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Investigation of a BAR Creep Constitutive Model Based on the Poyting–Thomson Model

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Abstract. Research on high-stress deep underground roadway has shown that the creep phenomenon of the surrounding rock has a significant impact on the stability of the underground roadway. This paper presents an innovative Bolt-Anchored Rock mass(BAR) creep constitutive model to simulate the coupling effect between rock bolts and the surrounding rock. Based on the Poyting-Thomson model, this paper preliminarily analyzes the creep constitutive equation, creep curve of the rock model without rock bolt support, as well as the mechanical properties of the rock bolt model. Subsequently, the two models are connected in parallel to simulate the stress state of the BAR, and the creep constitutive equation for BAR is established. A comparison between numerical methods and theoretical solutions demonstrates a high degree of consistency. The results indicate that the proposed creep constitutive model for BAR exhibits certain applicability. The findings of this study provide important theoretical support and reference for the design of BAR creep constitutive models in high-stress deep underground roadways, offering insights for future research on the mechanical theory and practical engineering applications of creep constitutive models.

Keywords: creep phenomenon, Poyting-Thomson model, rock bolt anchoring, mechanical analysis, constitutive model test

1. Introduction

As a critical supporting structure in underground tunnel engineering, the safety analysis of surrounding rock support is closely correlated with the highly coupled mechanical system of the rock mass and rock bolts. Under conditions such as high in-situ stress, elevated ground temperature, complex hydrogeological environments, and continuous disturbances, rock masses often exhibit significant nonlinear mechanical responses. Deformation and failure behaviors are particularly typical in soft rock and hard rock roadways [1,2]. Creep, a gradual deformation process occurring under sustained constant or varying stress, not only induces continuous deformation of the surrounding rock but may also lead to stress redistribution in support structures and performance degradation of anchoring systems. In severe cases, it can even trigger disasters such as roof collapse, bolt pull-out, and rockburst [3,4].

Currently, numerous existing studies have systematically analyzed the creep characteristics of rock masses under various loading conditions: To investigate the dynamic evolution of internal

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structural damage in rock masses, Lu et al. established a compression-creep coupled creep constitutive model [5]; To examine the creep development mechanism of sandy mudstone influenced by radial depth-dependent damage, Liu et al. developed a nonlinear creep constitutive model incorporating both initial radial depth damage and creep damage [6];To explore creep deformation, strength characteristics, and failure modes of rock masses under different confining pressures, Lu et al. formulated a nonlinear viscoelastic-plastic creep model [7]; To describe damage growth during instantaneous and constant-loading stages of rock creep, Tian et al. proposed a unified nonlinear creep-damage coupling model for rocks based on fractional calculus and statistical damage theory [8]. However, most of these existing models focus primarily on the behavior of the rock mass itself, failing to adequately consider the feedback effects of rock bolt support structures on the creep process.

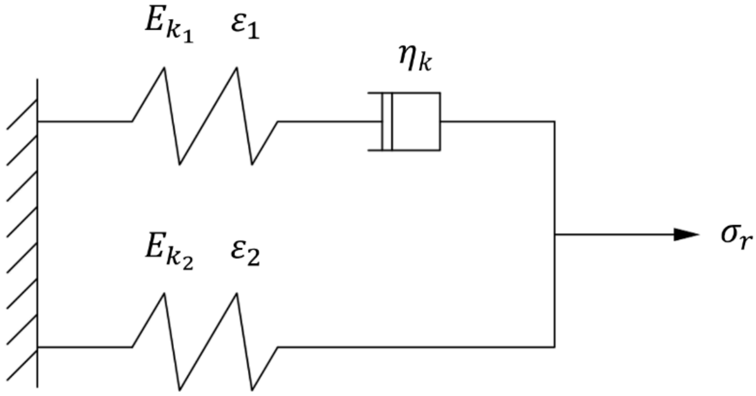
This study aims to investigate the coupling mechanism between rock bolts and surrounding rock by establishing a creep constitutive model for BAR. Initially, analytical derivation of theoretical formulas preliminarily clarified the mechanical properties and creep constitutive equation of the BAR model. Concurrently, numerical analysis implementing the constitutive equation in ABAQUS software demonstrated the reliability and feasibility of the proposed model. Finally, design strategies for underground engineering support systems based on the BAR creep constitutive model were proposed.

The creep constitutive model for BAR is developed based on the mechanical principles of the Poyting-Thomson model and rock bolts [9-11], formed by parallel connection of these theoretical models. This model exhibits three distinctive advantages: multi-field coupling capability for bolt-rock interaction, superior time-dependent behavior characterization, and separable structural response (bolts) from rock mass behavior. It can be effectively implemented into numerical simulation frameworks (e.g., finite element analysis in ABAQUS) for engineering scenario simulations. However, practical applications present significant challenges: the current model remains primarily theoretical in analyzing coupling mechanisms, with actual engineering parameters and simulation data yet to be determined. Future work should focus on developing multi-parameter evaluation models integrating in-situ conditions and advanced numerical analysis techniques.

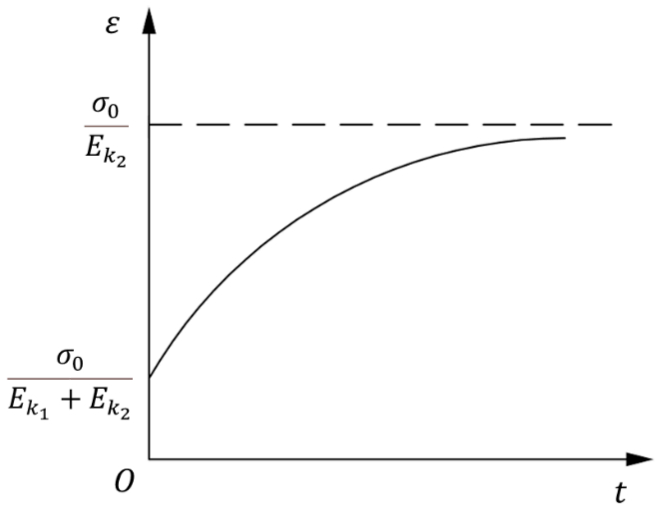
2. Method

2.1. Creep constitutive models for rock masses

The Poyting-Thomson model is constituted by parallel connection of a Maxwell element and an elastic component [12,13]. This rheological model demonstrates superior capability in characterizing both the primary creep stage and secondary creep stage of soft rock behavior. In this study, the Poyting-Thomson model is adopted to simulate the creep behavior of rock masses (Fig 1).



(a) Poyting-Thomson model



(b) Creep curve

Figure 1. Creep constitutive models for rock masses

From the parallel configuration, the governing equations are derived as:

$$\begin{cases} \sigma_r = \sigma_{k_1} + \sigma_{k_2} \; , \sigma_{k_1} = \sigma_\eta \\ \varepsilon_r = \varepsilon_{k_1} + \varepsilon_\eta = \varepsilon_{k_2} \end{cases} \tag{1}$$

where σ_r is the rock stress; σ_{k_1} , σ_{k_2} is the spring element stress; σ_η is the dashpot stress; ε_r is the rock strain; $\varepsilon_{k_1}, \varepsilon_{k_2}$ is the spring element strain; ε_η is the dashpot strain.

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$$\begin{cases} \dot{\varepsilon}_{k_1} = \frac{\dot{\sigma}_{k1}}{E_{k_1}} \\ \dot{\varepsilon}_{k_2} = \frac{\dot{\sigma}_{k2}}{E_{k_2}} \\ \dot{\varepsilon}_{\eta} = \frac{\sigma_{\eta}}{\eta_k} \end{cases} \tag{2}$$

where $\dot{\varepsilon}_{k_1}, \dot{\varepsilon}_{k_2}$ is the time derivative of spring element strain; $\dot{\sigma}_{k_1}, \dot{\sigma}_{k_2}$ is the time rate of spring element stress; E_{k_1}, E_{k_2} is the elastic modulus of spring element; $\dot{\varepsilon}_{\eta}$ is the strain rate of dashpot element; σ_{η} is the shear stress in dashpot element; η_k is the viscosity coefficient of dashpot element.

Substituting EQ.(2) into EQ.(1) yields the governing creep constitutive equation:

$$\dot{\sigma}_r + \frac{E_k}{\eta_k}\sigma_r = (E_{k_1} + E_{k_2})\dot{\varepsilon}_r + \frac{E_{k_1}E_{k_2}}{\eta_k}\varepsilon_r \tag{3}$$

2.2. Rock bolt model

The stress-strain behavior of rock bolts is idealized as an elastoplastic model [14,15], mechanically represented by a spring element (elastic response) and a friction slider (plastic yield) connected in series (Fig 2).

When $\sigma_b < \sigma_s$,the rock bolt is in the elastic state:

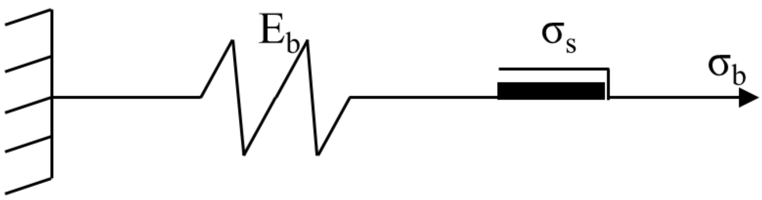
$$\sigma_b = E_b\varepsilon_b \tag{4}$$

where σ_b is the rock bolt stress; E_b is the elastic modulus of rock bolt; ε_b is the rock bolt strain.

When $\sigma_b > \sigma_s$, the rock bolt is in the plastic state.

$$\sigma_b = \sigma_s \tag{5}$$

where σ_s is the post-yield stress of rock bolt.



(a) Ideal elastoplastic model

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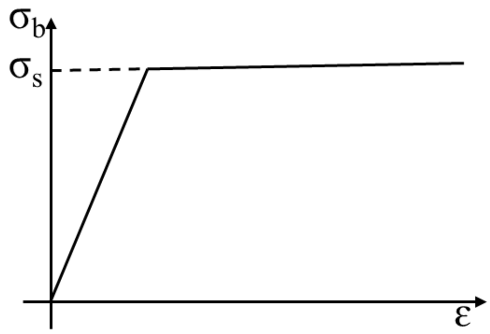

(b) Mechanical response curve

Figure 2. Rock bolt model

3. BAR creep constitutive model

To simulate the mechanical response of the BAR, the rock bolt and surrounding rock are modeled as parallel-connected elements (Fig 3). This configuration gives rise to two distinct mechanical states: the EE (Elastic-Elastic) state, where both components exhibit elastic behavior when the bolt stress σ_b remains below the yield threshold σ_s; and the EP (Elastic-Plastic) state, which occurs when σ_b exceeds σ_s, triggering plastic deformation in the bolt while the rock maintains its elastic properties. The complete rock-bolt interaction system can therefore be analytically solved by separately considering these two fundamental working conditions (EE and EP states). This modeling approach effectively captures the essential mechanical coupling characteristics between the reinforcing bolt and the viscoelastic rock mass under various loading conditions.

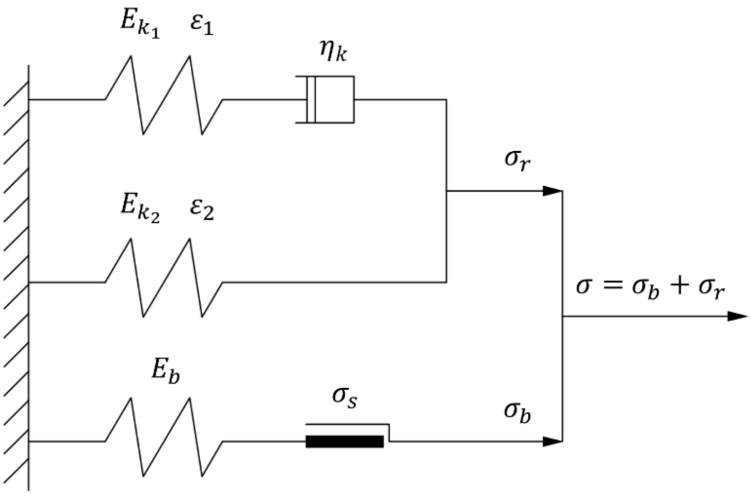

Figure 3. BAR creep constitutive model

3.1. Solution for EE state

When $\sigma_b < \sigma_s$:

DOI: 10.54254/2755-2721/2025.GL26041

$$\begin{cases} \sigma = \sigma_r + \sigma_b \\ \sigma_b = E_b \varepsilon_b \\ \dot{\sigma} = \dot{\sigma}_r + \dot{\sigma}_b \end{cases} \tag{6}$$

$$\begin{cases} \varepsilon = \varepsilon_r \\ \dot{\varepsilon} = \dot{\varepsilon}_r \\ \varepsilon = \varepsilon_b \end{cases} \tag{7}$$

Substituting EQ.(6) and EQ.(7) into EQ.(3),

$$\dot{\sigma} + \frac{E_{k_1}}{\eta_k}\sigma = (E_{k_1} + E_{k_2} + E_b)\dot{\varepsilon} + \frac{E_{k_1}(E_{k_2} + E_b)}{\eta_k}\varepsilon \tag{8}$$

During the loading stage, the model is subjected to tensile stress σ_0, with initial strain maintained by spring connections. Thus at t=0, the initial strain ε_0 is given by:

$$\varepsilon_0 = \frac{\sigma_0}{E_{k_1} + E_{k_2} + E_b} \tag{9}$$

When $t \to \infty$:

$$\varepsilon = \frac{\sigma_0}{E_{k_2} + E_b} \tag{10}$$

Substituting EQ.(9) and EQ.(10) into EQ.(8),

$$\varepsilon = \frac{\sigma_0}{E_{k_1} + E_b}\left(1 - \frac{E_{k_1}}{E_{k_1} + E_{k_2} + E_b} e^{\frac{-E_{k_1}\left(E_{k_2} + E_b\right)}{\left(E_{k_1} + E_{k_2} + E_b\right)\eta_k} t}\right) \tag{11}$$

3.2. Solution for EP state

When $\sigma_b > \sigma_s$:

DOI: 10.54254/2755-2721/2025.GL26041

$$\begin{cases} \sigma = \sigma_r + \sigma_b \\ \sigma_b = \sigma_s \\ \dot{\sigma} = \dot{\sigma}_b + \dot{\sigma}_r \end{cases} \tag{12}$$

Substituting EQ.(12) into EQ.(3),

$$\dot{\sigma} + \frac{E_{k_1}}{\eta_k}(\sigma - \sigma_s) = (E_{k_1} + E_{k_2} + E_b)\dot{\varepsilon} + \frac{E_{k_1}(E_{k_2} + E_b)}{\eta_k}\varepsilon \tag{13}$$

At the loading stage, the model is subjected to tensile stress σ_0 . The initial strain ε_0 at t=0 is expressed as:

$$\varepsilon_0 = \frac{\sigma_0 - \sigma_s}{E_{k_1} + E_{k_2}} \tag{14}$$

When $t \to \infty$:

$$\varepsilon = \frac{\sigma_0 - \sigma_s}{E_{k_2} + E_b} \tag{15}$$

Substituting EQ.(14) and EQ.(15) into EQ.(3),

$$\varepsilon = \frac{\sigma_0 - \sigma_s}{E_{k_2} + E_b}\left(1 - \frac{E_{k_1} - E_b}{E_{k_1} + E_{k_2}} e^{\frac{-E_{k_1}\left(E_{k_2} + E_b\right)}{\left(E_{k_1} + E_{k_2} + E_b\right)\eta_k} t}\right) \tag{16}$$

In summary, the BAR creep constitutive equation is derived as:

$$\varepsilon = \begin{cases} \frac{\sigma_0}{E_{k_1} + E_b}\left(1 - \frac{E_{k_1}}{E_{k_1} + E_{k_2} + E_b} e^{\frac{-E_{k_1}\left(E_{k_2} + E_b\right)}{\left(E_{k_1} + E_{k_2} + E_b\right)\eta_k} t}\right) \\ \frac{\sigma_0 - \sigma_s}{E_{k_2} + E_b}\left(1 - \frac{E_{k_1} - E_b}{E_{k_1} + E_{k_2}} e^{\frac{-E_{k_1}\left(E_{k_2} + E_b\right)}{\left(E_{k_1} + E_{k_2} + E_b\right)\eta_k} t}\right) \end{cases} \tag{17}$$

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4. Theoretical verification

4.1. Model establishment

The constitutive EQ.(3) and EQ.(17) were numerically verified using ABAQUS(Fig 4), with the computational model configuration shown in Fig 5. The simulation domain measures 100 m in length with a 5 m tunnel excavation radius. Material parameters for both surrounding rock and rock bolts (Table 1) were calibrated against uniaxial creep test data of soft rock [13], E_{k_1} =125MPa, E_{k_2} =208MPa, η_k =10GPa·h, σ_s =500MPa, E_b =650MPa. Boundary conditions include: vertical stress σ=2 MPa applied at the model top surface with lateral pressure coefficient λ=1, and normal constraints imposed on left/right boundaries and bottom boundary.

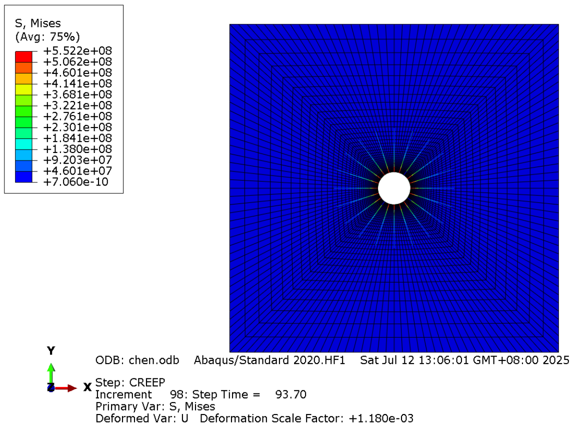

Figure 4. Numerical model of BAR

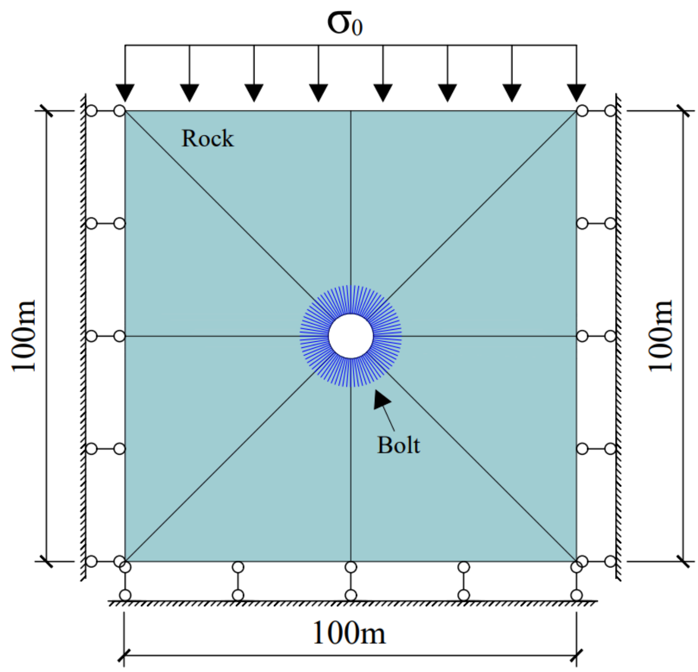

Figure 5. Schematic diagram of BAR

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Table 1. BAR parameters

Parameters	E_{k_1}	E_{k_2}	η_k	σ_s	E_b
Rock	125MPa	208MPa	10GPa·h	—	—
Bolt	—	—	—	500MPa	650MPa

4.2. Verification results

Comparative analysis between theoretical solutions and numerical simulations (Fig 6) reveals strong agreement in the time-dependent deformation trends of surrounding rock, demonstrating the model's robust capability in simulating creep behavior. The plotted results (x: time T/day; y: displacement y/mm) show excellent consistency during the initial phase (0-40 days), with minimal deviations (<0.5 mm absolute, <1% relative error at T=20 days). While slight error accumulation occurs over extended durations (150 days), the differences remain within acceptable limits (1.7% relative error between numerical [24.76 mm] and theoretical [24.35 mm] results at T=150 days). These findings confirm the model's high precision in capturing long-term time-dependent deformations and its reliability as an analytical tool for support system evaluation, meeting rigorous engineering accuracy standards. The validation process maintains original parameter precision (0.5mm/1.7% thresholds) and follows established visualization conventions for creep analysis.

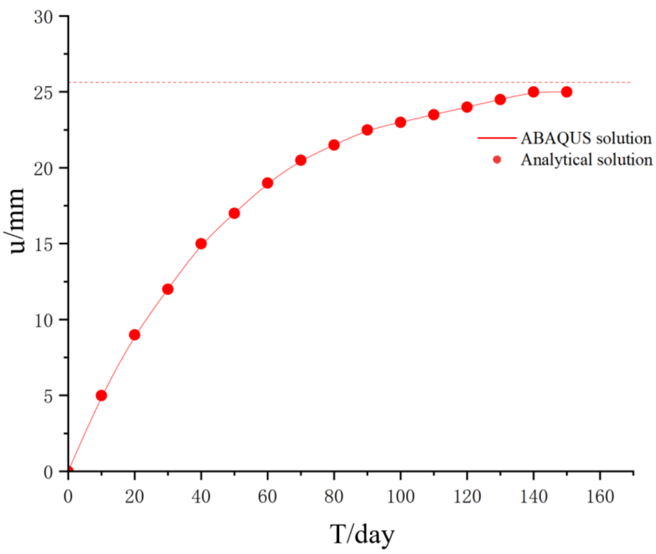

Figure 6. Numerical-theoretical solution validation

5. Conclusion

This project introduces an innovative creep constitutive model for BAR through theoretical and numerical analyses, investigating the coupling mechanism between rock bolts and surrounding rock. The main conclusions and recommendations are as follows:

(1) Based on the mechanical principles of the Poyting-Thomson model and bolt anchoring system, an innovative creep constitutive model for BAR was developed. The results indicate that as

time progresses, the creep-induced strain increases at a gradually decreasing rate, eventually approaching an asymptotic value;

(2) Comparative analysis between numerical simulations and theoretical solutions verifies the accuracy and reliability of the proposed creep constitutive equations. The model effectively captures the coupling interaction between bolts and surrounding rock in high-stress deep underground roadways;

(3) For practical engineering applications, the strain increments caused by rock creep can be estimated based on material properties, structural dimensions, and service life of deep underground excavations. However, due to time and experimental constraints, further validation through systematic long-term testing is required to refine the model;

(4) The current study focuses on theoretical formulation and numerical modeling without field validation. Future research should incorporate in-situ monitoring data to examine the influence of rock properties and environmental factors (e.g., temperature, confining pressure, and pore water pressure) on creep behavior, as well as to evaluate the applicability of the proposed constitutive model.

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