

Optimized Design of Crop Planting Strategies Based on Nonlinear Programming

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Abstract. The advancement of agricultural technology has facilitated large-scale production. Efficient land use, optimal crop selection, and strategic planting decisions are crucial for improving productivity, minimizing cultivation risks, and promoting sustainable rural economic development. This paper focuses on optimizing crop planting strategies in a rural area of northern China. Data preprocessing included encoding and standardizing variable types. The actual crop yields in 2023 were used as the projected sales volumes for subsequent years. Based on problem analysis, the task is modeled as a nonlinear programming problem. A nonlinear model was constructed using crop planting area SSS as the decision variable, with constraints including land area, crop rotation, and legume planting frequency. An algorithm was designed to solve the model. Initially, the nonlinear solver from Python's SciPy library was applied, later optimized using a genetic simulated annealing algorithm. Sensitivity testing showed high responsiveness. Under Scenario 1, the total revenue reached 10.516 million RMB; under Scenario 2, the total revenue reached 16.26 million RMB.

Keywords: Nonlinear programming, Genetic algorithm, Simulated annealing algorithm, Crop planting optimization

1. Introduction

Due to climatic constraints, the arable land in a certain rural area of northern China is only suitable for cultivating a single crop per year. The village possesses a total of 1,201 mu of farmland, distributed across 34 plots comprising flat dry land, terraced fields, hillside plots, and irrigated land. The former types are primarily suitable for growing grain crops, while irrigated land is better suited for rice and vegetables. In addition, the village operates 20 greenhouses, including both standard and intelligent greenhouses, which are used to cultivate vegetables, edible fungi, and double-season vegetables. To enhance productivity and management efficiency, crop rotation must be scientifically implemented. Specifically, legumes should be planted once every three years to improve soil fertility and prevent continuous cropping.

Assuming relative stability in crop yields, market prices, and cultivation costs in the coming years, this study uses 2023 as the baseline year. It considers scenarios involving unsold inventory and price reductions, aiming to develop an optimal crop planting strategy that maximizes revenue while minimizing potential economic losses from overproduction.

2. Symbol description

Table 1: Description of Symbols

Symbol	Definition
ω	Total revenue
j	Plot index
q	Season index
t	Year index
$x_{j tq}$	Crop ID planted in plot j , during season q , in year t
$i_{x_{jtq}}$	Binary indicator: 1 if crop x is planted; 0 otherwise
$P_{x_{jtq}}$	Price per unit of crop type x
$S_{x_{jtq}}$	Planting area for crop type x
$C_{x_{jtq}}$	Yield per mu for crop type x
α	Loss coefficient for unsold or discounted crops
S_j	Total area of plot j
$O_{x_{jtq}}$	Cultivation cost per mu for crop type x
$Y_{x_{jtq}}$	Expected sales volume of crop type x

3. Problem analysis

In constructing the model, several key factors are taken into account, including cultivation costs, selling prices, yield per mu, and expected sales volumes. Two decision variables are introduced: the planting area for each crop and whether excess crops should be sold at a discounted price. To ensure the feasibility and sustainability of the proposed strategy, constraints are established for plot area limits, crop rotation, and planting frequency.

The problem is solved using a hybrid approach that combines a genetic algorithm with simulated annealing. Both are heuristic search methods that emulate natural selection and the physical annealing process, respectively, to identify optimal solutions [1]. By employing these algorithms, this study is able to determine the optimal planting strategy while satisfying all predefined constraints.

4. Data preprocessing

The dataset contains a large volume of complex information, exhibiting typical characteristics of real-world agricultural data. To prepare for subsequent analysis and prediction tasks, foundational preprocessing steps were undertaken, including data cleaning, relational analysis, variable processing, and aggregation.

The detailed preprocessing steps are as follows:

1. Configured Pandas options to prevent automatic downgrading of data types, thereby enhancing processing efficiency.

2. Applied forward-filling methods to handle missing values after loading the dataset, ensuring data completeness.
3. Merged and consolidated the datasets to form a comprehensive data structure.
4. Defined custom functions to remove duplicate columns, standardize column names, and eliminate redundancy and ambiguity.
5. Coded the “plot type” using consecutive integers to simplify management and improve model accuracy and efficiency; linked this with arable land characteristics to support model analysis.
6. Supplemented statistical data for the first-season crops in intelligent greenhouses.

5. Model construction and solution

5.1. Model construction

The model proposed in this study aims to maximize the total agricultural revenue of the village across different plots, seasons, and years. It incorporates key factors such as cultivation costs, selling prices, actual yield versus target yield, and potential risks of overproduction or market price drops. By optimizing the objective function, the study seeks to determine the most advantageous crop planting strategy.

Objective Function:

$$\begin{aligned} \max \omega = & \sum_q \sum_j \sum_t (P_{x_{jtq}} \cdot \min \{S_{x_{jtq}} \cdot C_{x_{jtq}}, Y_{x_{jtq}}\} - S_{x_{jtq}} \cdot O_{x_{jtq}}) \\ & + \alpha \sum_q \sum_j \sum_t P_{x_{jtq}} \cdot \max \{(S_{x_{jtq}} \cdot C_{x_{jtq}}) - Y_{x_{jtq}}, 0\} \end{aligned} \quad (1)$$

Constraints:

1. Land Division Consistency: To avoid excessive fragmentation of plot areas, the 2024 land division scheme follows the 2023 layout of 87 designated plots.
2. Mixed Cropping Area Constraints: When multiple crops are cultivated within a single plot j , their combined planting area must not exceed the total area of plot j [3]. For open fields, the cumulative planting area cannot exceed 1,201 mu. Each greenhouse is limited to a planting area of no more than 0.6 mu.

$$\begin{cases} 0 \leq \sum_x S_{x_{jtq}} \leq S_j \\ \sum_x \sum_{j=1}^{34} S_{x_{jtq}} \leq 1201 \end{cases} \quad (2)$$

3. Continuous Cropping Constraint: The same crop type cannot be planted consecutively on the same plot in successive years.

$$x_{jtq} \neq x_{j,t+1,q} \quad (3)$$

4. Mixed Cropping Permissions: Mixed cropping is not permitted on open fields but is allowed in greenhouses.

$$\begin{cases} \sum_{j=1}^{34} i_{x_{jtq}} \leq 1 \\ \sum_{j=35}^{54} i_{x_{jtq}} \leq 2 \end{cases} \quad (4)$$

5. Legume Planting Frequency Constraint: At least one legume crop must be planted within every three-year cycle to support soil fertility.

$$\sum_{t-2}^t \sum_{x=1}^5 i_{x,jtq} + \sum_{t-2}^t \sum_{x=17}^{19} i_{x,jtq} \geq 1, (t \geq 2) \quad (5)$$

6. Crop Suitability for Dryland Fields: Flat dry land, terraced fields, and hillside plots may only support one season of grain crops per year, excluding rice.

$$i_{x,jtq} = \begin{cases} 1, j \in \{j|1 \leq j \leq 26, x \in Z\} \cap x \in \{x|1 \leq x \leq 15, x \in Z\} \cap q \in \{1\} \\ 0, else \end{cases} \quad (6)$$

7. Crop Suitability for Irrigated Land: Irrigated fields are suitable for either single-season rice or two-season vegetable cultivation. In the first season, various vegetables (excluding Chinese cabbage, white radish, and red radish) may be planted. The second season is restricted to only one among Chinese cabbage, white radish, or red radish.

$$i_{x,jtq} = \begin{cases} 1, j \in \{j|27 \leq j \leq 34, x \in Z\} \cap x \in \{x|17 \leq x \leq 34, x \in Z\} \cap q \in \{1\} \\ 1, j \in \{j|27 \leq j \leq 34, x \in Z\} \cap x \in \{x|35 \leq x \leq 37, x \in Z\} \cap q \in \{2\} \\ 0, else \end{cases} \quad (7)$$

$$\sum_{x=35}^{37} i_{x,jtq} \leq 1 \quad (8)$$

8. Crop Suitability for Standard Greenhouses: Standard greenhouses are suited for one season of vegetables and one season of edible fungi. The first season may include various vegetables, excluding Chinese cabbage, white radish, and red radish. Edible fungi, requiring lower temperatures and higher humidity, can only be planted in the second season.

$$i_{x,jtq} = \begin{cases} 1, j \in \{j|35 \leq j \leq 50, x \in Z\} \cap x \in \{x|17 \leq x \leq 34, x \in Z\} \cap q \in \{1\} \\ 1, j \in \{j|35 \leq j \leq 50, x \in Z\} \cap x \in \{x|38 \leq x \leq 41, x \in Z\} \cap q \in \{2\} \\ 0, else \end{cases} \quad (9)$$

9. Crop Suitability for Smart Greenhouses: Intelligent greenhouses support two vegetable planting seasons annually, excluding Chinese cabbage, white radish, and red radish.

$$i_{x,jtq} = \begin{cases} 1, j \in \{j|51 \leq j \leq 54, x \in Z\} \cap x \in \{x|17 \leq x \leq 34, x \in Z\} \cap q \in \{1, 2\} \\ 0, else \end{cases} \quad (10)$$

The first workflow focuses on cultivating specific crops under various conditions and selling them to generate total revenue. Taking into account the actual sales volume and planting costs, it accurately calculates the net profit of each crop.

The second workflow addresses the potential risks of unsold produce or price reductions when actual yields exceed target yields. By introducing a loss coefficient to quantify potential losses, it makes the net profit estimation more aligned with real economic conditions. It calculates the surplus yield and multiplies it by the loss coefficient to assess economic losses, reflecting sensitivity to

market fluctuations and emphasizing risk control. This highlights the importance of considering market demand and avoiding surplus risks when formulating planting plans.

Together, these two workflows form an objective function aimed at maximizing the total income of rural areas, taking into account yield, cost, and market risk. This paper explains the value ranges of certain variables used in the objective function:

Table 2. Description of Variable Ranges

Symbol	Value Range
j	$\{j 1 \leq j \leq 54, j \in \mathbb{Z}\}$
q	$\{q 1 \leq q \leq 2, q \in \mathbb{Z}\}$
t	$\{t 0 \leq t \leq 7, t \in \mathbb{Z}\}$
α	$\{\alpha 0 \leq \alpha \leq 7, \alpha \in \mathbb{R}\}$

5.2. Nonlinear algorithm design and solution

The modernization of agriculture has driven the development of crop planting optimization algorithms, which play a crucial role in improving agricultural productivity and economic returns. To maximize planting revenues for the years 2024 through 2030, this study proposes an optimization approach based on nonlinear programming. The goal is to enhance profitability by efficiently allocating planting areas, crop types, and seasonal scheduling. The crop rotation rule—which prohibits planting the same crop on the same plot in consecutive years—further confirms that the problem falls within the domain of nonlinear programming [4].

The algorithm is designed as follows:

First, collect information on plots and crops, including yield per mu, selling price, and expected sales volume. Standardize plot types and create dictionaries to store the relationships between plot types and yield per mu, crop codes and selling prices, and crop codes and expected sales volumes. To avoid ambiguities caused by nested subscripts in the model, convert subscripts into multi-level subscripts and create index sets for quick access to decision variables. Initialize all decision variables as zero vectors. Define the objective function to maximize profit. Set constraints such as land area and crop planting limitations. Use the minimize function from the SciPy library to solve the optimization problem, with the SLSQP (Sequential Least Squares Programming) method. Finally, extract and output the optimal solution.

5.3. Linear programming reformulation and solution

Due to the relatively slow computation speed of solving nonlinear problems, this study also attempts to reformulate the problem as a linear programming (LP) model to improve computational efficiency.

5.3.1. Algorithm overview

Linear programming is a mathematical technique used to solve optimization problems, particularly those involving linear objective functions and linear constraints [5][6]. Among various solution

methods for LP problems, the most well-known is the simplex method, an iterative algorithm that transitions from one feasible solution to another until the optimal solution is reached [7][8][9].

5.3.2. Solution procedure

Definition of Decision Variables:

Table 3. Description of Decision Variables

Variable	Description
$x[i,j,q,t]$	Area of crop j planted in plot i , during season q , year t
$plant[i,j,q,t]$	Binary variable: 1 if crop j is planted in plot i during season q , year t ; 0 otherwise
$z[j,t]$	Quantity of crop j exceeding expected sales volume in year t
$y[i,t]$	Binary variable: 1 if plot i has legumes planted in year t ; 0 otherwise

Read data related to land, crops, selling prices, expected sales volumes, and yield per mu. Standardize the land plot types and map each standardized type to a corresponding code. Organize the data into structures that are easy to process. Create a linear programming problem instance aimed at maximizing total revenue minus the loss from exceeding expected sales volumes. Add relevant constraints and call a linear programming solver to solve the model. Print all non-zero decision variables and their values, and calculate the total revenue.

5.4. Simulated annealing algorithm design and solution

To further enhance algorithmic performance, this study explores the use of the simulated annealing (SA) algorithm to search for a viable solution.

5.4.1. Algorithm overview

Simulated annealing is a global optimization algorithm based on principles of statistical mechanics. It mimics the physical process of annealing, wherein a system's temperature gradually decreases, thereby reducing the probability of accepting inferior solutions over time and guiding convergence toward a global optimum [10]. A key feature of SA is that it occasionally allows acceptance of worse solutions, which helps escape local optima and improves the chances of finding the global best solution [11]. As the system "cools," the algorithm becomes increasingly selective, ultimately converging on a satisfactory solution. The primary advantages of SA are its generality and insensitivity to the choice of initial solutions. However, its practical success depends on careful tuning of parameters such as the initial temperature, cooling schedule, and stopping criteria. Due to its randomness and global search capabilities, SA often entails higher computation time and complexity. In this study, SA is applied to solve a complex agricultural land allocation problem with the objective of maximizing total revenue [12][13].

5.4.2. Algorithm design and solution procedure

Begin by initializing the population through a function that generates an initial population consisting of a certain number of individuals. Each individual is a vector whose length equals the total number of decision variables, and whose elements are typically randomly generated real numbers. A fitness function is then used to evaluate the quality of each individual (candidate solution) and guide the

search direction. A decoding function is employed to convert the encoded solutions into a format that is easier to interpret and process. Selection operations choose better-performing individuals from the current population to advance to the next generation [14]. Crossover operations combine two parent individuals to produce a new offspring [15]. Mutation operations randomly alter individuals to increase population diversity. A simulated annealing algorithm is employed as the main optimization process. It seeks the global optimum by gradually reducing the temperature in a manner analogous to the annealing process in metallurgy [16].

5.5. Final results presentation

Upon executing the model, the optimal crop planting strategy for the years 2024 to 2030 was obtained. Under the scenario where excess yields result in unsold inventory, the total revenue from the planting plan is estimated at 10.516 million RMB. In contrast, when the excess yield is sold at a 50% discount based on the 2023 market price, the total revenue increases to 16.26 million RMB.

Authors' Contributions

Liyang Luo and Jiao Wei contributed equally to this work.

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