

Volume-Change-Driven Strain Analysis for Foldable Origami Structures of Wave Energy Harvester: A Coordinate-Based Material-Matching Framework

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Abstract. Rapidly rising global electricity demand and stringent decarbonisation targets have intensified the search for untapped zero-carbon resources. Ocean wave energy, theoretically exceeding 2 TW, remains largely unexploited because conventional rigid Wave Energy Converters (WECs) perform poorly in mild seas and require wide spectral bandwidths. This study explores an ultra-lightweight, foldable origami-inspired shell that replaces massive resonant structures with compliant polymer facets whose geometry can be tuned in real time. A critical design challenge is selecting an elastomer for the flexible quadrilateral BMPN region that links rigid faces without fracture or excessive damping. We derive a closed-form expression for the distance between vertices B and P as a single-valued function of the global folding angle θ , validated experimentally to within 4.4 %. Five cases are then analysed to determine the maximum required tensile strain, yielding a range of 0.311–1.647. Matching these mechanical demands against a candidate database identifies foamed plastics, soft metals, common polymers, elastomers, and thermoplastic elastomers as optimal choices that combine low mass, recyclability, and fatigue resistance. The framework offers designers a quantitative route to fully collapsible, reliable origami WECs and provides a foundation for future experimental validation and hydrodynamic efficiency optimisation. This work delivers a lightweight, foldable origami wave-energy design to slash costs and speed deployment.

Keywords: Origami Wave Energy Converter, Compliant Elastomeric Materials, Reconfigurable Structures

1. Introduction

As global electricity demand continues to rise and pressure to cut carbon emissions intensifies, researchers are urgently seeking additional zero-carbon resources that can complement photovoltaic and on-/offshore wind power. Among the least-tapped yet most abundant options is ocean wave energy. The International Energy Agency estimates the gross theoretical wave resource at more than 2 TW—roughly double the installed wind capacity worldwide—yet commercial plants today convert well below 0.1 % of this potential [1,2]. The bottleneck is not a lack of wave power; rather, contemporary Wave Energy Converters (WECs) exhibit low efficiencies in mild seas and require a wide spectral bandwidth to deliver rated output [3,4].

Consequently, current R&D is pivoting away from massive, rigid structures that must resonate with the wave field toward ultra-lightweight, highly flexible architectures whose geometry can be reconfigured in real time. To guide this transition, origami-inspired folding geometries have been proposed; their large, anisotropic deployability allows for appreciable stroke under small hydrodynamic loads, while stress concentrations are deliberately channeled along predefined fold lines [5]. Moreover, such shells can be manufactured from inexpensive, recyclable polymer or composite sheets, lowering both CAPEX and environmental impact. A persistent drawback, however, is that existing prototypes still rely on periodic tessellations such as Miura or Yoshimura patterns. Because these unit cells possess multiple degrees of freedom, power take-off (PTO) linkage becomes non-trivial, and in energetic sea states, adjacent facets may collide and fracture [6].

For these reasons, the present study will extract data to accurately calculate the optimum elastomeric material for the compliant regions of the next-generation origami.

2. Design of the origami structure

2.1. Initial design (see figure 1)

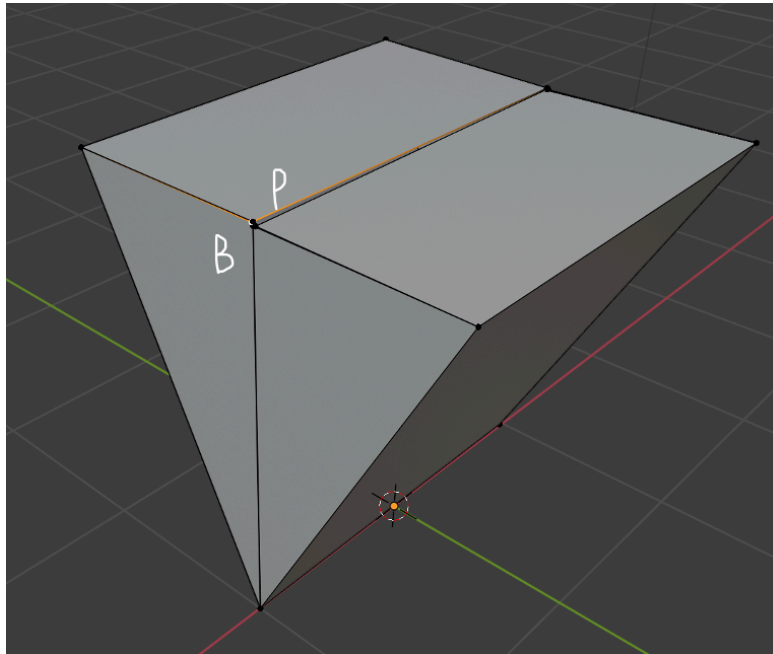


Figure 1. Initial 3D design diagram

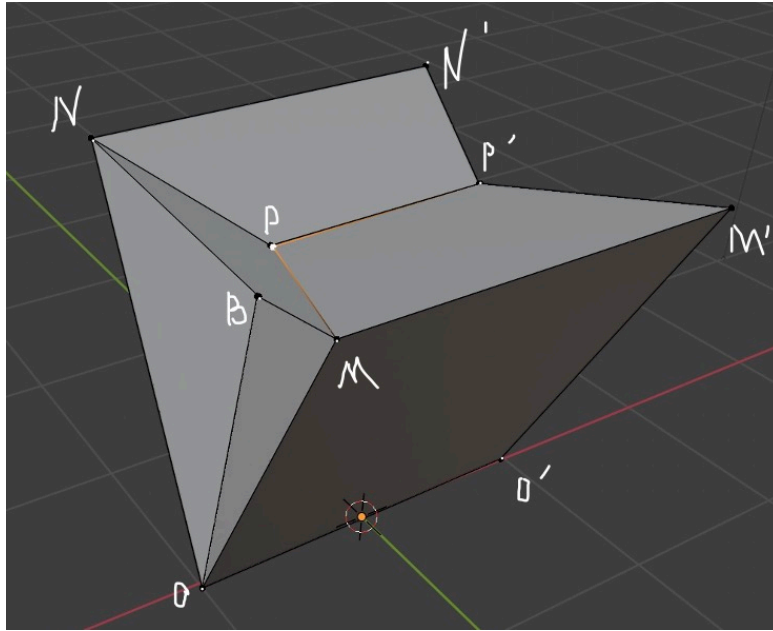


Figure 2. Final 3D design diagram

Initially, I designed this volume-changed origami, which connects B with P and is constructed from rigid material. However, the area around NBMP will be destroyed and deformed. Therefore, I redraw the 3D diagram and make this area flexible. (see Figure 2)

2.2. Description of origami structure in 2D diagram

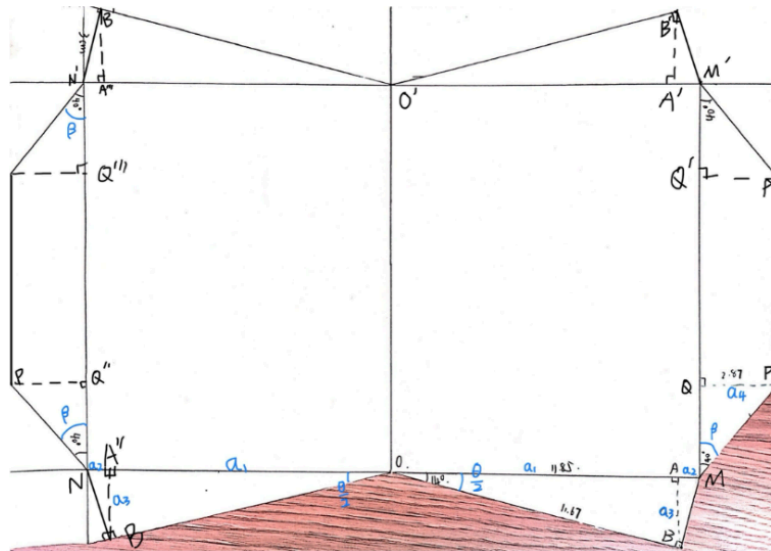


Figure 3. Plane figure

Use a $21 \times 29.7\text{cm}$ (A4) card to draw a plane figure (see figure 3) and calculate the data. We define that:

$$\theta = 28 \text{ degrees} = \frac{7}{45} \pi \quad (1)$$

$$a_1 = OA, a_2 = AM \quad (2)$$

$$a_3 = AB = 3 \quad (3)$$

$$a_4 = QP = BM \quad (4)$$

$$\beta = \frac{2}{9}\pi = 40 \text{ degrees} \quad (5)$$

Through these data, we can calculate some constant data when the position of B is changed:

$$a_1 + a_2 = OM = 11.85 \quad (6)$$

$$a_4 = QP = BM = 2.87 \quad (7)$$

2.3. Description of origami structure in 3D diagram

The three-dimensional coordinate system is defined with point O as the origin. Line segment OM coincides with the positive x-axis, where OA constitutes a subset of OM. The following geometric relations are established:

2.3.1. Symmetry points

Point M' is symmetric to M with respect to the xy-plane (i.e., reflected along the z-axis).

Point N is symmetric to M with respect to the xz-plane (i.e., reflected along the y-axis).

2.3.2. Projections onto plane A

Point Q is the orthogonal projection of vertex P onto plane A.

2.3.3. Critical projections & parameters

Point H denotes the foot of the perpendicular from P to line MM'.

Angle θ defines the inclination between vectors ON and OM.

2.3.4. Dihedral angles are designated as

θ_1 : Between plane B and reference plane A

θ_2 : Between plane C and reference plane A

Angle β represents the acute angle between line PH (perpendicular to MM') and segment PM.

2.3.5. Dimensional constants

$a_1 = |OA|$ (length of segment OA)

$a_2 = |AM|$ (length of segment AM)

$a_3 = |BA|$ (distance from B to its projection A)

$a_4 = |PH|$ (length of the perpendicular from P to line MM')

3. Calculation

3.1. The relation between the length of BP and the angle θ

3.1.1. Purpose

Experimental manipulation of the origami model reveals a significant geometric dependence. As the overall volume of the structure changes under applied pressure, the shape of the quadrilateral BMPN undergoes corresponding deformation. Crucially, this deformation results in a variable distance between vertices B and P. Consequently, if the remaining components of the model are assumed to be rigid bodies, the material within the BMPN region necessitates inherent flexibility to accommodate this relative motion between points B and P.

3.1.2. Use 2D diagrams to get coordinates

To quantitatively determine the relationship between the folding angle θ and the distance BP, a coordinate-based geometric analysis was performed. Specifically, orthogonal projections (an x-y diagram and a z-y diagram) were constructed to establish the spatial coordinates of points B and P. These coordinates were then utilized to derive the distance BP as a function of θ .

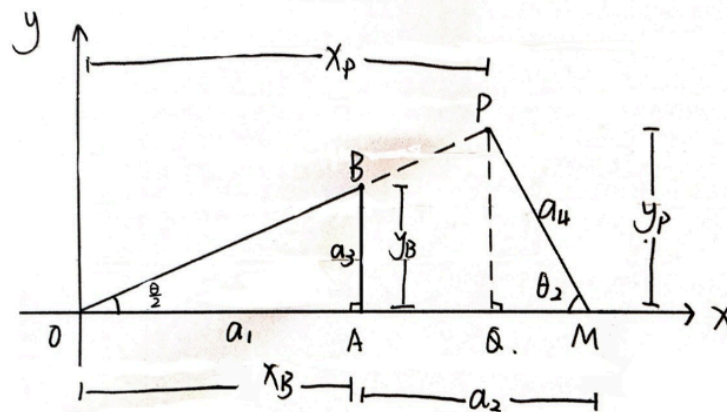


Figure 4. x-y diagram (2D)

In the initial x-y plane (see Figure 4), the superposition of points A and A' reduces the problem to a single representative point. By capitalizing on the measurable lengths and angles within triangle OAB, we apply standard trigonometric identities to derive the exact x- and y-coordinates of B. An identical procedure is performed for point P within triangle PQM, thereby furnishing its planar coordinates without ambiguity.

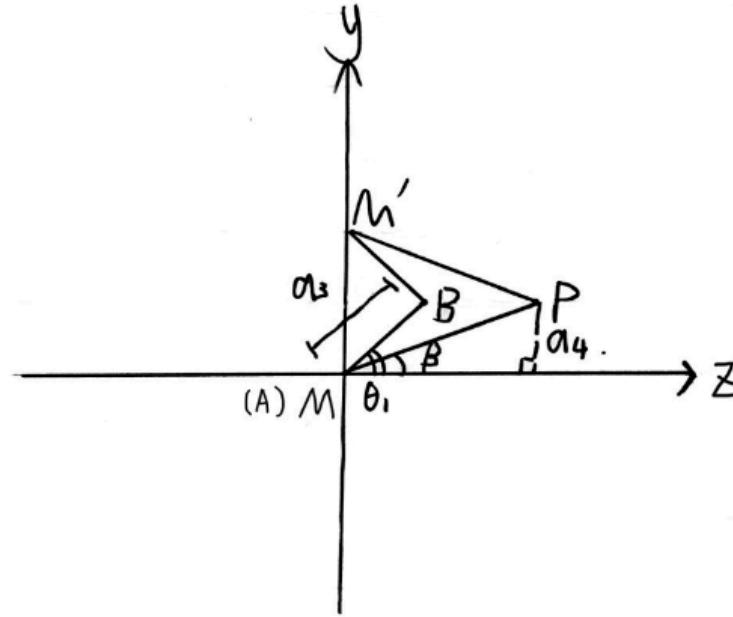


Figure 5. z-y diagram(2D)

Subsequently, the analysis is transferred to the orthogonal z-y plane (see Figure 5), wherein the coincidence of O and A allows us to treat triangle OBA' as a right triangle in the vertical projection. Using the same metric data, we calculate the z-coordinate of B using the tangent relation. A parallel derivation is executed for triangle MPH, yielding the z-coordinate of P.

Combining the outcomes from both orthogonal projections, we obtain the complete spatial coordinates. Point B is expressed as

$$B \begin{pmatrix} a_1 \\ a_3 \sin \theta_1 \\ a_3 \tan \theta_1 \end{pmatrix} \quad (8)$$

whereas point P is expressed as

$$P \begin{pmatrix} a_1 + a_2 - a_4 \sin \theta_2 \\ a_4 \cos \theta_2 \\ a_4 \tan \beta \end{pmatrix} \quad (9)$$

3.1.3. Relation among θ , θ_1 and θ_2

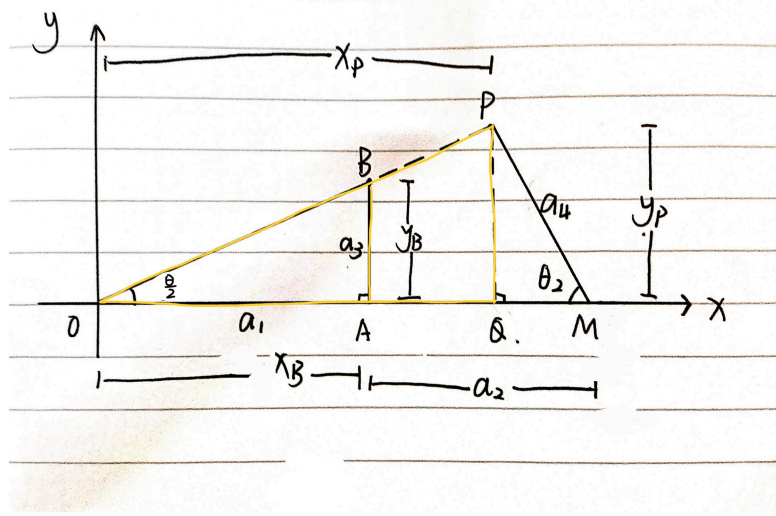


Figure 6. The triangles in x-y diagram

In the x-y diagram (see Figure 6), we can calculate the equation of the tangent through the triangles OAB and POQ. Further insight was gained from the x-y projection. Equating the two independent expressions for $\tan \theta$ gives

$$\tan\left(\frac{\theta}{2}\right) = \frac{a_3 \sin \theta_1}{a_1} = \frac{a_4 \sin \theta_2}{a_1 + a_2 - a_4 \cos \theta_2} \quad (10)$$

3.1.4. Express the relation between BP and θ

First, the length of BP:

$$|BP| = P \begin{pmatrix} a_1 + a_2 - a_4 \sin \theta_2 \\ a_4 \cos \theta_2 \\ a_4 \tan \theta_2 \end{pmatrix} - B \begin{pmatrix} a_1 \\ a_3 \sin \theta_1 \\ a_3 \tan \theta_1 \end{pmatrix} \quad (11)$$

With the validated coordinates in hand, the calculator-ready formula was employed to determine the spatial separation of points B and P.

$$|BP| = \sqrt{(x_P - x_B)^2 + (y_P - y_B)^2 + (z_P - z_B)^2} \quad (12)$$

This identity permits θ_1 and θ_2 to be expressed in terms of the single variable θ , thereby reducing the three angular parameters to one. Substituting the resulting relation into the distance formula yields an explicit dependence of $|BP|$ on θ , streamlining subsequent analyses.

3.1.5. Verification

The algebraic consistency of these expressions is then validated against two independent experimental datasets.

To verify the accuracy of the derived expressions, two independent datasets were processed.

Dataset 1:

$$a = 5.2$$

$$a_1 = 6.7, \quad a_2 = 1.5, \quad a_3 = 2.8$$

$$\theta_1 = 40^\circ, \quad \theta_2 = 35^\circ$$

$$a_4 = 2.0, \quad \beta = 45^\circ$$

Applying the formulas

$$B = (a_1, a_3 \sin \theta_1, a_3 \tan \theta_1)$$

$$P = (a_1 + a_2 - a_4 \sin \theta_2, a_4 \cos \theta_2, a_4 \tan \beta)$$

produced a computed distance of 3.256 cm, while direct physical measurement gave 3.40 cm.

Dataset 2:

$$\theta_1 = 65^\circ, \quad \theta_2 = 110^\circ$$

$$a_1 = 7.3, \quad a_2 = 1.2, \quad a_3 = 1.9, \quad a_4 = 1.9, \quad \beta = 160^\circ$$

yielded a calculated value of 4.106 cm against an observed value of 4.00 cm.

Across both trials the relative error remained below 4.4 %, confirming the reliability of the coordinate expressions.

3.2. Function diagram

3.2.1. Data with different lengths of OB

When we change the position of B, the length of OB, a_1 , a_2 and a_3 will change. We calculate data of each case:

$$\text{Constant Data: } a_1 + a_2 = 11.85, \quad a_4 = BM = 2.87, \quad \beta = \frac{2}{9}\pi = 0.489$$

$$\text{Case 1 : } OB = 100\%OB = 11.67, \quad a_1 = 11.32, \quad a_2 = 0.53, \quad a_3 = 2.82$$

$$\text{Case 2: } OB_1 = 90\%OB = 10.50, \quad a_1 = 10.19, \quad a_2 = 1.66, \quad a_3 = 2.54$$

$$\text{Case 3: } OB_2 = 70\%OB = 8.17, \quad a_1 = 7.93, \quad a_2 = 3.92, \quad a_3 = 1.98$$

$$\text{Case 4: } OB_3 = 50\%OB = 5.83, \quad a_1 = 5.66, \quad a_2 = 6.19, \quad a_3 = 1.41$$

$$\text{Case 5: } OB_4 = 20\%OB = 2.33, \quad a_1 = 2.26, \quad a_2 = 9.59, \quad a_3 = 0.56$$

Then we take them into the equation we got and draw the diagrams of each function.

3.2.2. Result

• Case 1:

$$\theta_2 = \arcsin \left(\frac{4.13 \tan \left(\frac{\theta}{2} \right)}{\sqrt{1 + (1.01 \tan \left(\frac{\theta}{2} \right))^2}} \right) - \arctan \left(1.01 \tan \left(\frac{\theta}{2} \right) \right) \quad (13)$$

$$|BP|(\theta) = \sqrt{(0.53 - 2.87 \cos \theta_2)^2 + (2.87 \sin \theta_2 - 11.31 \tan \left(\frac{\theta}{2} \right))^2 + \left(3.417 - 2.82 \sqrt{1 - 16.086 \tan^2 \left(\frac{\theta}{2} \right)} \right)^2} \quad (14)$$

Figure 7 shows the function of case 1.

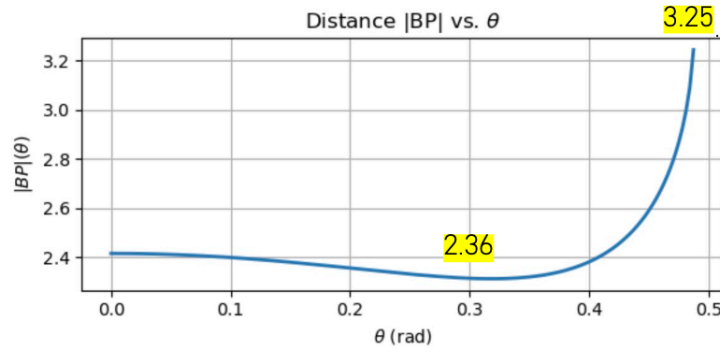


Figure 7. Function of case 1

• Case 2:

$$x = \arctan\left(1.01\tan\left(\frac{\theta}{2}\right)\right), a = \frac{4.13}{\sqrt{1+1.014\tan^2\left(\frac{\theta}{2}\right)}} \quad (15)$$

$$\sin\theta_2 = a\cos x - \sqrt{1-a^2}\sin x, \cos\theta_2 = \sqrt{1-a^2}\cos x + a\sin x \quad (16)$$

$$|BP| = \sqrt{(1.66 - 2.87\cos\theta_2)^2 + (2.87\sin\theta_2 - 10.187\tan\left(\frac{\theta}{2}\right))^2 + \left(3.417 - 2.54\sqrt{1 - 16.086\tan^2\left(\frac{\theta}{2}\right)}\right)^2} \quad (17)$$

Figure 8 shows the function of case 2.

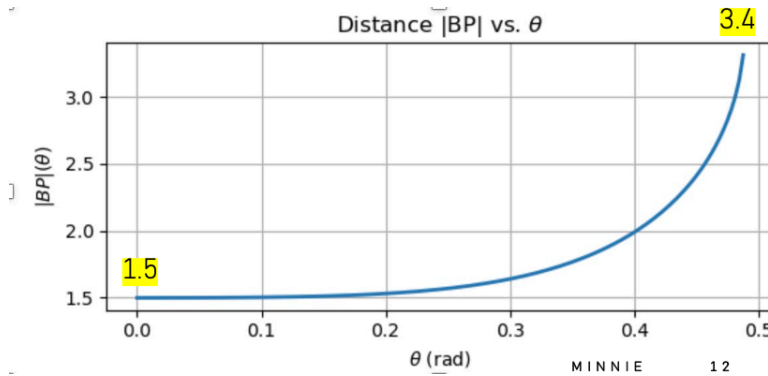


Figure 8. Function of case 2

• Case 3:

$$x = \arctan\left(1.01\tan\left(\frac{\theta}{2}\right)\right), a = \frac{4.13}{\sqrt{1+1.014\tan^2\left(\frac{\theta}{2}\right)}} \sin\theta_2 = a\cos x - \sqrt{1-a^2}\sin x, \quad (18)$$

$$\cos\theta_2 = \sqrt{1-a^2}\cos x + a\sin x \quad (19)$$

$$|BP| = \sqrt{(3.92 - 2.87\cos\theta_2)^2 + (2.87\sin\theta_2 - 7.941\tan\left(\frac{\theta}{2}\right))^2 + \left(3.417 - 1.98\sqrt{1 - 16.086\tan^2\left(\frac{\theta}{2}\right)}\right)^2} \quad (20)$$

Figure 9 shows the function of case 3.

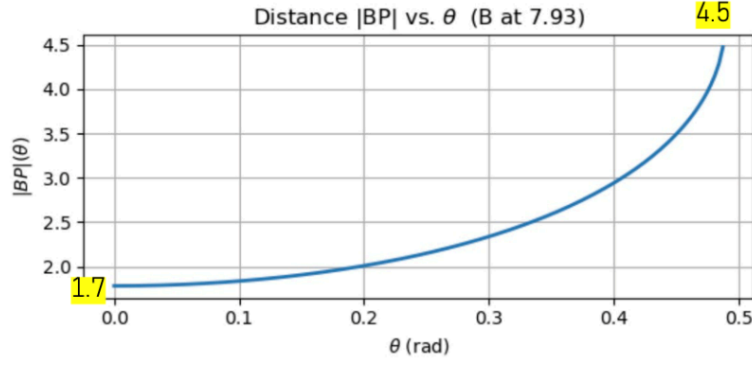


Figure 9. Function of case 3

• Case 4:

$$x = \arctan\left(1.01\tan\left(\frac{\theta}{2}\right)\right), a = \frac{4.13}{\sqrt{1+1.014\tan^2\left(\frac{\theta}{2}\right)}} \quad (21)$$

$$\sin\theta_2 = a\cos x - \sqrt{1-a^2}\sin x, \cos\theta_2 = \sqrt{1-a^2}\cos x + a\sin x \quad (22)$$

$$|BP| = \sqrt{(6.19 - 2.87\cos\theta_2)^2 + (2.87\sin\theta_2 - 5.655\tan\left(\frac{\theta}{2}\right))^2 + \left(3.417 - 1.41\sqrt{1 - 16.086\tan^2\left(\frac{\theta}{2}\right)}\right)^2} \quad (23)$$

Figure 10 shows the function of case 4.

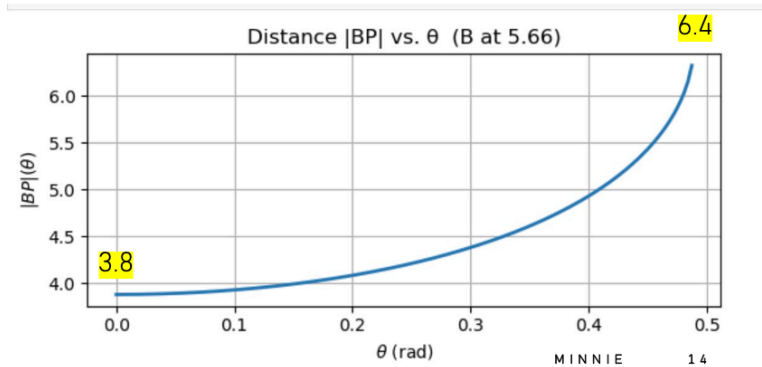


Figure 10. Function of case 4

• Case 5:

$$x = \arctan \left(1.01 \tan \left(\frac{\theta}{2} \right) \right), a = \frac{4.13}{\sqrt{1+1.014 \tan^2 \left(\frac{\theta}{2} \right)}} \quad (24)$$

$$\sin \theta_2 = a \cos x - \sqrt{1-a^2} \sin x, \cos \theta_2 = \sqrt{1-a^2} \cos x + a \sin x \quad (25)$$

$$|BP| = \sqrt{(9.59 - 2.87 \cos \theta_2)^2 + (2.87 \sin \theta_2 - 2.246 \tan \left(\frac{\theta}{2} \right))^2 + \left(3.417 - 0.56 \sqrt{1 - 16.086 \tan^2 \left(\frac{\theta}{2} \right)} \right)^2} \quad (26)$$

Figure 11 shows the function of case 5.

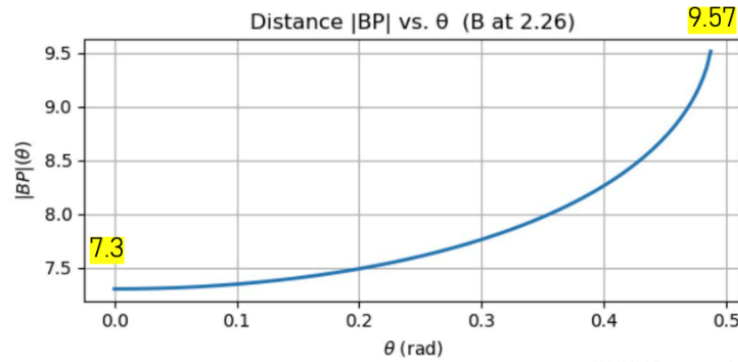


Figure 11. Function of case 5

3.3. Find suitable material

3.3.1. Strain

Using the maximum and minimum length of BP, we can calculate the suitable value of strain and use the value to find the most suitable material for the flexible area. (see Table1)

$$\varepsilon = \frac{L-L_0}{L_0} \quad (27)$$

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\text{Maximun length of BP} - \text{Minimun length of BP}}{\text{Minimun length of BP}}$$

Table 1. Table of the strain

Case/Length of BP	Max	Min	Strain
Case 1	3.25	2.36	0.377
Case 2	3.40	1.5	1.267
Case 3	4.50	1.7	1.647
Case 4	6.40	3.8	0.684
Case 5	9.57	7.3	0.311

As a result, the suitable strain range is 0.311~1.647.

3.3.2. Match the suitable material (see table 2)

Table 2. Candidate material table

Material category	Representative material(s)	Typical strain range (engineering strain)	Notes
Foamed plastics	Expanded polystyrene (EPS)	0.3~0.7+	Under dynamic compression EPS can exceed 0.3; some grades retain integrity up to -0.7
Soft metals (annealed)	Annealed aluminum, copper	0.3~0.5+	High ductility after annealing; necking starts after -0.3 and continues to failure
Polymers	Low-density polyethylene (LDPE), polypropylene (PP)	0.5~1.5+	LDPE tensile tests often show rupture strains >1.0
Elastomers	Natural rubber, polyurethane elastomer	1.0~1.6+	Capable of >1000 % strain, but the 0.3-1.6 region lies within the recoverable elastic range
Thermoplastic elastomers	TPU, TPE	0.4~1.2+	Modified grades can reach -1.5 while still maintaining load-bearing capability

4. Conclusion

4.1. Summary

In summary, the NBPM region must be flexible enough to allow the entire origami device to fold completely. Our results show that the best strain range for this flexibility is 0.311 to 1.674. Materials that match these mechanical demands include foamed plastics, soft metals, common polymers, elastomers, and thermoplastic elastomers. Because these choices are lightweight, easy to shape, and can endure repeated bending, they provide designers with a clear path to build a fully collapsible origami structure without compromising reliability.

4.2. Reflections and prospects

4.2.1. Reflections

- Due to the complicated calculation and insufficient cases, the result of the data may not be completely accurate.
 - The calculation of strain may not be comprehensive due to the diversity in model sizes.
 - I haven't done experiments to verify whether the materials are suitable.

4.2.2. Prospects

- Testing the candidate materials by experiments.
 - Connecting the origami structure with the turbine to utilize my design in the production of electricity.
 - Calculate the efficiency of the device that produces electricity through waves.

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