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Dynamic Modeling of Rolling Bearings under Dynamic Contact Conditions

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Abstract. This research investigates the dynamic characteristics of rolling bearings under time-varying contact conditions, proposing a dynamic contact model integrating periodic damping and stiffness variations. Leveraging Hertz contact theory, the static stiffness formulation between rolling elements and raceways is derived and extended to a time-dependent stiffness framework. Combined with the fault excitation model of the local defect of the rolling bearing, a 2-degree-of-freedom dynamic equation including dynamic damping, variable stiffness and fault additional displacement was constructed. The motion equations were solved by the Runge-Kutta method, and the vibration responses of defect-free and defective bearings under static and dynamic contact conditions were compared and analyzed. The results show that the dynamic model can more accurately reflect the fluctuation characteristics of vibration displacement and acceleration; With defects included, the amplitude of vibration acceleration increased significantly to 150 m/s², verifying the effectiveness of the acceleration sensor in bearing health monitoring. This study provides theoretical support for dynamic characteristic analysis and fault diagnosis of rolling bearings.

Keywords: Rolling bearings, Dynamic contact, Dynamic model, Vibration

1. Introduction

Rolling bearings are the core components of rotating machinery, and their health directly affects the reliability and lifespan of the equipment's operation. As the bearings run for a long time, they tend to develop defects such as cracks and pitting [1]. If local faults of the bearing can be detected early, repairs can be arranged to avoid catastrophic accidents and reduce economic losses. Therefore, fault diagnosis of bearings is of great significance to practical application and production.

Some research has been conducted on the monitoring and modeling of bearing faults. Liu et al. [2] used piecewise functions to simulate the additional displacement of local faults and quadratic functions to fit the local deformation at the defect edge. The results showed that the defect width had little effect on the acceleration waveform. Cao et al. [3] discussed the effects of single and multi-point defects on the vibration of cylindrical rollers. Atef et al. [4] modeled the conventional rectangular defect as an arc-shaped defect and used a semi-sine function to simulate displacement variations. Li et al. [5] analyzed the multi-point failure of the outer ring of the ball bearing and represented the impact of the failure using the axis trajectory diagram. Cui et al. [6] simulated the

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effect of bearing vibration variation under local defect propagation and verified the accuracy of the model by comparing it with the signals from the loading degradation test. Petersen et al. [7] defined the local defect displacement as a constant and estimated the defect size using the response signal under the influence of the local defect. Qin et al. [8] considered the coupling effect of the rolling element in the defect on the normal rolling element. Sawalhi et al. [9] analyzed the local fault double pulse phenomenon and simulated the pulse using the signal. In addition, some researchers have also discussed the effects of bearings on systems such as RV reducers [10], inner and outer ring corrugations [11], and rotor systems [12]. However, prior research predominantly assumes fixed contact parameters, neglecting the time-varying nature of stiffness and damping, thereby introducing discrepancies between theoretical models and real-world operating conditions.

In response to the above issues, a dynamic model of raceway bearings under dynamic contact conditions was established, and the dynamic responses of rolling bearings under normal operating conditions and local fault conditions were discussed respectively. It deepens the theoretical understanding of the dynamic behavior of rolling bearings and provides practical solutions for fault warning and health management in engineering practice.

2. Dynamic modeling method

2.1. Dynamic contact damping and stiffness model

The contact damping and stiffness inside the bearing are not fixed constants during periodic rotation, so in dynamic contact conditions, the fixed damping and stiffness are changed to dynamic contact damping and stiffness. Dynamic contact damping can be expressed as

$$\mathrm{c_d}\left(\mathrm{t}\right) = \mathrm{c_s} + \mathrm{A_c}\sin\left(2\pi\,\mathrm{f_c}\mathrm{t}\right) \tag{1}$$

In the formula: $c_d\left(t\right)$ is damping under dynamic contact conditions, c_s is damping under static contact conditions, A_c and f_c are the amplitude and frequency of the damping wave, respectively.

For ball bearings, the contact form between the ball and the inner and outer raceways of the bearing is ball-ball contact form. According to Hertz contact theory, the static contact stiffness between the inner and outer raceways of a ball-bearing can be expressed as

$$K_{i/o} = \left(\frac{\pi^2 k^2 E_h^2 \Sigma}{4.5\Gamma..^3\Sigma_\rho}\right) \tag{2}$$

In the formula: K_i and K_o are the static contact stiffness between the rolling elements and the inner and outer raceways, respectively; k, Γ and Σ are the elliptic parameters and the first and second types of full elliptic integrals, respectively; E_h Is the equivalent stiffness; $\sum\rho$ is the sum of the curvatures of the contact pair.

The contact between the rolling elements and the inner and outer raceways is simplified as a spring damping system, as shown in Figure 1. The expression for the total static contact stiffness between the ball and the inner and outer raceways of the normal bearing is

$$K_s = \frac{1}{\left[\frac{1}{K_i^n} + \frac{1}{K_o^n}\right]} \tag{3}$$

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In the formula: n is the load-deformation index. For ball bearings, n is taken as 1.5.

Conventional stiffness models usually fix the contact stiffness at a fixed value. This paper proposes adding a time variable to the stiffness model. The time-varying stiffness model is formulated as

$$K_d(t) = K_S + A_K \sin(2\pi f_K t)) \tag{4}$$

In the formula: $K_d(t)$ is the stiffness under dynamic contact conditions, K_s is the stiffness under static contact conditions, A_K and f_K are the amplitude and frequency of stiffness fluctuation, respectively.

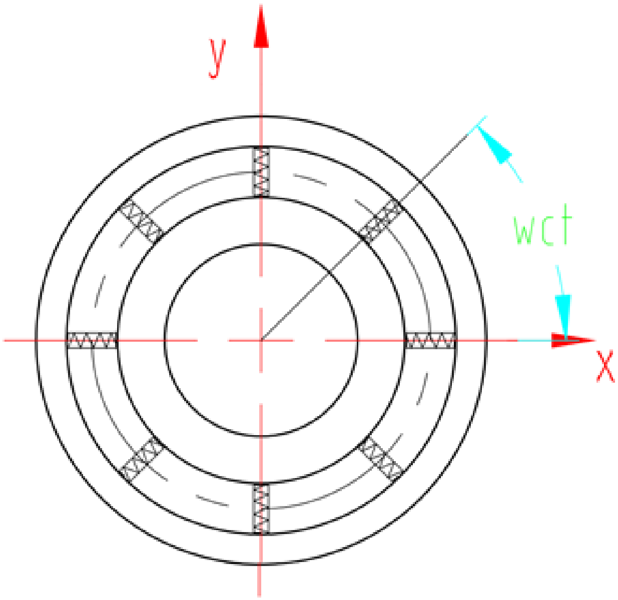

Figure 1: A simple diagram of the spring-damping of a rolling bearing.

2.2. Fault excitation model

When the bearing operates for a long time, the raceways of the bearing are affected by fatigue cyclic loads, resulting in spalling damage on the raceway surface. To investigate the impact of spalling damage on the bearing's operating signals, a fault excitation model needs to be established to enable the mechanism analysis of local faults. In this paper, the local defect is simplified to a rectangular shape, as shown in Figure 2.

When the rolling elements pass through the local defect, the rolling elements will deviate from the normal motion track, and this deviation distance is called the additional displacement. Defect-induced displacement is modeled using a semi-sinusoidal function, which can be written as

$$H(\theta_j) = H_d \sin\left(\frac{\theta_j - \theta_0}{\theta_d}\pi\right) \tag{5}$$

Where: H_d is the depth of the local defect, θ_0 is the entry angle of the local defect, θ_j is the position angle of the JTH rolling element relative to the center of the rolling element, θ_d is the

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circumferential angle of the raceway where the local defect is located, whose value is determined by the length of the rolling element and can be expressed as

$$\theta_d = arcsin\left(\frac{L_d}{d_o}\right) \tag{6}$$

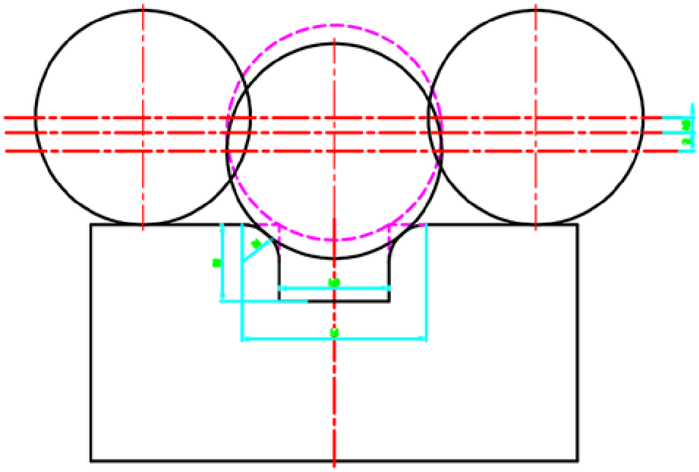

Figure 2. Local damage stiffness.

2.3. Establish the equations of motion

A 2-degree-of-freedom dynamic model of the ball bearing is established by integrating dynamic damping of the bearing, variable stiffness and local faults on the raceway surface. The differential equation of the bearing motion can be expressed as

$$\begin{cases} m\ddot{x} + c_d\dot{x} + F_x = W_x \\ m\ddot{y} + c_d\dot{y} + F_y = W_y \end{cases} \tag{7}$$

Where m is the mass of the inner ring of the bearing and the rotating shaft, W_x and W_y are the external forces of the inner ring of the bearing in the x and y directions respectively, F_x and F_y are the components of the contact force between the rolling elements and the raceway in the x and y directions respectively, which can be expressed as

$$\begin{cases} F_x = \sum_{j=1}^{N_b} K_d \lambda_i \delta_j^n \cos\theta_j \\ F_y = \sum_{j=1}^{N_b} K_d \lambda_i \delta_j^n \sin\theta_j \end{cases} \tag{8}$$

In the formula: N_b is the number of rolling elements; λ_i used to determine whether the scroll enters the local defect area, if it enters the defect area, $\lambda_i = 1$; then, if it does not enter the defect area, $\lambda_i = 0$. δ_j denote the contact deformation of the JTH rolling element, which can be expressed as

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$$\delta_j = x\cos\theta_j + y\sin\theta_j - C_r - H \tag{9}$$

3. Result analysis

To obtain the vibration response signal of the rolling bearing, the Runge-Kutta method was used to solve the motion differential equation. The simulation step size is. $\Delta t = 10^{-6}\,s$ The inner ring of the bearing rotates with the spindle, the outer ring is fixed, the rotational speed of the inner ring is, the initial displacement and velocity of the inner ring are 0 m and 0 m/s, respectively. $n = 2\times10^{3}\,rpm$. The geometric parameters of the rolling bearing are shown in Table 1.

Table 1. Geometric Parameters of Rolling Bearings.

Parameter	Value	Parameter	Value
Inner raceway diameter(mm)	49.912	Outer raceway diameter(mm)	80.088
Pitch circle diameter(mm)	65	Rolling element diameter(mm)	15.081
Number of scroll elements	8	Radial clearance of the raceway(μm)	1
Quality of the inner ring and the rotating shaft(Kg)	0.6	Contact Angle(°)	0

3.1. Defect-free bearing vibration response comparison

Select the rolling bearing parameters given above for the experiment and compare the bearing's response under static and dynamic contact conditions. The stiffness and damping under dynamic contact conditions compared to static contact conditions are shown in Figure 3.

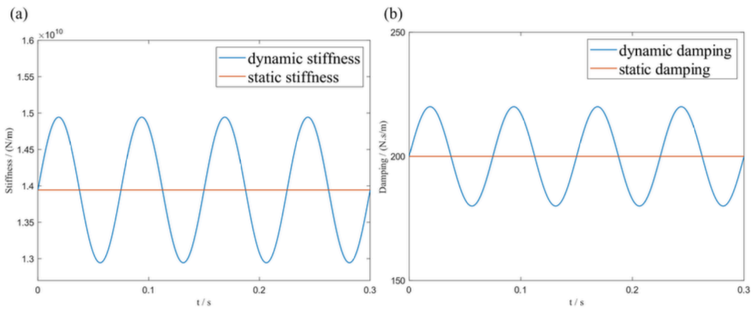

Figure 3. (a) Stiffness comparison, (b) damping comparison.

Solve the differential equations of motion to obtain the vibration displacement and acceleration of the bearing under normal operating conditions without defects, as shown in Figures 4 and 5. After the operation stabilizes, it can be seen that the amplitudes of vibration displacement and acceleration under dynamic contact conditions fluctuate, while the amplitudes of vibration under static conditions remain stable. Therefore, under defect-free conditions, dynamic stiffness and damping are closer to the actual situation.

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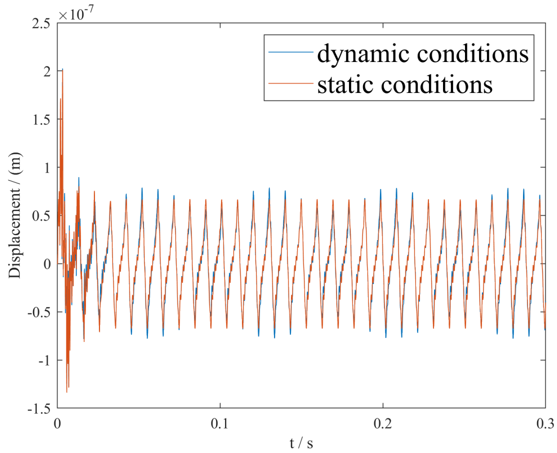

Figure 4. Defect-free bearing vibration displacement.

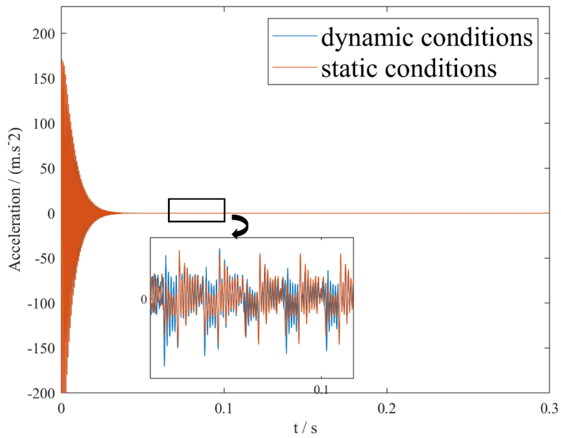

Figure 5. Vibration acceleration of the defect-free bearing.

3.2. Comparison of vibration responses of bearings with defects

Select a local fault with a length of 0.5mm, a width of 0.2mm, and a defect center Angle at 135 degrees from the X-axis. Vibration displacements and accelerations of the bearing with local defects are shown in Figures 6 and 7. Similarly, compared with the response signal under the static contact condition, the signal under the dynamic contact condition presents a fluctuating form. When compared with the vibration signal of the defect-free bearing, it was found that the amplitude of the vibration displacement changed little, while the amplitude of the vibration acceleration increased to 150 m/s^2 when local defects occurred. Therefore, to monitor the operational health status of the bearing, the monitoring effect of the acceleration sensor is better.

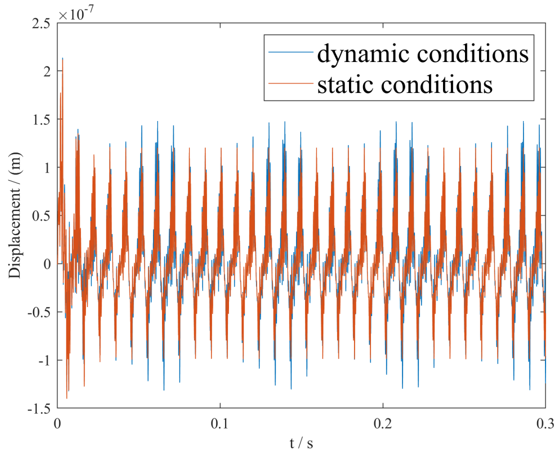

Figure 6. Vibration displacement of the bearing with defects.

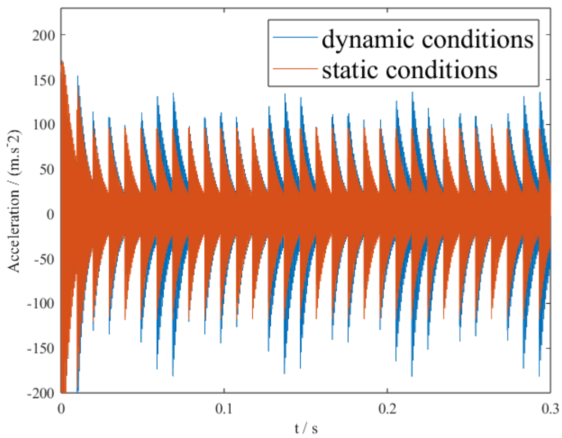

Figure 7. Vibration acceleration of the bearing with defects.

4. Conclusion

This paper systematically analyzes the vibration response characteristics of rolling bearings under dynamic contact conditions by establishing dynamic contact damping, stiffness models and fault

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excitation models and inputting them into the bearing dynamics equation. The simulation results show that the dynamic model can describe the nonlinear vibration behavior of the bearing during operation more accurately than the traditional static model. Especially in the case of local defects, the amplitude of vibration acceleration is significantly increased, highlighting the advantage of the dynamic model in fault sensitivity. The study confirmed that the use of acceleration signals to monitor bearing status is more sensitive and practical. In the future, the accuracy of dynamic models can be further verified in combination with experiments, and the dynamic behavior of bearings under multi-fault coupling and complex conditions can be explored to provide a theoretical basis for the development of intelligent diagnostic technology.

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