11: Scatter diagram of correlation between section coefficient of rod 17 and displacement of each node

Statistical analysis revealed that Rod 4 exhibited significant negative correlations between its cross-sectional area reduction coefficient and horizontal displacements at Nodes 5, 8, 9, and 10, while no discernible correlations were observed with horizontal displacements at Nodes 3 and 4. For Rod 17, under the combined loading pattern, its cross-sectional area coefficient demonstrated distinct correlations with horizontal displacements at Nodes 3 and 4. Specifically, the horizontal displacement at Node 3 increased with the cross-sectional area coefficient, indicating a positive correlation, whereas at Node 4, the horizontal displacement decreased with increasing cross-sectional area coefficient, showing a significant negative correlation.

Comparative analysis highlights that under vertical-only loading (Pattern 1), vertical displacements were predominantly governed by members resisting vertical forces, such as Rod 4 acting as the primary load-bearing component. Conversely, under combined vertical-horizontal loading (Pattern 2), horizontal forces activated lateral load-transfer mechanisms, enhancing the correlation between Rod 17's cross-sectional area coefficient and horizontal displacements. This suggests that Rod 17's stiffness is critical for resisting lateral deformation. Simultaneously, Rod 4's cross-sectional area coefficient correlated with both vertical and horizontal displacements, reflecting its multi-directional load-transfer capability under combined loading conditions.

Synthesis of correlation statistics from both loading patterns indicates that under single-axis vertical loading, Rod 4 exhibited highly concentrated sensitivity to target displacements, with low structural redundancy and pronounced influence of parameter uncertainty on responses. In contrast, under combined loading, sensitivity became distributed across multiple members, where concurrent parameter variations jointly influenced nodal displacements. This redistribution reduced the dominance of single-parameter effects, enhanced structural robustness, and improved tolerance to localized uncertainties through load-path diversification.

5. Conclusions

To address the requirements for structural safety assessment of bridges, this study proposes a truss bridge damage identification method integrating numerical simulation and probabilistic statistics, based on the OpenSees nonlinear finite element analysis platform and Bayesian theory. The methodology achieves probabilistic inversion of damage parameters by fusing numerical simulation data with field measurements. Key conclusions are summarized as follows:

(1) The consistency between empirically measured cross-sectional area coefficients and posterior mean values demonstrates the method's multi-source data fusion capability. Even under the influence of 5% standard deviation in observational errors, the framework maintains high parameter inversion accuracy. The probabilistic assessment results provide quantitative decision-making support for bridge structural health monitoring.

(2) Sensitivity analysis reveals the stiffness contribution of structural members under varying loading conditions, with distinct members influencing displacement responses in different directions. This characteristic establishes a direct basis for structural parameter prioritization, design optimization, and uncertainty quantification. For damage identification, this property enables adaptive loading pattern design to enhance information screening efficiency and optimize Bayesian updating performance.

(3) Combined horizontal-vertical loading activates multi-directional load-transfer mechanisms, generating richer response data that enables Bayesian inversion to comprehensively capture nonlinear relationships between parameters and displacement metrics. This approach improves the comprehensiveness and reliability of parameter identification while offering precise guidance for structural optimization under complex loading scenarios. Additionally, it enhances rejection sampling efficiency and reduces solution multiplicity in inversion outcomes.

(4) The low acceptance rate (0.65%–0.74%) observed in the rejection sampling method employed in this study highlights limitations in sampling efficiency. To address this, Markov Chain Monte Carlo (MCMC) algorithms could be adopted in subsequent research to enhance sampling performance and refine inversion outcomes.

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