

Prediction of Tee Erosion Rate Based on Response Surface Analysis Method

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Abstract. To investigate tee erosion under multi-factor coupling and enhance tee service life for safe, efficient operation, a numerical simulation was conducted using Fluent software. The study examined erosion patterns under varying inlet flow velocities, sand concentrations, and particle sizes, specifically analyzing the coupling effects of these three factors. Results indicate that tee erosion rate increases with higher flow velocity and greater sand concentration, while exhibiting a non-monotonic relationship with particle size—decreasing initially before rising. Analysis of the factor coupling revealed flow velocity as the most significant influence on erosion rate, followed by sand concentration and particle size. A regression equation relating erosion rate to these key parameters was derived. This simulation comprehensively accounts for both single-factor and multi-factor coupling effects on erosion, providing theoretical insights for extending pipeline service life and ensuring operational safety.

Keywords: Tee, corrosion rate prediction, Response surface analysis, mock

1. Introduction

Erosion is the phenomenon in which solid particles carried in a fluid collide with the surface at a certain angle and speed, causing material loss to the solid surface [1]. The erosion problem has always been one of the main problems plaguing the development of oil and gas gathering and transportation, and as the oilfield exploitation progresses to the later stage, the amount of sand produced in the oilfield is increasing, but due to the reduction of oil pressure, the oilfield usually does not install a sand filter system to ensure stable production [2]. According to the British Eyre estimate, erosion wear accounts for about 8% of the total wear of industrial production [3], and the economic losses caused by erosion corrosion in China are also serious. According to statistics, the economic losses caused by erosion corrosion in the industry account for 9% of the economic losses caused by all corrosion [4].

As the most important energy transportation mode at present, pipeline transportation cannot be ignored by erosion and corrosion. The pipeline erosion corrosion process is very complex, it is related to the material composition of the pipe wall, the flow medium in the pipeline, the nature of solid particles and other factors, in addition to the pipe structure and the sudden change of flow direction is also an important cause of pipeline erosion wear, the study shows that [5] the erosion

rate at the fluid steering can reach dozens of times of the straight pipe section. In pipeline transportation, it is mainly elbows and tees that assume the role of fluid steering.

At present, scholars at home and abroad have studied the erosion and wear of elbows, while tee, as an important part of the pipeline system, has less research on the erosion and wear of solid particles. Only Farokhipour A, Mansoori Z, Xu Liyun, Qian Dongliang and others [6-8] have carried out numerical simulation and theoretical research, and the study only found the most serious location of three-way erosion and did not systematically study the main influencing factors of erosion. Compared with the elbow, the flow field and solid particle movement of the tee are more complex and changeable, and it is more likely to produce erosion effect, therefore, the erosion law of the tee under the action of different influencing factors is studied, the most serious position of erosion is predicted, the relationship between the movement trajectory of solid particles and erosion of the pipeline is analyzed, and the erosion mechanism of the tee is obtained, which provides suggestions for the protection of the tee components, which is of great significance for the safety of pipeline operation and reducing economic losses.

2. Mathematical model

2.1. Geometry

The tee configuration is depicted in Figure 1. With an inner diameter of $D=40$ mm, the main pipe extends $10D$ to ensure fully developed flow, while the combined length of both lateral branches totals $20D$ ^[9]. This dimensional design stabilizes turbulence effects on erosion rates. The present study investigates branch-originating erosion phenomena in this shunt-type tee configuration.

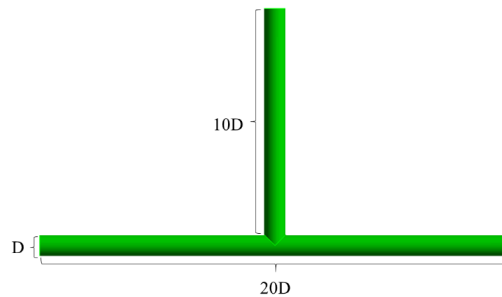


Figure 1. Structural parameters of tee

2.2. Turbulence model

When the Reynolds number exceeds a certain value, the flow state of the fluid will change from laminar to turbulent. This study focuses on regions where the direction of the flow field changes, the flow state is turbulent, and the flow state is assumed to be stable. This yields the continuous momentum equation and the fluid phase equation, as shown in equations (1) and (1):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (1)$$

$$\frac{\partial}{\partial t} (\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot \left\{ \mu [(\nabla \vec{u} + \nabla \vec{u}) - \frac{2}{3} \nabla \cdot \vec{u} I] \right\} + \rho \vec{g} + \vec{F}_D \quad (2)$$

In the formula:

ρ is the continuous phase density, kg/m³; $\mathbf{u}$ is the instantaneous velocity vector, m/s; p is pressure, Pa; μ is viscosity, Pa·s; $\mathbf{I}$ is the unit tensor; $\mathbf{g}$ is gravity, N; F_D is an additional source term for the relative continuous phase action of particles.

In the elbow, tee, blocking tee and other parts where the flow direction and structure change, the flow is more complex and accompanied by a certain secondary vortex flow, so the correct selection of liquid phase turbulence model to describe the flow field of the continuous phase is the basis and premise of the two-phase flow erosion numerical simulation.

The turbulence models provided by Fluent include standard k- ϵ models, Spalart-Allmaras models [9], RNG k- ϵ models, Realizable k- ϵ models, k- ω models, Reynolds stress models, etc., and different turbulence models have their corresponding scope of application.

Among them, based on the zero equation model, the k- ϵ model adds two variables: turbulent energy transfer equation and dissipation rate transport equation. RNG k- ϵ has the advantages of good applicability, moderate calculation amount and ensures calculation accuracy, so it is widely used in engineering practice.

The RNG k- ϵ model accurately simulates turbulent flows exhibiting secondary flow phenomena and demonstrates robust performance across low to high Reynolds number conditions. Consequently, this turbulence model was selected for the present study.

2.3. Boundary condition settings

The continuous phase is liquid water, the density is 998kg/m³, the pipe wall material is 20# carbon steel, the density is 7500kg/m³, the pipe wall is set as the wall boundary and the rebound condition is set, and the properties are static wall and non-slip wall. The viscosity of the continuous phase liquid is 1.00×10⁻³Pa·s, the inlet boundary conditions adopt velocity inlet, the outlet is free outflow, and the inlet and outlet are all using escape (Escape) conditions [10].

The discrete phase is a quartz stone particle with a density of 2650kg/m³. The coupling calculation of the liquid phase continuous phase and the solid dispersed phase was carried out by biphasic coupling, and after the convergence was stabilized by simulation calculation, the solid dispersed phase was post-processed, and the liquid phase flow field was observed.

2.4. Tee meshing

In order to ensure the accuracy of erosion simulations while saving computational resources, a mesh independence study is required to determine the minimum mesh density required, ensuring that the CFD solution results are independent of mesh density. The meshing method of hexahedral structured mesh is adopted, and the boundary layer network at the pipe wall is encrypted, and the mesh gradient rate is 1.2, and the number of layers is 5. The tee meshing results are shown in Figure 2:

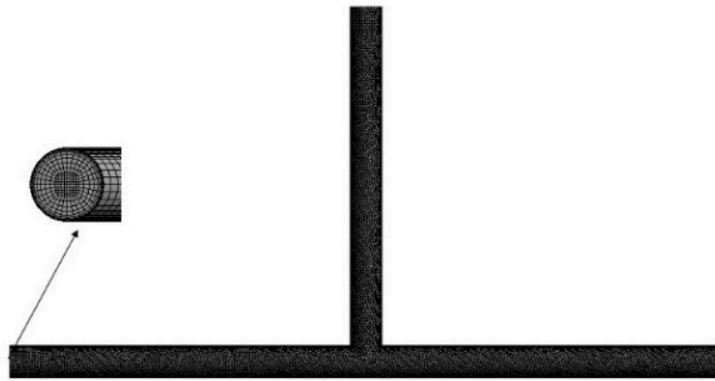


Figure 2. Tee grid structure division

The model with different mesh numbers is simulated and calculated, and the relationship between the maximum erosion rate and the number of meshes is shown in Figure 2.

According to Figure 3, it can be seen that when the number of grids is small, the maximum erosion rate will change significantly when the number of meshes is gradually increased, but when the number of grids increases to about 1,260,000, the maximum erosion rate changes very little with the increase of the grid and tends to be stable. Therefore, under the premise of ensuring the calculation accuracy, in order to reduce the computer calculation time as much as possible, the division method with the number of grids is 1260000.

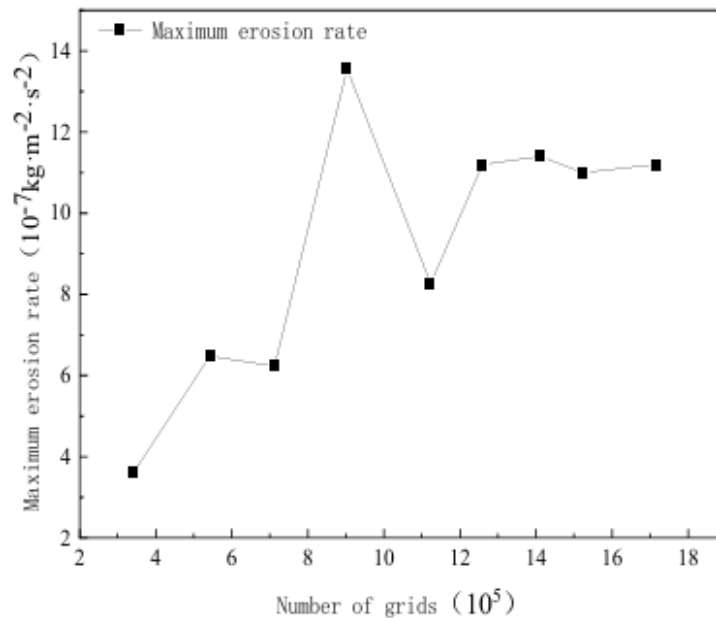


Figure 3. Curve of maximum erosion rate with grid number

3. Three-way erosion simulation results and analysis

3.1. Flow rate impact on erosion

Within this section, under fixed operational parameters of 150 μm solid particle size and 0.15 kg/s particle mass flow rate, six discrete inlet velocities 3 m/s, 5 m/s, 8 m/s, 10 m/s, 15 m/s, and 20 m/s were systematically examined to quantify tee erosion wear rates. This experimental matrix elucidated the comprehensive quantitative relationship between liquid flow velocity and resultant tee erosion rates.

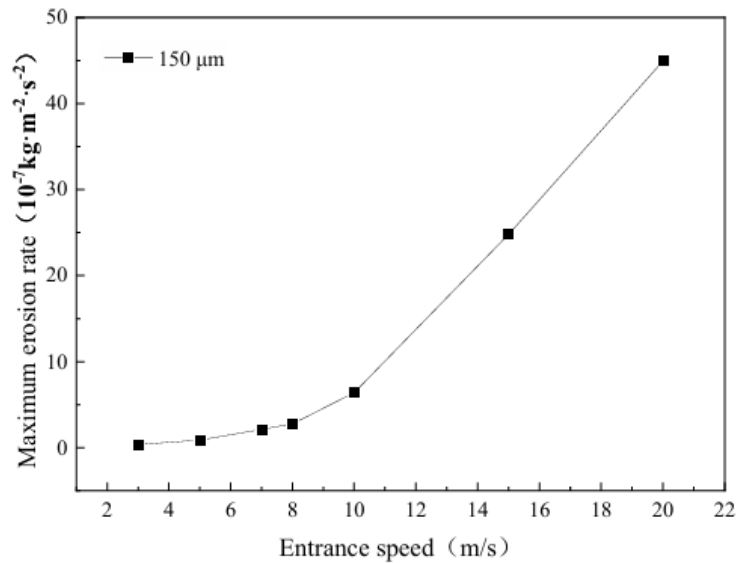


Figure 4. Effect of flow velocity at the inlet of a type flow tee on maximum erosion rate

Figure 4 illustrates the maximum erosion rate of the tee across varying flow rates. The data reveals that the erosion rate exhibits a nearly quadratic growth trend with increasing inlet flow velocity. The predominant factors governing pipe wall erosion are the impact velocity and collision frequency of solid particles per unit time. Under constant solid particle size and mass flow rate, the number of particle-wall collisions per unit time remains stable. However, as inlet flow velocity increases, the kinetic energy imparted to solid particles during momentum transfer with the liquid phase fluid consequently rises. This results in higher-velocity particle impacts against the pipe wall, causing progressively more severe erosion damage.

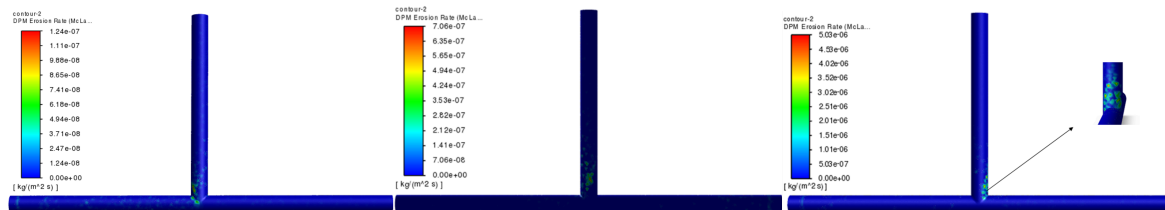


Figure 5. The erosion rate outline diagram of A-type flow T-shirts at different flow rates

The visualization reveals that as the inlet flow rate progressively increases, the region of maximum erosion wear gradually shifts towards the inlet of the main outlet pipe. At lower inlet velocities, solid particles acquire minimal kinetic energy, rendering them unable to overcome the

influence of the secondary flow at the tee junction. Consequently, these particles undergo repeated collisions with the pipe wall at the neck region of the branch pipe inlet side, leading to localized erosion wear. However, with continued elevation of the inlet flow rate, the inertia of the solid particles intensifies significantly. Under these conditions, neither the secondary flow effect at the tee intersection nor the drag force exerted by the outlet pipe can substantially alter the inertial trajectory of the particles. As a result, the particles directly impact the neck of the tee structure, positioning the area of maximum erosion wear closer to the entrance of the main pipe section.

3.2. Effect of particle size on erosion

Under the working conditions of inlet flow rate of 8m/s and particle mass flow rate of 0.15kg/s, six solid particle sizes of 50 μ m, 100 μ m, 150 μ m, 200 μ m, 250 μ m and 300 μ m were selected to study the erosion wear rate of tee, and the overall law of solid particle size to erosion rate was obtained.

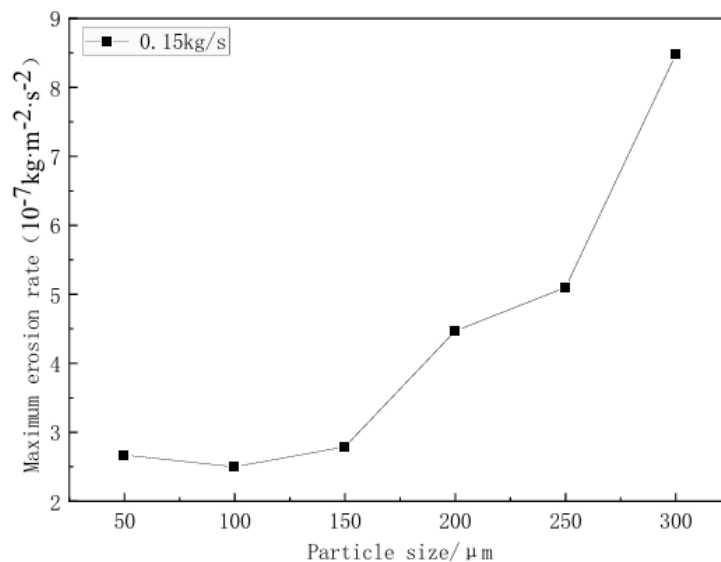


Figure 6. Effect of solid particle size on maximum erosion rate in a type flow tee

Figure 6 depicts the variation in the tee's maximum erosion rate with solid particle size, revealing an initial decrease followed by an increase. The minimum erosion rate occurs at a particle size of 100 μ m, indicating this value represents the critical particle size for tee erosion.

Consequently, under identical inlet flow conditions, varying solid particle sizes alter the dominant forces governing their motion within the liquid phase. Smaller particles, significantly influenced by fluid traction, are predominantly directed by the strong secondary flow at the tee intersection, impacting the outlet pipe's inlet side. Conversely, larger particles, dominated by inertial force, primarily strike the tee shoulder adjacent to the outlet pipe. Particles of medium size experience a more balanced interplay between inertial force and secondary flow effects. This reduces both the frequency and number of collisions at these two critical locations, resulting in the minimal observed maximum erosion rate.

3.3. Effect of particle mass flow on erosion

Under the working conditions of inlet flow rate of 8m/s and solid particle size of 150 μ m, six solid particle mass flow rates of 0.05kg/s, 0.10 kg/s, 0.15 kg/s, 0.20 kg/s, 0.25 kg/s and 0.30 kg/s were

selected to study the erosion wear rate of the tee, and the erosion wear under different particle mass flow rates was obtained.

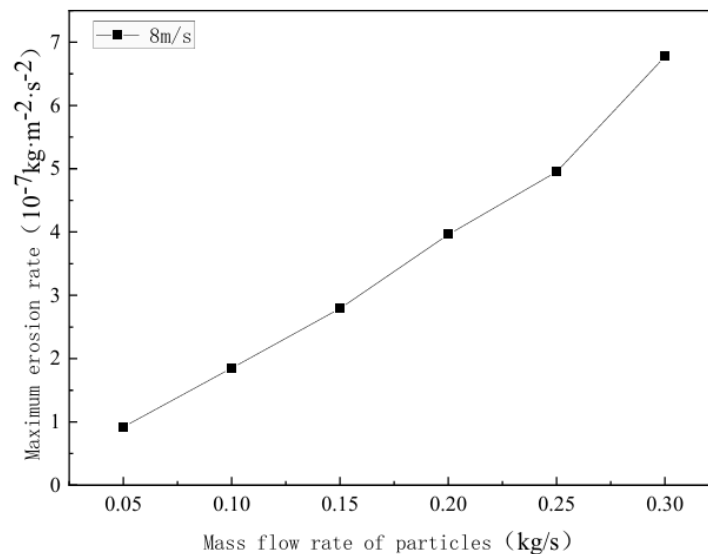


Figure 7. Effect of solid particle mass flow rate on the maximum erosion rate of a tee

Figure 7 depicts the maximum erosion rate of tees across varying particle mass flow rates. The data reveals a linearly proportional increase in the tee's maximum erosion rate with rising particle mass flow. This relationship stems from the fact that particle impact velocity and collision frequency per unit time primarily govern pipe wall erosion. Within the tee, the fluid dictates impact velocity. Consequently, when inlet fluid velocity is constant, the kinetic energy of particles striking the wall remains constant. Particle mass flow rate, however, solely governs the number of wall impacts per unit time. Given that collision count exhibits a linear relationship with erosion rate, the erosion rate consequently increases linearly with particle mass flow.

4. Response surface method coupling study

4.1. Mathematical model of the response surface method

Response Surface Method (RSM) is an experimental statistical method developed internationally in the 90s of the 20th century, which focuses on nonlinear problems and studies the impact of the interaction coupling of various factors on non-independent variables in the experimental process, which not only considers the random error of the experiment, but also uses multiple quadratic regression equations to fit complex implicit relationships, thereby expressing the relationship between multiple factors and response values.

Set the erosion rate (Y_1) as the response value, and the three factors affecting the erosion rate, flow rate (X_1), mass flow rate (X_2), and particle size (X_3) are the variables to be investigated. The response surface experiment was designed using Design-Expert software, and 17 experiments were required to select 3 test variables. The test protocol is shown in Table 1.

Table 1. Experimental scheme and results

Serial number	X ₁ Flow rate(m/s)	X ₂ Mass flow(kg/s)	X ₃ Particle size(μm)	Y ₁ Erosion rate (10 ⁻⁷ .kg.m ⁻² .s ⁻²)
1	8	0.15	150	2.79
2	8	0.15	200	4.47
3	3	0.15	150	0.368
4	15	0.15	300	42.4
5	8	0.2	100	3.34
6	15	0.3	150	49.6
7	3	0.3	150	0.697
8	8	0.3	100	5.01
9	3	0.05	150	0.126
10	3	0.15	300	0.838
11	8	0.3	50	5.25
12	8	0.2	150	3.96
13	8	0.05	300	2.83
14	15	0.05	150	8.27
15	8	0.3	300	17
16	15	0.15	50	11.2
17	8	0.05	50	0.871

The experimental data in Table 1 were fitted using Design-Expert software to obtain the regression equation:

$$\begin{aligned}
 Ln(Y_1) = & -4.75630 + 0.605654X_1 + 10.70295X_2 + 0.003094X_3 + 0.048912X_1X_2 \\
 & -0.000022X_1X_3 + 0.000558X_2X_3 - 0.015117X_{12} - 12.18372X_{22} + 0.00000620103X_{32}
 \end{aligned} \quad (3)$$

4.2. Analysis of variance and significance experiments

The analysis of variance and significance test for the above regression equations are shown in Table 2

Table 2. Analysis of variance of maximum erosion rate response surface

	Sum of squares	Degree of freedom	Mean Square	F value	P value	Significance
Model	40.87	9	4.54	260.64	< 0.0001	Significant
A- Inlet flow rate	30.48	1	30.48	1749.28	< 0.0001	Significant
B- Mass flow	6.33	1	6.33	363.57	< 0.0001	Significant
C- Particle size	2.87	1	2.87	164.98	< 0.0001	Significant
AB	0.0055	1	0.0055	0.3183	0.5902	Not significant
AC	0.0008	1	0.0008	0.0447	0.8386	Not significant
BC	0.0003	1	0.0003	0.0190	0.8942	Not significant
A ²	1.04	1	1.04	59.45	0.0001	Significant
B ²	0.1333	1	0.1333	7.65	0.0279	Not significant
C ²	0.0262	1	0.0262	1.50	0.2597	Not significant
Residual	0.1220	7	0.0174			
Total deviation	40.99	16		S/N:57.368	R ² _{Adj} =0.9970	

From Table 2, it can be seen that the $P < 0.0001$ of the established model is significant, indicating that the model is reasonably designed, and the adjustment coefficient $R^2_{Adj} = 0.9970$ indicates that 99.70% of the changes in the maximum erosion rate come from the flow rate, mass flow rate and particle size. Signal-to-noise (S/N) = 57.3628 > 20, indicating that the model has high fitting degree and good prediction accuracy, so it indicates that this response surface model is reliable. Through the analysis of the variance, the larger the variance value, the better the effect on the erosion rate, so the primary and secondary orders of the influence of the three factors on the erosion rate are $X_1 > X_2 > X_3$, that is, flow rate > mass flow > particle size. As shown in Table 2, the primary term A, B, C, and the quadratic terms A_2 and B_2 had a significant effect on the results ($p < 0.05$), while the quadratic term C_2 and the interaction terms AB, AC, and BC had no significant effect on the results ($p > 0.05$).

4.3. Response surface analysis

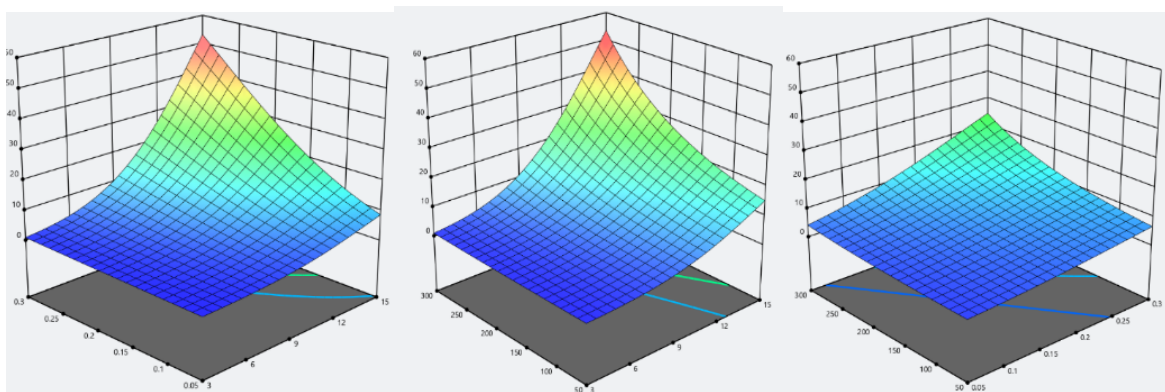


Figure 8. Response surface of the impact of the interaction of three factors on the erosion rate

Figure 8 indicates that erosion rate is significantly influenced by both flow velocity and particle concentration when particle size is fixed, with velocity exerting a substantially greater effect; erosion rate increases with both parameters. Similarly, under fixed particle concentration, erosion rate increases markedly with flow velocity and particle size, although velocity again dominates the influence. In contrast, when flow velocity is fixed, variations in particle size and concentration

exhibit minimal impact on erosion rate. Overall, the data demonstrates a clear hierarchy of influence: flow velocity has the strongest effect on erosion rate, followed by particle concentration, with particle size having the least significant impact.

5. Conclusion

(1) The influence of key factors on tee erosion rate exhibits distinct trends:

Flow Velocity: Erosion rate increases quadratically with higher inlet flow velocity. However, velocity impacts the erosion rate sensitivity to particle size and particle concentration.

Particle Size: Erosion rate displays a non-monotonic relationship with particle size, featuring a critical size where erosion is minimized. This critical size decreases with increasing flow velocity (e.g., 150 μm at 3 m/s vs. 100 μm at 8 m/s). At high velocity (15 m/s), erosion increases linearly with particle size.

Particle Concentration: Erosion rate increases linearly with particle concentration. This linear trend remains consistent regardless of variations in flow velocity or particle size.

Erosion Location: For small particles, maximum erosion occurs near the outlet junction. For large particles, the primary erosion zone shifts to the tee intersection.

(2) The relationship between inlet flow rate, particle size, and particle mass flow rate and erosion wear rate is expressed by the response surface method, and the formula of the erosion wear rate of the tee is $\text{Ln}(Y_1)=4.75630+0.605654X_1+10.70295X_2+0.003094X_3+0.048912X_1X_2-0.000022X_1X_3+0.000558X_2X_3-0.015117X_{12}-12.18372X_{22}+0.00000620103X_{32}$.

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