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The Interplay of Structural Stiffness and Mechanical Vibrations in Multi-Level Constructions

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Abstract. This paper explores the structural dynamics and vibration of a model can be used in varying types of structures. Structural dynamics has a wide range of applications like civil engineering, mechanical engineering, transportation, aerospace. When designing aircrafts, it is necessary to consider the disadvantages that comes from vibration due to its mass, stiffness and many inherent characteristics. For example, miscalculation of the influences of the separation surface joints of the aircrafts can be fatal, showing the significance of carefulness in calculation and consideration for theories and methods of structural dynamics. More application will be to be mentioned later on in the paper including large bridges, high-rise buildings and so on.

Keywords: MDOF analysis, structural dynamics, Lumped mass, building vibration, Tuned Mass Damper.

1. Introduction

1.1. An overview of a spring-mass model with 3DOF

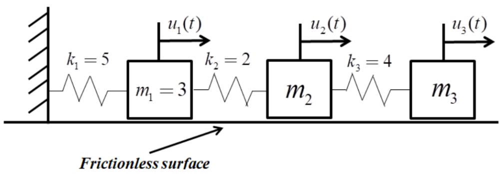

Figure 1. Three lumped masses with 3-DOF

Figure 1 shows that three lumped masses, m1, m2, and m3, connected with three springs, k_1 , k_2 , and k_3 , are placed on a frictionless surface, where

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$$m_1 = 3\ k_1 = 5\ k_2 = 2\ k_3 = 4$$

The unit of stiffness coefficient k_{ij} are kip/in. and that for m_i ’s are kip-sec²/in.

1.2. Motion analysis

Assume the displacement of m_1 , m_2 , and m_3 are u_1 , u_2 , and u_3 respectively.

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{1}$$

Hence, the acceleration can be obtained by calculating the second derivative of displacement:

$$a = \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} \tag{2}$$

m_1 is connected with a spring fixed in the wall, k_1 , providing a spring force, $k_1 u_1$, to the left. There is a spring, k_2 , connecting m_1 and m_2 , so m_1 is also under the effect of a spring force, $k_2(u_1 - u_2)$, pointing to the right, while a spring force, $k_2(u_2 - u_1)$, to the left is acting on m_2 . m_2 is also attached with a spring, k_3 , that is fixed on m_3 , so there is a spring force, $k_3(u_2 - u_3)$, away from the wall acting on m_2 . Since m_3 is only connected with one spring, so it's only experiencing one spring force, $k_3(u_3 - u_2)$, to the left. Hence, free body diagram of each mass can be built:

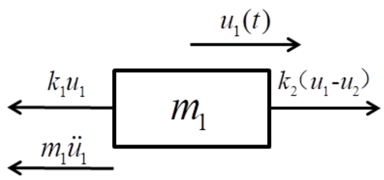

Figure 2. Free body diagram ofm_1

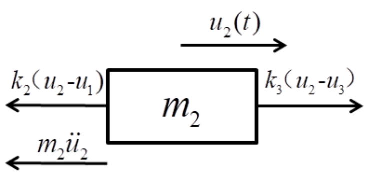

Figure 3. Free body diagram ofm_1

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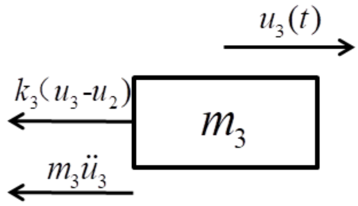

Figure 4. Free body diagram ofm_1

Since the system is undamped, the equation of motion is in the form of Equation (3)

$$m\ddot{u} + ku = u(t) \tag{3}$$

where m is mass, k is stiffness of spring, u is the displacement of mass, and $u(t)$ is the external force of the mass. By substituting the mass, spring force, and external force of each lumped masses analyzed in Figure 2, Figure 3, and Figure 4, the equation of motion of each mass can be expressed as

Equation 4 for m_1 :

$$m_1\ddot{u}_1 + (k_1 + k_2)u_1 - k_2u_2 = u_1(t) \tag{4}$$

Equation 5 for m_2 :

$$m_2\ddot{u}_2 - k_2u_1 + (k_2 + k_3)u_2 - k_3u_3 = u_2(t) \tag{5}$$

Equation 6 for m_3 :

$$m_3\ddot{u}_3 - k_3u_2 + k_3u_3 = u_3(t) \tag{6}$$

Based on Equation (4), Equation (5), and Equation (6) and the known value, matrix of the lumped masses M, external force F, and stiffness K of the system can be formed:

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{7}$$

$$F_{external} = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \tag{8}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_2 & k_3 \end{bmatrix} \tag{9}$$

Hence, we can get the matrix form of the equation of motion, Equation (10), of this system:

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$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -4 \\ 0 & -4 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \tag{10}$$

1.3. Two unknown masses

It is given that one mode of vibration is

$$\phi_1 = \begin{Bmatrix} 0.44444 \\ -0.66667 \\ 1.00000 \end{Bmatrix} \tag{11}$$

to determine the two unknown masses, it is effective to use eigenvalue equation can to connect the mass and mode of vibration.

$$[k - \omega^2 m]\phi = 0$$

Based on Equation (7), Equation (9), and the given eigenvector ϕ_1, the eigenvalue equation can be expressed as

$$\begin{bmatrix} 7-3\omega^2 & -2 & 0 \\ -2 & 6-m_2\omega^2 & -4 \\ 0 & -4 & 4-m_3\omega^2 \end{bmatrix} \bullet \begin{bmatrix} 0.44444 \\ -0.66667 \\ 1.00000 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

Equation (12) can be simplified as

$$\left[\begin{matrix} 7-3A & -2 & 0 \\ -2 & 6-m_2A & -4 \\ 0 & -4 & 4-m_3A \end{matrix} \right] \bullet \left[\begin{matrix} 0.44444 \\ -0.66667 \\ 1.00000 \end{matrix} \right] = \left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right], \text{ where } A = \omega^2 \tag{13}$$

To calculate A, the first row of the $[k - \omega^2 m]$ matrix is used

$$0.44444\,(7 - 3A) + 2 \times 0.66667 + 0 = 0 \tag{14}$$

This leads to

$$A = 3.33335$$

Then substitute A to the second and third row of $[k - \omega^2 m]$ matrix respectively to calculate m_2 and m_3

$$(-2) \times 0.44444 - (6 - 3.33335m_2) \times 0.66667 - 4 \times 1.00000 = 0 \tag{15}$$

$$0 - 4 \times (-0.66667) + (4 - 3.33335m_3) \times 1.00000 = 0 \tag{16}$$

It turns out that

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$$m_2 = 4 \;\; m_3 = 2$$

1.4. Three natural frequencies

The natural three frequencies of this system can be obtained by using frequency equation:

$$\left| k - \omega^2 m \right| = 0 \tag{17}$$

where k is stiffness of the system, ω is the natural frequency of the system and m is mass. Then, substitute Equation (7) and Equation (9) to get the frequency equation of this system:

$$\begin{bmatrix} 7-3\omega^2 & -2 & 0 \\ -2 & 6-4\omega^2 & -4 \\ 0 & -4 & 4-2\omega^2 \end{bmatrix} = 0 \tag{18}$$

By replacing ω^2 with B, Equation (18) can be expressed as

$$\begin{bmatrix} 7-3B & -2 & 0 \\ -2 & 6-4B & -4 \\ 0 & -4 & 4-2B \end{bmatrix} = 0 \tag{19}$$

Solve the matrix equation:

$$(7-3B)\left[(6-4B)(4-2B)-16\right] - (-2)\left[-2(4-2B)-0\right] + 0 = 0$$

Hence

$$B_1 = 0.219 \;\; B_2 = 3.333 \;\; B_3 = 2.281$$

Therefore, by calculating the square root of each B, the natural frequencies of this system are

$$\omega_1 = 0.49 \;\; \omega_2 = 1.83 \;\; \omega_3 = 1.51$$

2. Shear frame model with 3DOF

2.1. Overview

To showcase the varying application of mechanical vibration and for better understanding of its existence and function in a multi degree of freedom system, it is appropriate to use a three-story building with three degrees of freedom as an example and analyze its dynamic behavior

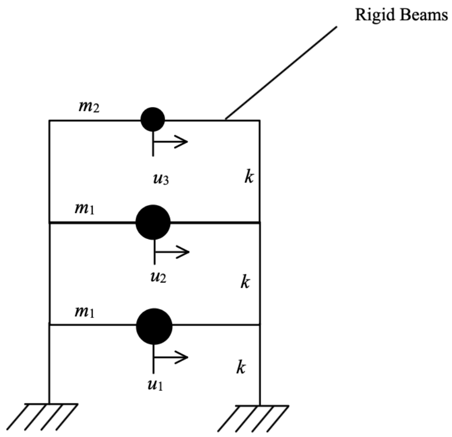

Figure 5. Three story shear-frame building with 3-DOF

Figure 5 illustrates the 3-DOF system with lumped mass in the middle of each floor. Several information is given: the story stiffness k is 326.33 kips/inch, which is the stiffness sum for the floor. The lowest natural vibration frequency for the system, ω , is 18.4883 rad/s, the two lowest eigenvectors are:

$$\phi_1 \begin{bmatrix} 0.8024 \\ -1.3898 \\ 1.6048 \end{bmatrix} \phi_2 \begin{bmatrix} 1.6011 \\ 0 \\ -1.6011 \end{bmatrix} \tag{20}$$

2.2. Analysis

One suitable approach to analyze its different masses in each floor and its natural vibration frequency with eigenvectors is to use the eigenvalue equation:

$$(K - \omega_{n^2} M)\phi_n = 0 \tag{21}$$

For better clarity, it is algebraically manipulated into

$$K\phi_n = \omega_{1^2} \bullet M\phi_1 \tag{22}$$

Where K is the stiffness matrix that represents the elastic force and M is the mass matrix that represents the mass distribution of the system. It should appear to be diagonal because the mass of the structure is concentrated at the nodes, and each mass is associated only with its corresponding degree of freedom, without interaction with other degrees of freedom. Both matrices are not given but we can break it down by first deriving the equation of motion from Figure 1.

For mass1 on the first floor, the force acting on it includes the spring force and the force from the rigid beam connecting to the mass above. For mass 1 on the second floor, the forces include the spring force from the first floor and third floor. For mass 2 on the third floor, the force acting on it is the spring force from only the 2nd floor. Together, the equation of motion for the entire building can be expressed as

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$$\begin{gathered} m_1\ddot{u}_1 + 2ku_1 - ku_2 = 0 \\ m_1\ddot{u}_2 - ku_1 + 2ku_2 - ku_3 = 0 \\ m_2\ddot{u}_3 - ku_2 + ku_3 = 0 \end{gathered} \tag{23}$$

For more in-depth analysis, please refer to [1,2]. Equation (23) can be simplified into the matrix form

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = 0 \tag{24}$$

With the new matrices and all known values, it is brought back to the eigenvalue equation

$$\begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \begin{bmatrix} 0.8024 \\ -1.3898 \\ 1.6048 \end{bmatrix} = 18.4883^2 \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} 0.8024 \\ -1.3898 \\ 1.6048 \end{bmatrix} \tag{25}$$

And further simplified into

$$\begin{bmatrix} 70.1588 \\ 121.521568 \\ 70.1588 \end{bmatrix} = \begin{bmatrix} 274.2741509m_1 \\ 475.0575958m_1 \\ 548.5483018m_2 \end{bmatrix} \tag{26}$$

As a result of isolating the two masses m1 and m2, they are found to be

$$M = \begin{bmatrix} 0.2558 \\ 0.2558 \\ 0.127899 \end{bmatrix} \tag{27}$$

The two m1 values match each other, as they should, proving the accuracy of the result.

The second lowest natural vibration frequency of the system can be obtained similarly, using the Eigenvalue equation and isolating the variable

$$\begin{gathered} K\phi_n - \omega n^2 M\phi_n = 0 \\ K\phi_2 = \omega 2^2 \bullet M\phi_2 \end{gathered} \tag{28}$$

$$\begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \begin{bmatrix} 1.6011 \\ 0 \\ -1.6011 \end{bmatrix} = \omega 2^2 \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} 1.6011 \\ 0 \\ -1.6011 \end{bmatrix}$$

$$\begin{bmatrix} 1044.941904 \\ 0 \\ -522.470952 \end{bmatrix} = \omega 2^2 \begin{bmatrix} 0.40956138 \\ 0 \\ -0.2047790889 \end{bmatrix} \tag{29}$$

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Theoretically, the result from the square root can be both positive and negative, but considering that this is an application question, the negative value will not be taken into account

$$\omega_2 = \sqrt{\frac{1044.941904}{0.40956138}} = 50.51\ rad/sec \tag{30}$$

$$\omega_2 = \sqrt{\frac{-522.470952}{-0.2047790889}} = 50.51\ rad/sec \tag{31}$$

The final value of the second lowest natural vibration frequency is calculated to be 50.51 radian per second, which corresponds to the other result that is calculated using a different set of data, suggesting the accuracy of the calculation.

3. Shear frame model with 2DOF

3.1. Overview

The mass and stiffness properties of the idealized two-story frame are shown in Figure 6.

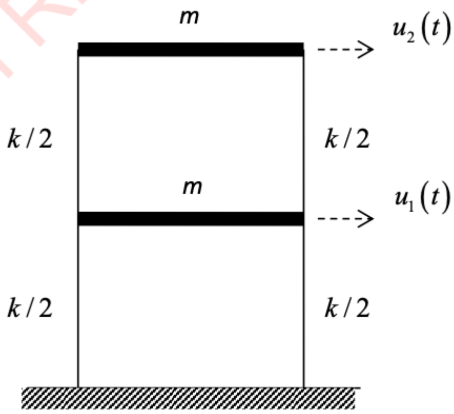

Figure 6. Two-story frame property

3.2. Formulate the equation of motion

To find the equation of motion, it's very important to consider it as lots of SDOF system,

The forces acting on upper mass:

$$m[u1]''(t) + k(2u1 - u2(t)) = 0$$

$$= m[u1]''(t) + 2ku1 - ku2(t) = 0$$

The forces acting on lower mass:

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$$m[u2]''(t) + k(u2 - u1(t)) = 0$$

$$= \ m[u2]''(t) + ku2 - ku1(t) = 0$$

Therefore, the equation of motion

$$\begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \begin{bmatrix} \ddot{u}_1(t) \\ \ddot{u}_2(t) \end{bmatrix} + \begin{bmatrix} 31.54 & -15.77 \\ -15.77 & 15.77 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3.3. Compute the two natural frequencies (1and 2) and the mode shapes

Using the formula

$$K^2 - w^2 M = 0$$

to the eigenvector

$$1 : K^2 - w^2 M = 0$$

$$det(k\ \hat{}\ 2 - w\ \hat{}\ 2M) = 0$$

$$= det\left(\begin{bmatrix} 31.5 & -15.77 \\ -15.77 & 15.77 \end{bmatrix} - w\ \hat{}\ 2 \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}\right)$$

$$= (31.54 - 50w\ \hat{}\ 2)(15.77 - 50w\ \hat{}\ 2) - (-15.77)(-15.77) = 0$$

$$= (31.54 - 50w\ \hat{}\ 2)(15.77 - 50w\ \hat{}\ 2) - (15.77)\ \hat{}\ 2$$

$$w1 = 6.8231$$

$$w2 = 17.8622$$

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3.3.1. Mode shape

$$k - mw\ \hat{}\ 2 = 0$$

$$\begin{bmatrix} 31.54 - 50w^2 & -15.77 \\ -15.77 & 15.77 - 50w^2 \end{bmatrix} \begin{bmatrix} 1 \\ u_2(t) \end{bmatrix} = 0$$

$$\phi = \begin{bmatrix} 1 \\ u_2(t) \end{bmatrix}$$

$$\phi 2 = \begin{bmatrix} 1 \\ -0.618 \end{bmatrix}$$

Put back into the formula and to find the mode shape:

Using the initial conditions given below, determine the undamped displacement response of the structure.

$$u\left(0\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\ u'\left(0\right) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\gamma = \gamma\left(0\right)\cos\left(\omega_n t\right) + \frac{\dot{\gamma}(0)}{\omega_n}\sin\left(\omega_n t\right)$$

$$\phi = \begin{bmatrix} 1 & 1 \\ 1.618 & -0.618 \end{bmatrix}$$

$$\phi^{-1} = \begin{bmatrix} 0.276 & -0.447 \\ 0.724 & 0.447 \end{bmatrix}$$

and that,

$$\boldsymbol{v} = \phi Y$$

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$$y(0) = v(0)\phi \ \hat{} \ -1$$

$$y'(0) = v'(0)\phi \ \hat{} \ -1$$

$$= [0.894]$$

$$[-0.894]$$

By orthogonality and using the undamped equation of displacement, we could simplify the MDOF system,

The undamped system is subjected to a suddenly applied force at the first-floor mass:

Derive equations for the lateral floor displacements as functions of time, and

$$K = \phi \ \hat{} \ t\ k\phi$$

$$k1 = \phi 1 \ \hat{} \ t\ k\phi 1$$

$$k2 = \phi 2 \ \hat{} \ t\ k\phi 2$$

$$k1 = \begin{pmatrix} 1{,}1618 \end{pmatrix} \begin{pmatrix} 31.54 & -15.77 \\ -15.77 & 15.77 \end{pmatrix} \begin{pmatrix} 1 \\ 1.618 \end{pmatrix}$$

$$k2 = \begin{pmatrix} 1{,}1618 \end{pmatrix} \begin{pmatrix} 31.54 & -15.77 \\ -15.77 & 15.77 \end{pmatrix} \begin{pmatrix} 1 \\ -1.618 \end{pmatrix}$$

$$P = \phi \ \hat{} \ t\ p(t)$$

$$P1 = (1{,}1.618) \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

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$$P2 = (1, -0.618)\begin{bmatrix}10\\0\end{bmatrix}$$

$$y = P/K\ (1 - cosw(t))$$

$$y1 = 0.45886(1 - cos6.8231t)$$

$$y2 = 0.17527(1 - cos17.8622t)$$

3.4. Solution

$$v = y\phi = \begin{bmatrix}1 & 1\\1.618 & -0.618\end{bmatrix}\begin{bmatrix}y1\\y1\end{bmatrix}$$

$$= \begin{bmatrix}0.45886\,(1 - cos68231t) + 0.17527(1 - cos17.8622t)\\0.742\,(1 - cos6.8231t) - 0.108(1 - cos17.8622t)\end{bmatrix}$$

The story drift in the second story as a function of time,
which equals the product of stiffness,

$$v2 - v1 = [0.742(1 - cos6.8231t) - 0.103(1 - cos17.8672t)] - [0.4589(1 - cos6.8231t) + 0.17527$$

$$(1 - cos17.8622t)]$$

$$= 0.28314(1 - 6.8231t) - 0.28327[1 - cos17.862]$$

The story shear in the 2nd floor is

$$F = (v2 - v1)k$$

$$= 15.77(0.28314(1 - cos(6.8231t)) - 0.28327(1 - cos(17.8662t))$$

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$$= [4.4651261 - cos(6.8231t)] - [4.46712(1 - cos(17.8622t))]$$

Assuming the stiffness proportional damping ($C = \alpha K$) with modal damping ratio corresponding to the first mode given as§1=0.05. Find the damping matrix C and the
modal damping ratio for the second mode (§2).
First, we need to find the value of a1

$$\xi_1 = \frac{\alpha K_1}{2M_1\omega_1}$$

Plug in numerical data

$$0.05 = \frac{21.793a}{2 \bullet 0.4682 \bullet 6.823}$$

$$q = 0.0466$$

And there for, the second mode:

$$E_2 = \frac{ak_2}{2M_2\omega_2}$$

$$E_2 = \frac{0.0466 \times 57.055}{2 \bullet 0.1988 \bullet 7.8622} = 0.8505$$

Using the value of$\alpha = 0.01466$, the damping matrix is computed as

$$C = ak$$

$$= 0.0466 \begin{bmatrix} 31.54 & -15.77 \\ 15.77 & 15.77 \end{bmatrix} = \begin{bmatrix} 1.4698 & -0.7349 \\ 0.7349 & 0.7349 \end{bmatrix}$$

The key reason that the modal superposition method works for an undamped (MDOF) system is due to the orthogonality of the mode shapes with respect to the mass and stiffness matrices.

In an undamped MDOF system, the dynamic response can be expressed as a combination of independent modal responses. The orthogonality of the mode shapes allows the system of coupled

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equations of motion to be separated into a set of independent SDOF systems. Each of these independent systems corresponds to a specific mode of vibration, and their responses can be superimposed to find the total response of the system.

It's proved by the condition of orthogonality,

$$\begin{cases}\phi_m^T m\phi_n = 0 \\ \phi_m^T k\phi_n = 0\end{cases}(m \neq n)$$

If $m = n$

$$\phi_m^T m\phi = \phi_m^T m\phi_n$$

$$\gamma(0) = \frac{\phi_m^T mV(0)}{\phi_m^T m\phi_n}$$

$$\dot{\gamma}(0) = \frac{\phi_m^T m\dot{V}\left(0\right)}{\phi_m^T m\phi_n}$$

The key assumption for the modal superposition method to work for a damped MDOF system is that the damping matrix must allow for the decoupling of the equations of motion and it's also related to orthogonality. This means that the damping matrix should be such that the system can still be represented in terms of independent modal coordinates.

Which,

$$C_n = \Phi_n^T C_b \Phi_n$$

And for the Proportional Damping Expression, it represents the damping matrix in terms of the mass matrix M, stiffness matrix K, and a proportional damping coefficient a and b, ensures the orthogonality of the mode shapes allows the system to be treated as separated modes.

$$C_n = \alpha_b \phi_n^T M\left[M^{-1}K\right]^b \Phi_n$$

4. Application

4.1. Working principles

There are many applications of structural dynamics in real buildings, such as for TMD system. TMD system is Turned Mass Damper, which is a type of damper especially used in tall buildings, which works by a combination of a heavy object and a spring or hydraulic device. When a building is

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shaken by an external force such as a strong wind or an earthquake, the TMD will, through its own mass and motion, create a force in the opposite direction of the shaking of the building, thereby reducing the shaking of the tall building.

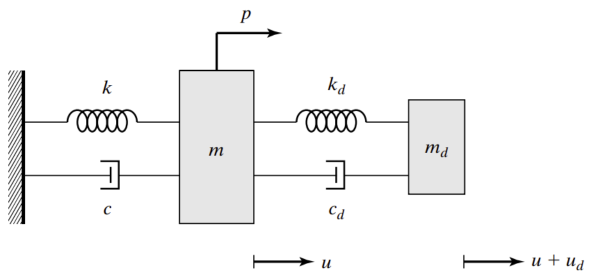

Figure 7. Tuned mass damper system

Specific to the working principle, when a strong wind comes, the damper will first detect the impact of the wind on the building and the degree of shaking caused by the wind. Then, through a computer-controlled device, the dampers use suspended cables, springs and hydraulics to absorb the shaking from the building. This means that if the wind blows from a certain direction, the damper will swing in the opposite direction, reducing the shaking of the building. This reverse motion can effectively offset the impact of strong winds or earthquakes on buildings and ensure the stability and safety of buildings.

4.2. Examples

For example, for Shanghai Tower. Shanghai Tower, located at Lujiazui Financial and Trade Zone, is a huge high-rise landmark skyscraper in Shanghai, currently the tallest building in China and the third tallest building in the world. Shanghai Tower is mainly used for office, hotel, business, sightseeing and other public facilities; The main building is 127 floors above ground, the building height is 632 meters, so under this circumstance the stability and safety is very important for Shanghai tower,because if the building is higher the impact by wind and earthquakes will become greater. And in the Shanghai Tower, there are also have TMD system to reduce the shaking, which is called "Shanghai Eye" located between 125th and 126th floors, it is made of weights and slings, similar to a giant compound pendulum, it's a beautiful sculpture and the architects built it into a concert hall. This is a wind damper weighing 1,000 tons, which is currently the heaviest damper in the world, accounting for about 0.118% of the weight of the building. The giant damper is suspended inside the building by 12 steel cables, each 25 meters long. And this TMD system will reduce building motion, which induced by moderate and high winds to approximately one-half of what the building motion would have been without the system operating. It can reduce the shaking of the building by 45%. During the landing of Typhoon Lekima in 2019, the unilateral swing of the damper was more than 50 centimeters, and the instantaneous peak value even reached 70 centimeters, which greatly reduced the shaking of the building and ensured the stability and safety of the building.

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Figure 8. “Shanghai Tower”

Figure 9. “Shanghai Eye”

Furthermore, here is the John Hancock Tower:

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Figure 10. “John Hancock Tower”

Figure 11. Location of TMD system

Completed in 1975, the John Hancock Tower is the first completed fully operation installation and the first major building featuring a TMD in the USA. After base building was complete, the windows falling from the John Hancock Tower garnered the press, but the frame stability and sway under wind loads is the main reason what the TMD system is installed, it aims to mitigate the response induced by wind action,

Hired specifically to reduce the building response during wind events, a dual Tuned Mass Damper system to increase structural damping, with one 275-ton mass damper installed at each end of the 58th floor of the building. When excessive motion is detected, the two TMDs are activated in opposing directions to tame the natural twisting of the building’s elongated shape.

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4.3. The evolution

4.3.1. John Hancock Tower

The TMDs simultaneously work together to reduce the sway in the narrow direction and also in opposition to reduce the twisting of the building. Each mass block is comprised of lead contained in a steel coffer box. In operation, the masses may move up to six feet with an operating cycle of about seven seconds. Each damper has the dimensions of 5.2 x 5.2 x 1 m, riding on a 9 m track. The mass is fitted with hydraulic dampers that are enabled when the floor acceleration exceeds 0.003 g. This solution proved to be a very effective method reducing the motion of the building by 40% to 50%.

4.3.2. Shanghai Tower

The Shanghai Tower TMD was no different, but with two special features: one aesthetic, and the other technical.

For the aesthetic feature, the cables that suspend the thousand-ton steel mass – are visible from an observation area. The device extends through five floors at the top of the building, and designers chose to expose some of its mechanics to the public and shows the tower's remarkable engineering.

For the technical feature, the Shanghai Tower TMD primarily uses another unique mechanism, namely eddy current. The bottom of the steel pendulum is fitted with 1800 brick-sized neodymium iron boron magnets. A large sheet of copper sits on the floor over which the pendulum is suspended. The copper neither attracts nor repels the magnets and doesn't make physical contact with them, but electromagnetic effects create a drag force as the pendulum moves laterally over the copper. Just as a hydraulic piston can be adjusted to create more or less resistance, the amount of frictionless drag this set-up creates can be adjusted by varying the distance between the magnets and the copper sheet. This innovative TMD delivers a 43% reduction in building acceleration under windy conditions.

The improvement throughout the years is that we are able to put aesthetic elements into the design and making it become one of the famous sites to visit in Shanghai.

In terms of reducing the buildings acceleration under windy condition, the tuned mass damper doesn't really have a great improvement, with both building having around 40% reduction in building acceleration. For more in-depth analysis on the applications mentioned, please refer to [3-6].

4.4. Discussion

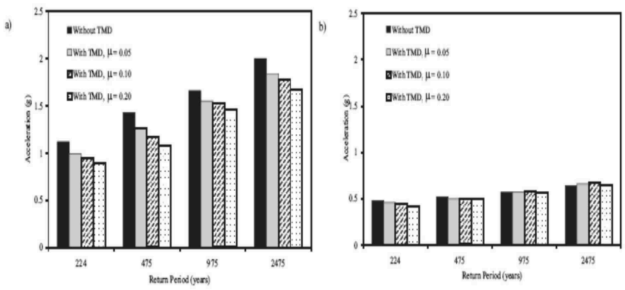

Figure 12. Mean peak absolute floor acceleration, 10-Story building with and without TMD: a) elastic response, b) inelastic response

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Figure 12 Mean Peak Absolute Floor Acceleration, 10-Story Building with and without TMD: a) Elastic Response, b) Inelastic Response

The benefit of using TMD system in tall buildings is obvious. As shown in the graph, with the implication of TMD in building, the elastic and inelastic response of the mean peak absolute floor acceleration is effectively reduced. With the advantages of TMD is Capable of significantly reducing dynamic response of linear structures, its construction is simple, and no need for external power source or sophisticated hardware, the implication of TMD had success made buildings taller.

However, there are also disadvantages of using TMD system in buildings. TMD require a relatively large mass, large space for installation, furthermore, it usually undergo large relative displacements and require large clearances and need to be mounted on a smooth surface to minimize friction and facilitate free motion. These features made TMD only can be used in buildings that occupies a large area of land. Moreover, make the damper itself can be a more complicated process than build the building.

Using different damper materials will result in different size and effects of the damper. Since then, we believe that the future of tuned mass damper is focusing on reducing the effect of the disadvantages of the damper and full utilize its advantages.

5. Conclusion

This study underscores the fundamental principles and practical applications of structural dynamics, highlighting their pivotal role in modern engineering. By analyzing dynamic systems through mathematical modeling, including multi-degree-of-freedom (MDOF) frameworks, the study demonstrates the accuracy and efficiency of vibration analysis in ensuring structural stability. Applications such as Tuned Mass Dampers (TMDs) in high-rise buildings like the Shanghai Tower and John Hancock Tower illustrate the innovative integration of theoretical principles into practical solutions. These systems significantly mitigate vibrations caused by external forces, enhancing the safety and comfort of occupants. While TMD systems have proven effective, challenges remain, including their size, cost, and space requirements. Future advancements in material science and engineering design may help overcome these limitations, enabling the development of more compact and efficient vibration mitigation systems. This progress will continue to push the boundaries of architectural design and structural safety, paving the way for taller, more resilient structures worldwide.

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All the authors contributed equally to this work and should be considered as co-first author.

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