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Empirical Analysis of Dynamics in U.S. Stock-Bond Relation

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Abstract. In this paper, we explore the dynamic relationship of the U.S. stock and bond market from 2000 to 2024, with the stock market represented by the S&P 500 Index and the bond market by the U.S. 10-year Treasury Note yield. We examine the fluctuations in the stock-bond correlation across different economic cycles by utilizing a series of statistical methods, including Pearson and Spearman correlation coefficients, the rolling window method, and the DCC-GARCH model, revealing a shift from negative to positive correlations in this period, and identify key drivers of these changes. Additionally, we analyze the lead-lag relationship of stock and bond returns with the VAR model and Granger causality tests, showing a bidirectional relationship with both markets exhibiting lead-lag effects.

Keywords: Stock-Bond Correlation, Lead-Lag Relationship, Macroeconomic Factors.

1. Introduction

The relationship of stock and bond markets has long been a subject of significant interest, as it determines how assets are distributed and risks are managed within a portfolio. This relationship can change and affect diversification strategies, especially in the 2008 financial crisis, when the correlation turned negative, helping to reduce overall portfolio risk by serving as a hedge tool during equity market downturns. This paper aims to study the evolution of the U.S. stock-bond correlation from 2000 to 2024, a period that encompasses significant economic fluctuations. To analyze this relationship, we use Pearson and Spearman correlation coefficients to assess static relationships, the rolling window method to capture time-varying dynamics, and the DCC-GARCH model to enhance the estimation of conditional correlations. Then, we analyze several macroeconomic factors such as monetary policy, inflation, economic growth, and investor sentiment to explain the underlying reasons for the fluctuations in the stock-bond correlation. Moreover, we use the VAR model to explore the lead-lag relationship between stock and bond markets and apply Granger causality tests to determine the direction and strength of this relationship, finding that stock returns generally lead bond returns, while bond returns also influence stock returns. These analyses offer a valuable insight into the dynamics of stock-bond relationships and their implications for investment.

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2. Literature review

In early studies, Bollerslev, Engle, and Wooldridge used static rolling windows and EWMA methods to estimate stock–bond correlation [1]. To make dynamic measures, Engle and Kroner introduced multivariate GARCH models such as the VECH and BEKK, but they were too complex for practical use [2]. Later, Engle put forward the more flexible dynamic conditional correlation (DCC) model that considers time-varying correlations and allows for vector autoregressive structures [3]. Also, his empirical results show sharp changes in stock-bond correlations when dynamics shift during financial crises, where uncertainty and risk aversion matter. Then, Ilmanen and Andersson highlighted inflation expectations, macroeconomic volatility, and investor sentiment as key drivers of correlation changes [4,5]. At the same time, there were many studies focused on lead-lag relations between stock and bond markets too. Kwan showed that stock returns lead and bond returns lag in incorporating firm-specific information [6]. Downing, Underwood, and Xing explained this phenomenon by pointing out that the pace of information incorporation into stock prices is faster in transparent markets [7]. Likewise, Hong, Lin, and Wu showed that next-period stock returns predict investment-grade and high-yield bonds [8]. Since these studies mainly focus on specific aspects of the stock-bond relationship or research on particular methods, we develop a more comprehensive and systematic analysis by integrating various approaches to study this relationship and incorporating macroeconomic factors to analyze the reasons behind the relationship.

3. Data and summary statistics

We analyze the evolving relationship between the U.S. stock and bond markets from 2000 to 2024. To study the stock-bond correlation, we first conduct stationarity and ARCH tests to ensure the data is stable and exhibits heteroscedasticity, then apply Pearson and Spearman correlation coefficients, the rolling window method, and the DCC-GARCH model to examine the correlation dynamics. For the lead-lag relationship, we perform stationarity and cointegration tests first, followed by the VAR model and Granger causality tests to investigate the bidirectional influence between stock and bond returns.

3.1. Data source

We select the daily closing prices of the S&P 500 Index and U.S. 10-year Treasury Note Yield data from January 1, 2000 to November 31, 2024 from Investing.com as statistical samples. Because the data of these two decades covers various economic conditions, including growth, crises, and recovery, it enables us to make a comprehensive analysis. Additionally, the consistent period and daily frequency provide us with stable model parameters and a precise understanding of market movements.

3.2. Indicators

We use the S&P 500 Index as the stock indicator, which tracks the stock prices of the 500 largest U.S. companies, representing about 80% of total market capitalization. Its broad coverage makes it a reliable gauge of the U.S. economy and overall market performance. Meanwhile, we choose the U.S. 10-year Treasury Note as the treasury indicator, serving as a key benchmark in the debt market. Its yield reflects the U.S. government's borrowing cost and influences interest rates across financial instruments, making it a crucial indicator of market and economic conditions.

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3.3. Descriptive statistics

Table 1. Descriptive statistics

Statistics	Stock_Close	Bond_Close	Stock_Return	Bond_Return
mean	2111.0636	3.2740	-0.0001	-0.0003
std	1241.7915	1.3048	0.0123	0.0602
min	676.5000	0.5120	-0.1038	-0.4650
max	6090.2700	6.7900	0.1362	0.3290

The above table shows the descriptive statistics of the stock and bond data, and the following figure displays the original data of the S&P 500 Index and the U.S. 10-year Treasury Note, along with the returns of these two indicators.

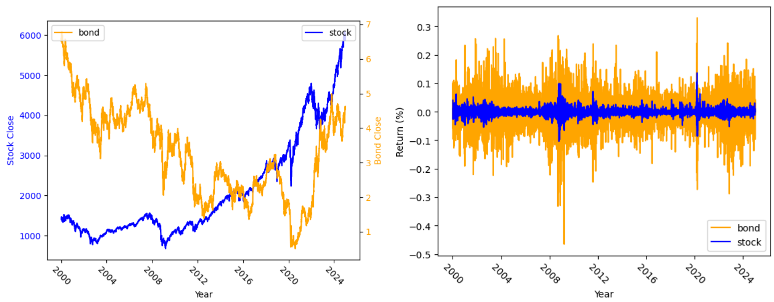

Figure 1. Data of the S&P 500 Index and the U.S. 10-year Treasury Note

4. Methodology and empirical analysis

4.1. Stock-bond correlation

4.1.1. Prerequisite tests

Before constructing a model to estimate the correlation between stocks and bonds, it is necessary to ensure that the data satisfies the basic assumptions of the model. The DCC-GARCH model requires the time series to be stationary and to exhibit ARCH effects. Therefore, we perform an Augmented Dickey-Fuller (ADF) test to examine the stationarity and an ARCH-LM test to test for heteroscedasticity.

The result of the ADF test shows that the p-values for both stock and bond returns are below the 0.05 significance level, indicating that the null hypothesis of a unit root is rejected and both series are stationary. Similarly, the ARCH-LM test reveals p-values below 0.05 for both series, rejecting the null hypothesis of no ARCH effects and confirming the presence of conditional heteroscedasticity.

Table 2. Results of the ADF test and the ARCH-LM test

Category	ADF Statistic	LM Statistic
Stock_Return	-14.269***	1795.805***
Bond_Return	-59.683***	547.853***

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

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4.1.2. Empirical methods and results

We successively use three methods—static correlation, rolling window correlation, and the DCC-GARCH model—to calculate and progressively optimize the correlation analysis.

First, we calculate the static correlation with Pearson and Spearman correlation coefficients. The Pearson correlation measures the linear relationship between raw data and is more affected by outliers and distribution; while the Spearman correlation is based on data ranks and is better suited for situations with outliers or non-normal distributions. Suppose we have two sets of data $X = \{x_1, x_2, \ldots, x_n\}, \quad Y = \{y_1, y_2, \ldots, y_n\}$. We rank them respectively to get $R_X = \{r_{x_1}, r_{x_2}, \ldots, r_{x_n}\}, \quad R_Y = \{r_{y_1}, r_{y_2}, \ldots, r_{y_n}\}$. The Pearson correlation coefficient and Spearman correlation coefficient are calculated by $r = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}}$ and $\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)} \ (d_i = r_{xi} - r_{yi})$.

Second, compared to the static annual Pearson and Spearman correlation coefficients, the rolling window method calculates correlations dynamically. It can better capture the short-term changes and volatility characteristics of the stock and bond markets. Suppose we have two sets of data $X = \{x_1, x_2, \ldots, x_n\}, \quad Y = \{y_1, y_2, \ldots, y_n\}$, with a window size of $w = 250$ days. For each time point t , the correlation from $t - w + 1$ to t is calculated. The Pearson and Spearman correlation coefficient at time t are given by $r_t = \frac{\sum_{i=t}^{t+w-1}\left(x_i - \bar{x}_t\right)\left(y_i - \bar{y}_t\right)}{\sqrt{\sum_{i=t}^{t+w-1}\left(x_i - \bar{x}_t\right)^2 \sum_{i=t}^{t+w-1}\left(y_i - \bar{y}_t\right)^2}}$ and $\rho_t = 1 - \frac{6\sum_{i=t}^{t+w-1}\left(r_{x_i} - r_{y_i}\right)^2}{w(w^2-1)}$.

Third, since the rolling window method is highly influenced by the subjective choice of window size, whereas the DCC-GARCH model can generate parameters automatically, the latter one can reflect the complex dynamic correlation more accurately. Its mathematical Formulas are as below:

$$r_t | I_{t-1} \sim N(0, \ H_t)$$

$$r_{i,t} = \mu_i + \epsilon_{i,t}, \quad i = 1,2$$

$$h_{i,t} = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$$

$$z_{i,t} = \frac{\epsilon_{i,t}}{\sqrt{h_{i,t}}}, \quad i = 1,2$$

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$$Q_t = (1 - a - b)\overline{Q} + aZ_t Z_t^{'} + bQ_{t-1}$$

$$R_t = D_t^{-1/2} Q_t D_t^{-1/2}$$

Constraints:

$$(\omega_i > 0)$$

$$(\alpha_i \geq 0), (\beta_i \geq 0)$$

$$(\alpha_i + \beta_i < 1)$$

The static correlation results show that both the Spearman and Pearson correlation coefficients of the overall period are negative, indicating a negative relationship between the stock and bond markets. Furthermore, the stock-bond correlation shows significant fluctuations from 2000 to 2024, with consistent trends across all three methods. From 2000 to 2003, the correlation remained negative and gradually strengthened, reaching its lowest point in 2003. Between 2004 and 2006, the correlation weakened and briefly became positive. During 2007 and 2009, the correlation declined sharply, hitting its lowest level amid the 2008 financial crisis. From 2010 to 2020, the correlation stayed generally negative, with occasional positive peaks in 2013, 2015, and 2018. From 2021 to 2024, the correlation exhibited an upward trend with notable positive peaks in 2021, 2023, and 2024. To compare these three methods, the static correlation captures the annual relationship, the rolling window method details short-term fluctuations, and the DCC-GARCH model more accurately captures the volatility, as reflected in its greater fluctuations.

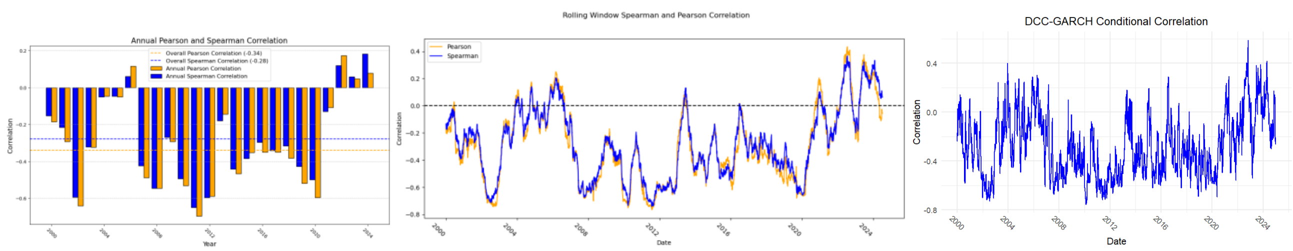

Figure 2. Correlations of the stock and bond returns

4.1.3. Results analysis

4.1.3.1. Macroeconomic drivers

First, monetary policy. Under an accommodative monetary and low interest rate policy, stocks and bonds tend to rise together. Under a tightening monetary and rising interest rate policy, stocks and bonds usually decline together. Second, inflation. During low-medium-stable inflation regimes, both stocks and bonds tend to rise together. As inflation rises, stocks increase while bonds fall due to rising interest rates. In periods of high inflation or heightened inflation volatility, stocks and bonds are likely to decline simultaneously. Third, economic growth. When economic growth is strong, stock prices generally rise, while higher interest rates place downward pressure on bond prices. When economic growth slows, stock prices tend to fall, while increased demand for safe-haven assets pushes bond prices higher. In times of severe recession or systemic risk, investors may sell both stocks and bonds. Fourth, the investor’s emotion. Under moderate optimism, stocks tend to rise while bonds decline. Under moderate pessimism, stocks tend to fall while bonds rise. Under extreme optimism, such as when accommodative monetary policy coincides with robust economic recovery, stocks and bonds may rise together. Under extreme pessimism, such as during a financial crisis or liquidity shortage, stocks and bonds may fall together.

4.1.3.2. Periods analysis

Due to the dot-com bubble crisis in 2000 and the 9/11 terrorist attacks in 2001, economic growth stagnated, leading to a flight to safety that pushed risk capital from stock markets into bond markets. As a result, from 2000 to 2003, the negative correlation between U.S. stock returns and bond returns notably strengthened. Then, since the U.S. economy recovered from the recession, stock returns rose, and with the Fed's gradual interest rate hikes, inflation was effectively controlled, causing bond returns to rise. Consequently, from 2004 to 2006, there was a period of higher stock-bond correlation, with at times even a positive correlation. The subprime mortgage crisis in 2007 and the global financial crisis in 2008 triggered a severe economic recession, prompting investors' flight to safety, which caused stock returns to fall while bond returns rose. After 2009, the government implemented stimulus policies to support recovery and lift equities. Meanwhile, due to quantitative easing and low interest rates, bond returns remained high, resulting in a lower correlation between stocks and bonds. From 2012 to 2020, with low interest rates and moderate economic growth, the correlation remained low. However, in 2013, 2015, and 2018, the Fed's tapering and interest rate hikes increased market uncertainty, leading to declines in both markets, which in turn raised the correlation. In 2020, the COVID-19 pandemic triggered an economic recession, causing the stock market to plunge due to consumer panic, while the bond market acted as a safe haven asset. From 2021 to 2024, as the economy gradually recovered and with the Fed’s extremely accommodative monetary policies, the stock-bond correlation gradually turned positive.

4.2. The lead-lag relation between stock and bond

4.2.1. Prerequisite tests

We have already done the ADF test in the previous part to confirm the stationarity of each time series. Next, we apply the Engle-Granger two-step method to test for cointegration, determining the existence of a long-term equilibrium relationship and avoiding the problem of spurious regression. According to the test results, the residual p-value is 0.00, less than 0.05. Therefore, the null hypothesis is rejected, indicating a long-term cointegration relationship between the time series.

4.2.2. Empirical methods and results

We use the Akaike Information Criterion (AIC) to select the optimal model by balancing goodness of fit and model complexity. By Comparing AIC values at different lag lengths, we discover that the

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minimum AIC occurs at a lag length of ten. So, we choose number ten as the optimal lag for the VAR model. The VAR model is constructed in Equation (1) and (2) as follows:

$$\text{Stock Return}_t = \alpha_1 + \sum_{i=1}^{10} \beta_{1i} \cdot \text{Stock Return}_{t-i} + \sum_{i=1}^{10} \beta_{2i} \cdot \text{Bond Return}_{t-i} + \epsilon_t \quad (1)$$

$$\text{Bond Return}_t = \alpha_2 + \sum_{i=1}^{10} \beta_{1i} \cdot \text{Stock Return}_{t-i} + \sum_{i=1}^{10} \beta_{2i} \cdot \text{Bond Return}_{t-i} + \nu_t \quad (2)$$

Based on the VAR model estimation results, we can observe that the p-values of both bond returns on stock returns and stock returns on bond returns are less than 0.05 in one lag period. Therefore, the null hypothesis is rejected, demonstrating the existence of a bidirectional lead-lag relationship between stock and bond returns.

Table 3. Results of the VAR model

VARIABLES	Lag Order 1	Lag Order 2	Lag Order 3	Lag Order 4	Lag Order 5
Stock Return	-0.081***	-0.011	0.023*	-0.029**	-0.013
	(-6.039)	(-0.819)	(1.740)	(-2.224)	(-0.945)
Bond Return	0.012***	0.003	0.005*	-0.002	0.003
	(4.257)	(1.264)	(1.657)	(-0.726)	(1.011)
VARIABLES	Lag Order 6	Lag Order 7	Lag Order 8	Lag Order 9	Lag Order 10
Stock Return	-0.034**	0.016	-0.018	0.041***	-0.019
	(-2.521)	(1.209)	(-1.355)	(3.047)	(-1.436)
Bond Return	-0.005	0.000	-0.000	-0.005	-0.002
	(-1.644)	(0.151)	(-0.082)	(-1.653)	(-0.630)

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

We conduct the orthogonalized impulse response analysis to isolate the effects of shocks from each variable by removing contemporaneous correlations, allowing for a clearer interpretation of their dynamic impacts. Its result shows that stock returns have a significant short-term influence on bond returns, while the impact of bond returns on stock returns is relatively modest. The interaction between the two markets is concentrated in the short term, with shocks transmitting quickly and effects dissipating over time.

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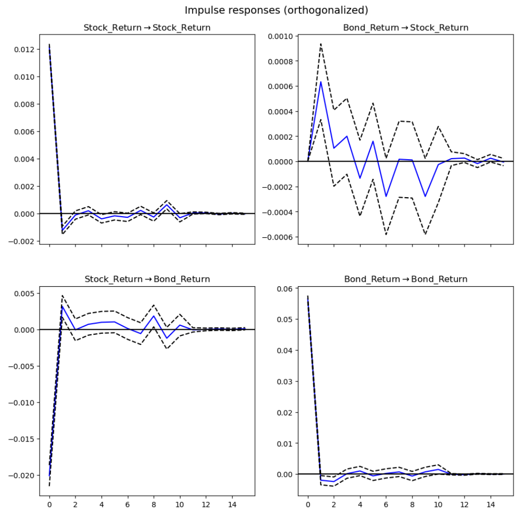

Figure 3. Results of orthogonalized impulse response analysis

4.2.3. Validity tests

We conduct the CUSUM test to assess the stability of the VAR model. The result of the test shows that the p-value is greater than 0.05. Therefore, the null hypothesis cannot be rejected, indicating that the model coefficients are stable over time. Next, we use the Ljung-Box Q-test to examine whether the residuals of the VAR model satisfy the white noise assumption. The result of the test shows that the p-value is greater than 0.05. Therefore, the null hypothesis cannot be rejected, indicating that the residuals are white noise. These two tests confirm the validity of the parameter estimation.

Table 4. Results of the CUSUM test and the Ljung-Box Q-test

Test	Statistic	p-value
CUSUM	0.861	0.449
Ljung-Box Q-test	0.290	1.000

The Granger causality test assesses whether past values of one variable help predict future values of another. The test results show p-values less than 0.05 in both directions, so we can reject the null hypothesis, indicating a bidirectional causal relationship with stock returns exerting a stronger predictive influence on bond returns.

Table 5. Results of the Granger causality test

Direction	SSR F Test	SSR Chi2 Test	Likelihood Ratio Test	Parameter F Test
Stock->Bond	2.9273***	29.3715***	29.3029***	2.9273***
Bond->Stock	2.0653**	20.7222**	20.6880**	2.0653**

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

5. Economic significance

The above research helps investors optimize asset allocation, diversify risk effectively, and maintain stable returns when the market fluctuates. Investors can develop profitable strategies based on stock-bond correlation. When the correlation is negative, they can implement long-short strategies. Additionally, by monitoring macroeconomic indicators, investors can forecast changes in stock-bond correlation and adjust their strategies accordingly. Furthermore, the lead-lag relationship between the stock and bond markets offers arbitrage opportunities, allowing investors to position ahead of market adjustments for potential gains. In terms of risk management, negative correlation helps hedge risk, while positive correlation may increase portfolio risk.

6. Conclusion

This study investigates the dynamic correlation and lead-lag relationship between the U.S. stock and bond markets from 2000 to 2024. By integrating Pearson and Spearman coefficients, the rolling window method, and the DCC-GARCH model, we find that the stock-bond correlation was predominantly negative from 2000 to 2020, then shifting toward a noticeable positive correlation in recent years. The fluctuation of the correlation is further explained by major economic events and government policies leading to changes in macroeconomic conditions. Furthermore, we use the VAR model and Granger causality tests to reveal a bidirectional lead-lag structure between stock and bond markets, with stock returns showing a more pronounced leading role. These findings highlight the evolving interaction between the two markets and offer insights for portfolio allocation and risk management.

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