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Modeling of Leg Coupling Dynamics and Intelligent Optimal Control for Quadruped Robots

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Abstract. In recent years, quadruped robots have received many attention due to excellent adaptability in complex terrains,and the key to their stable locomotion lies in gait coordination. But,the traditional central pattern generator (CPG) models often face challenges such as high reliance on manual experience for tuning coupling parameters and poor adaptive capability. To address this problem, this study proposes a control method integrating coupling dynamics modeling and intelligent optimization. And,a four-leg coupling dynamics model based on Hopf nonlinear oscillator is constructed, in which coupling matrix describes inter-leg phase relationships. The matrix is automatically optimized by incorporating a genetic algorithm and implementing global search with a phase synchronization stability metric as the fitness function. Simulation results show that the optimized coupling parameters significantly improve the phase coordination ability of the four-leg oscillators. This effective eliminates phase deviations under natural dynamics, and greatly enhances both gait synchrony and stability.And so,the study contributes to the autonomous adaptability of quadruped robots by providing a data-driven global optimization framework for their gait control.\

Keywords: Quadruped robot, Central pattern generator, Genetic algorithm, Hopf oscillator

1. Introduction

Quadruped robots are robotic models bio-inspired by mammals of advanced quadrupedal species. They adopt the general anatomical structure of animals that move on four legs, including joint-like leg movements and central nervous system control. Unlike robotic arms or wheeled robots, they feature a design of four legs, a more complicated mechanism of balance control, and a more actuated structure resulting in higher flexibility and development potential. Quadruped robots have gained a good amount of attention in robotics and due to good agility and balance properties have lately been used on tasks of complex-terrain exploration, operation on high terrains, environmental sensing, and short-range assisted transportation. On a quadruped robot, accurate prediction and control of its leg movements is important. The bio-inspired CPG provides a more effective means for prediction than the common PID-based control structure. However, in existing CPG models parameters have to be tuned on the base of manual experience due to lack of an adaptive optimization mechanism for the coupling parameters among different leg structures. Intelligent optimization can be introduced into predictive process for improving its efficiency and accuracy.

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In this regard, it becomes very attractive to find an optimization approach applicable to engineering application, which can automatically update the coupling parameters and enhance the gait stability. CPG models meet difficulties in manual parameter settings, inadequate model convergence, and low adaptability for different gait generation. Therefore, we apply some intelligent optimization approaches, including the genetic algorithm (GA), to conduct a global search and an adaptive optimization of the coupling matrix and the key parameters in the models. This approach effectively enhances the stability and robustness of CPG output signal so that the robot can keep the behavioural coordination of gaits, despite random disturbances, or on another terrain, or changing initial conditions. The optiarithmetized algorithm process reduces the dependence on the human experts for parameter tuning and completed the higher level of the system automation and intelligence in the control system of the quadruped robots.

Ultimately, we hope to create a CPG control architecture using biologically inspired design and engineered practicality which gave quadruped robot systems greater gait stability, scalability, and environmental adaptability, a foundation to build of which deep learning, neural network control, etc., may be integrated to/from in the future.

2. Model description

This study takes a quadruped robot as its platform. The locomotion of quadruped robots is based on the gaits of quadrupedal animals, where diverse motion patterns are achieved by adjusting leg phases. For quadrupedal animals, there are always changes in the periodic motion patterns of their legs across various actions and states during movement, such as walking, running, and turning. For example, cats and dogs, the most common quadrupedal animals, exhibit distinctly different motion patterns in their four legs during running versus walking. These periodic and variable motion patterns are termed gaits.

When moving at low speeds, such animals cat and dog, walk, moving the four legs up and down in turn, so that each foot is assured of support; when impelled to move faster they trot, swinging, to reduce the number of legs in contact with the ground, pairs of legs diagonally opposite and in succession; if further urged they gallop, swinging the fore and hind feet in pairs but never at the same time, at a kind of high-speed, rat-tat rhythm. Although different from one another, these three gaits have in common the feature that in walking with all four legs the actual phase relation of the movements is such as to adjust automatically itself to whatever speed and environment calls for the four limbs in action. That indicates that the animal has no control over any one leg when they walk but instead is using an internal rhythm generator that allows the legs to work together in a coordinated manner with synchronized phase differences [1].

As far as biology is concerned, this rhythmic generation is due to the CPG in the spinal cord. In other words, the 'gaits’ of animals really emanate from the behaviour of the ensemble of coupled oscillators. By modifying the coupling clearly the robot can produce different gaits and even walk smoothly into a run. A Central Pattern Generator or “CPG” is an interconnected set of neurons that can produce stable periodic signals due to its internal structure alone without involvement of sensory input. Different patterns of connection between the neurons effectively produce certain definite phase relations and thus varying patterns of motion. Thus the difference between the two and another gait is really the same phenomenon merely reorganised. It is for this reason that CPGs are one of the essential ingredients in a gaitswitching organism.

These robot models are very much inspired by the construction and action of the CPG; a kind of similar system capable of periodic motions on its own is constructed by way of mathematical means. The most common way of realizing this is using nonlinear oscillators to create a coupled oscillator

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network. “By representing CPG neurons with nonlinear oscillators and coupling several oscillators, it is possible to reproduce the phase coordination displayed between biological legs”. Hence, a quadruped robot will be modeled by four oscillators, each in charge of periodically moving one leg. These coupled oscillators now “work together with appropriate phase lags: by parametrizing the strength of coupling between the oscillators and the lags between their phases, the robot can produce different gaits and switch from one gait to another in a smooth and flexible way [2].

Subsequently, a four-leg coupled oscillator model is constructed, and its output simulates the gait rhythm for the quadruped robot.

A single-leg model is first addressed. As mentioned above, each leg of the quadruped robot can be represented by a nonlinear oscillator, and a single oscillator corresponds to one "oscillator unit". At the single-leg level, the oscillator must feature periodicity (the continuously repeating periodic trajectory for leg swing), stability (the ability to return rapidly to its original rhythm after minor disturbances), and controllability (the adjustable frequency, phase, and amplitude to motion requirements). During its dynamic evolution, a nonlinear oscillator often forms a closed trajectory, known as a limit cycle. This means the system can maintain bounded and stable periodic motion in the absence of additional feedback. This study adopts the Hopf adaptive frequency oscillator (HAFO) with a limit cycle attractor as the foundation for the single-leg model. This model generates stable periodic output while adjusting its intrinsic frequency in response to external rhythmic signals. It is highly suitable as a gait generation unit for quadruped locomotion. Its standard form is as follows:

$$x = \left(\mu - \left(x^2 + y^2\right)\right)x - \omega y \tag{1}$$

$$y = \left(\mu - \left(x^2 + y^2\right)\right)y - \omega x \tag{2}$$

Where ω is the instantaneous frequency of the oscillator; μ is the control amplitude, which determines the radius (r) of the limit cycle; (x, y) is the system status and phase, for which $r = \sqrt{x^2 + y^2}$ is true; x and y correspond to the forward-backward and upward-downward swing phases of the leg structure, respectively.

Oscillator Code

```
for i = 1:steps-1
for leg = 1:4
r2 = x(leg,i)^2 + y(leg,i)^2;
coupling_x = sum(K(leg,:) .* (x(:,i)' - x(leg,i)));
coupling_y = sum(K(leg,:) .* (y(:,i)' - y(leg,i)));
dx_dt = mu*(1 - r2)*x(leg,i) - omega*y(leg,i) + coupling_x;
dy_dt = mu*(1 - r2)*y(leg,i) + omega*x(leg,i) + coupling_y;
x(leg,i+1) = x(leg,i) + dx_dt * dt;
y(leg,i+1) = y(leg,i) + dy_dt * dt;
end
end
```

Substituting x and y yields $r = \left(\mu - r^2\right)r$

DOI: 10.54254/2755-2721/2026.AS31311

From that we can analyse the amplitude evolution of the system, growing when $r < \sqrt{\mu}$ and decaying when $r > \sqrt{\mu}$, until $r = \sqrt{\mu}$ is reached. A limit circle exists. In other words, the Hopf oscillator keeps a fixed period and amplitude because of its dynamics, independent from additional control, which is part of the reason it is so pertinent for bioinspired gait control.

In actual operation, an external forcing term is typically introduced to better align with realistic mechanical environments, for example: $\sin(\Omega_t + \phi_0)$

This ensures that the oscillator can automatically adjust its frequency in response to external rhythms while preserving the stability of the limit cycle [3] [4]. Figure 2 shows the limit cycle trajectory.

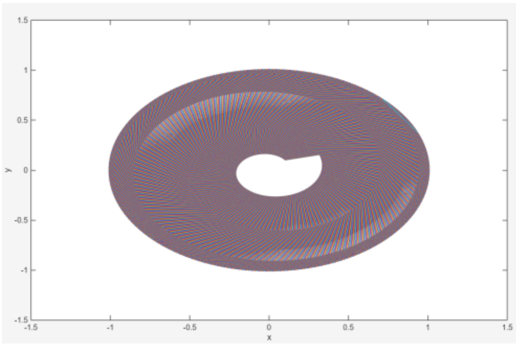

Figure 1. Limit cycle trajectory

A single leg periodic model based on a single Hopf oscillator is developed. However, a single leg model generates the trajectory of a periodic motion of only one leg; it does not produce automatically the required innocent trot or walk of the robot, i.e. coordinated gaits or trotting. In fact, for a quadrupedal animal (or in general) a gait is not simply the superposition of the four legs' independent movements; it is a stable relationship of the phases of the several legs relative to each other. Hence there is some "coupling" (influence of each on the other) that must be included in the expansion of a four leg gait based on a single leg model. Also, the phase configuration is fixed, and relative changes among the topologically different states can occur.

In biological systems this coordinating ability is a result of the aforementioned CPG consisting of a number of neuronal rhythmic units being weakly coupled to one another in varying degrees, producing rhythmic outputs phase locked, i.e., with a stable configuration of phase to the limb muscles. For example, when cats or dogs switch speed or patterns of movement, so the phase relationships among their four legs change allowing them to switch between different gait patterns.

The essential differences between these gaits are due to certain combinations of phase differences. To this end the four oscillators must exchange information by means of coupling terms, in such a way as to allow the robot to change the gait and remain stable in the same manner as real animals do. The system will then tend to settle in the correct phase pattern [5].

The coupling approach leads to a phase transition in the system from the “independent oscillation” of the single oscillators to the “cooperative oscillation” of the multiple oscillators. The oscillators are no longer isolated, but rather the frequency, phase and amplitude of each oscillator is affected by the others. A careful choice of the coupling matrix K allows the four oscillators to settle in stable states of phase relationships, generating a gait. Moreover, different coupling configurations produce different patterns of phase locking, leading to different robot types being seen waddling.

The coupling matrix K is thus a symmetric 4x4 matrix, each entry Kij indicating the strength of the coupling from leg i to leg j, or in other words how much each oscillator can "influence" another

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phase. Although in general inter-leg interactions are bidirectional so a symmetry is assumed, i.e.,Kij = Kji. It can also be seen from K that as K gets larger, the coupling strength between each leg gets larger, and thus the stronger the attraction of the phase of the two coupled oscillators, making it easier for them to synchronize or remain at a fixed phase difference. When a particular coupling term becomes high, the corresponding two legs quickly "stick" to a fixed relative phase, resulting in a particular gait. It follows that the coupling matrix not only determines the interaction of oscillators, but also generates a particular type of gait. That is to say, the elements of K itself can be modified, and thus the robot mathematically "chooses" what kind of motion to do; therefore this is a very simple and highly controllable means of changing gait and gait switching.

3. Algorithm description

This study is concentrated on the matrix K containing purely 6 continuous variables that correspond to conformities among the 4 legs. Gait stability is a very nonlinear output, depending on the complexity of the oscillator behavior, the phase difference evolution, and disturbance propagation, so gradient methods of explicit differentiation are unclear, and classical gradient methods are useless in searching this complex and high-dimensional space. An intelligent optimization-based approach is suggested. In this study we implement that a GA evaluates each parameter's performance through a fitness function. This is especially applicable to complex imaging structures which are likely to have many local optima. The GA has a stochastic search processes and utilizes evolutionary selection techniques successfully exploring vast parameter spaces and not falling into poor local maximums. With different random initial phase conditions, we can find coupling matrices which perform well in different initial states, this will give a K which is more general and robust and achieve better gait stability [6].

To quantify the synchronization level among the four legs, a stability index is adopted:

$$S(K) = \frac{1}{T}\sum_{i=1}^{T}\sum_{a<b}\left(x_a(t_i) - x_b(t_i)\right)^2 \tag{3}$$

If the four legs are highly synchronized, their trajectories will closely align, resulting in a smaller value for S(K). Conversely, if phase imbalance or desynchronization occurs, this index will increase. The optimization objective is to find the coupling matrix that minimizes S(K) [7].

The solution process using the GA follows classical evolutionary steps. The overall procedure is as follows:

Population Initialization: Randomly generate multiple sets of coupling matrix parameters K to form an initial solution set.

Fitness Evaluation: Calculate the stability index S(K) for each individual, which serves as its fitness value.

Selection Mechanism: Select individuals with better performance from the current population to provide "genetic material" for the next generation.

Crossover Operation: Pair selected individuals and generate new candidate solutions by exchanging parameters.

Mutation Step: Randomly perturb certain parameters with a small probability to enhance diversity within the search space.

Update the population and repeat the above process, until the generation limit or convergence criteria are met.

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4. Application and comparison

The GA is used to find the matrix K that minimizes the stability index S(K). The stability function is defined as follows:

$$S(K) = \frac{1}{T} \sum_{i=1}^{T} \sum_{a<b} \left(x_a(t_i) - x_b(t_i) \right)^2 \tag{4}$$

Rationale of the stability function: Calculate the squared phase errors for the oscillators at each time instant t and then average these values over time. The GA code shown below.

```
nVars = 6;
lb = 0.0*ones(1,nVars);
ub = 2.0*ones(1,nVars);
fitnessFunc = @fitnessWrapper;
options = optimoptions('ga',...
'PopulationSize',30,...
'MaxGenerations',20,...
'PlotFcn', {@gaplotbestf});
[bestopt, funcvalue] = ga(fitnessFunc, nVars, [], [], [], [], lb, ub, [],
options);
```

The initial population size is 30, and evolution occurs for 20 generations. The stability index S(K) is implemented for the four legs in the matrix K. The diagonal pairs are Oscillator 1 with Oscillator 2, and Oscillator 3 with Oscillator 4. Subsequently, the coupling is applied;

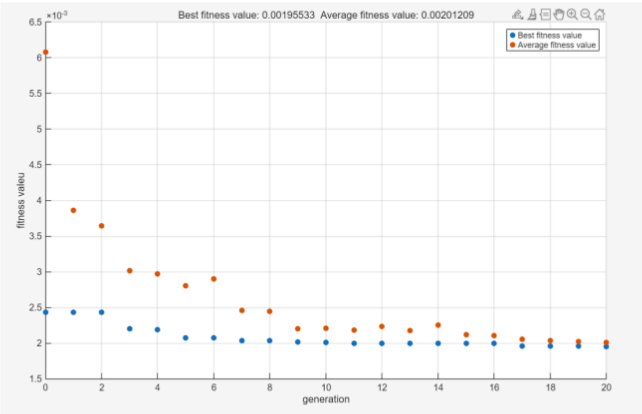

Figure 2. Graph of generation number vs. fitness value

The optimal value for the coupling matrix K is obtained.The resulting K is then transformed into the coupled matrix and substituted into the four-leg oscillator model.The transformation method is as follows:

DOI: 10.54254/2755-2721/2026.AS31311

```
K_opt =

[          0         best_opt(1)   best_opt(2)     best_opt(3);

       best_opt(1)       0         best_opt(4)     best_opt(5);

       best_opt(2) best_opt(4)       0             best_opt(6);

       best_opt(3) best_opt(5)  best_opt(6)          0         ];
```

Figure 3. Matrix transformation method

The transformation yields:

```
K_opt =

 [     0         1.9701      1.8628      1.9858

     1.9701        0         1.9531      1.9662

     1.8628      1.9531        0         1.7861

     1.9858      1.9662      1.7861        0       ]
```

Figure 4. Optimized coupling matrix

In the non-coupled case, K is a corresponding 4x4 zero matrix.
The following presents a comparison before and after coupling:

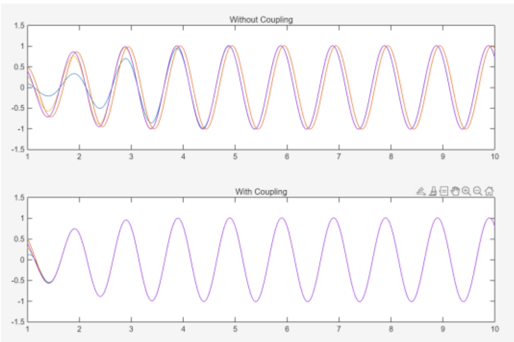

Figure 5. Comparison before and after coupling

Before GA optimization, the four legs could maintain a certain stable rhythm under the system's inherent natural dynamics; however, a phase deviation of approximately 1/11 of a cycle persists between them, and a complete gait synchronization is not achieved. After the GA search for the optimal coupling matrix, the phase errors among the oscillators converge significantly to near zero, and the previous phase offsets are essentially eliminated. Overall, the optimized coupling structure effectively enhances the inter-leg phase attraction, resulting in a higher degree of coordination and synchrony in the final gait [8].

5. Discussion

A coupled oscillator model is used as a basis for improving phase consistency of its gaits through intelligent optimization technology. This work favours a data-driven approach to automatically tune

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the coupling structure, so that the system is capable of producing stable gait outputs for a wider variety of starting states or perturbations, compared with prior CPG control methods that are heavily reliant on experience to choose coupling parameters or phases to lag behind each other. Prior work manually dictates inter-leg phase relationships such as roughly a half-cycle difference for diagonals or 90° difference for trotting. manually set coupling strengths do not always ensure convergence or stability in more complicated cases. On the other hand, since GA in this paper can search for a coupling matrix that operates towards attraction of phases and confluence of the legs based on the actual dynamics of the system, resulting gait is much more robust. Moreover, S(K) explicitly focuses on a difference in the end-effector trajectory between oscillators as opposed to things used in previous work (frequency error, phase-locking time, energy function). As a result, it lend itself more intuitively to the characteristics of synchronisation, more conducive to the optimisation process, and finally, low cost and high scalability.

Although the model appears to be stable behaving in simulation, there are still aspects of it that require further study. For instance, our current CPG is completely feedforward in nature. There is no perception of ground contact nor do we consider the joint torque feedback observed in real animals' nervous systems. Likewise, we still need to validate that generalization across a greater domain of initial phases exists. Future work may explore the feasibility of incorporating sensory feedback loops, adaptive parameter adjustment of certain aspects of the model as well as how to react to specific types of perturbation on such models if they are realized on physical robots.

Overall, this study validates the feasibility of automatically deriving multi-leg coupling relationships through intelligent optimization. It provides a new direction for the further development of CPG-based gait generation methods.

6. Conclusion

This work is devoted to gait generation for quadruped robots with coordinated gaits. Starting from a dynamic coupled model with four Hopf oscillators, it improves the stability of the entire phase through optimized coupling links. The first part concentrates on periodic motion accounting of a single leg oscillator, then expands to coupled legs with inter-leg effects for the four legs. It explains how to physically understand the role of the coupling matrix K for attraction and repulsion linking with phase of each leg, thus laying down the mathematical basis for the generation of various typical gaits. GA is employed to search and optimize those coupling parameters thereby improving the coordination capability of the oscillators. A stability index S(K) which quantitatively sums up the weighted phase errors of the four oscillators over the entire time period is formulated. The index allows quantifying the degree of synchronization achieved for various coupling topologies. Using multi-generational iteration, crossover, and mutation operations, the GA converges to the optimal coupling parameters. Results show that this procedure reduces the phase error between the legs significantly. The optimal parameters obtained in this way are then reconstituted into a symmetric coupling matrix and plugged into the oscillator. Additional comparison experiments show that without coupling the phase of each leg does follow to some extent, but changes independently, leading it to drift; with optimized coupling the system rapidly converges to a continuous and stable rhythm, and the phase relationships between the four legs are obeyed to a much higher degree, gaining in gait coordination. In short, the "coupling modeling + intelligent optimization" method improves uniformity of gait synchronization performance greatly in quadruped robots. Moreover it indicates strong extensibility and can likely be applied to other multi-joint systems, and also bio-inspired motion control as well.

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