

Analysis of a Bayesian Parameter Updating Scheme for Truss Structures Based on Sensitivity and Fisher Information

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Abstract. Truss structures are widely used in actual bridge systems. During a long period of service life, the aging process of the bridges and the influence of the environment cause the measurements to be susceptible to noise during observations, thus reducing the ability of the data to effectively limit the posterior distribution of the model parameters. To overcome these issues, a numerical model based on OpenSees is adopted as the forward model to identify the parameter-response sensitivity matrix using the finite difference method. Under the assumption that the measurements follow a Gaussian distribution, a Fisher information matrix is introduced to characterize the information content of different candidate schemes for ranking and selecting loading and measurement schemes. Then, the Bayesian parameter updates based on the selected representative schemes are performed, while posterior degeneracy for the parameters is calculated using Monte Carlo sampling methods. The efficacy of the selected schemes is demonstrated by comparing the ability to reduce uncertainties and parameter correlations.

Keywords: Truss structure, Bayesian updating, sensitivity matrix, fisher information matrix

1. Introduction

Trusses are widely used in bridges and structural designs because they have well-understood load paths and good material efficiency. However, in the course of the structural life, some degradation in material performance, construction errors, and environmental influences might cause variations in the basic structural parameters, affecting structural safety. Updating structural parameters based on monitoring information has become a very important method for improving the reliability of structural analysis. Bayesian updating has become a very important method in structural mechanics for carrying out probabilistic identification, as it offers a way of incorporating prior knowledge into structural analysis.

Research has focused heavily on the computational algorithms associated with Bayesian updating and posterior analysis but has not focused as much on the collection of observation data, including how to design loading and measurement schemes. With respect to typical civil engineering practice, if sufficient care is not taken in selecting the combined loading and measurement schemes, the observations might be only weakly sensitive to the parameters of interest. Even with a formal, established Bayesian updating methods the posterior contraction could be limited or be adversely

affected in terms of state identifiability or stability. Therefore, it is valuable to have a means of quantitatively evaluating and screening the informativeness of loading and measurement schemes, and via this quantification an improvement in update efficiency can be achieved.

To address the above issues, this paper focuses on information screening and scheme evaluation prior to Bayesian updating. The approach taken is that prior to obtaining actual observation data, different combinations of loading and measurement methods will be quantitatively compared for their effectiveness in constraining and identifying target parameters from the perspective of numerical models and prior uncertainty information. This information will be useful in establishing a screening ordering among the candidate loading and measurement schemes. It is important to note that truss structures are typically analyzed with numerical models and obtaining explicit expressions are typically difficult, in this case, an OpenSees numerical model is chosen as the forward solver and the finite difference perturbation method is employed to estimate sensitivity matrix of model response with respect to model parameters. The parameters of interest will be based on the linear and Gaussian assumptions of noise, where posterior updating is involved as part of the updating process, the Fisher information matrix will be employed as a measure of parameter-constraining capacity or effectiveness, and in this way it is proposed that the loading and measurement schemes will be ranked and screened for future monitoring.

Ranking information doesn't mean filtering out invalid information. Consequently, further in this paper, standardized screening and decision criteria based on sensitivity and the Fisher information matrix will be established here. This paper will create a set of rules for interpretability and engineering. This paper connects the levels of sensitivity to the scales of noise so as to discriminate between schemes that are highly recommended, acceptable schemes and schemes that should be discarded. The impact of the structural parameter uncertainty on the sensitivity conclusions is discussed along with the influence of the choice of assessment nominal point. Further changes in the noise levels are also discussed. Further, an analysis is done of the local sensitive decision making which causes risk and countermeasure. Moreover, it evaluates the suggested method against recently published adaptive psychological screening work on the identical candidate set and under already given computational adjustment to authenticate the advantages and defects for its screening effectiveness and interpretability.

2. Theory

2.1. Observation model and notation

Let $\theta \in \mathbb{R}^n$ denote the vector of structural parameters to be identified. For a given loading case and measurement configuration, the observation vector $y^{\text{obs}} \in \mathbb{R}^m$ is obtained. The forward numerical model is denoted as $f(\theta)$, which represents the mapping relationship from parameters to response. This study assumes additive, zero-mean Gaussian measurement errors [1,2].

$$y^{\text{obs}} = f(\theta) + \varepsilon, \varepsilon \sim \mathcal{N}(0, \Sigma_\varepsilon) \quad (1)$$

Where ε denotes the measurement noise and Σ_ε is the corresponding error covariance matrix. The covariance structure explicitly encodes measurement accuracy and noise scale, which are subsequently incorporated into the information-based evaluation [1].

2.2. Basic form of Bayesian updating

Given the observations vector y^{obs} , the posterior distribution $p(\theta|y^{\text{obs}})$ follows Bayes' theorem [2]

$$p(\theta|y^{\text{obs}}) \propto p(y^{\text{obs}}|\theta)p(\theta) \quad (2)$$

Here $p(\theta)$ denotes the prior distribution. Under the zero-mean Gaussian error assumption in Section 2.1, the likelihood can be written, up to a normalization constant, in an exponential form [1]

$$p(y^{\text{obs}}|\theta) \propto \exp\left[-\frac{1}{2}(y^{\text{obs}}-f(\theta))^T \Sigma_\epsilon^{-1}(y^{\text{obs}}-f(\theta))\right] \quad (3)$$

Given prior samples θ_i , approximate posterior statistics can be obtained using importance weights w_i constructed from the likelihood of each sample [1,3]. The weights are proportional to the exponential term in the likelihood [1].

$$w_i = \exp\left(-\frac{1}{2}\left[(y^{\text{obs}}-f(\theta_i))^T \Sigma_\epsilon^{-1}(y^{\text{obs}}-f(\theta_i))\right]\right) \quad (4)$$

To avoid scale issues, normalized weights \tilde{w}_i are used [1,3].

$$\tilde{w}_i = \frac{w_i}{\sum_{k=1}^N w_k} \quad (5)$$

Based on the normalized weights, the effective sample size (ESS) is introduced to quantify the degree of weight degeneracy. It reflects the number of equivalent independent samples effectively contributing to the posterior estimate [3]

$$\text{ESS} = \frac{1}{\sum_{i=1}^N \tilde{w}_i^2} = \frac{\sum_{i=1}^N w_i^2}{\sum_{i=1}^N w_i^2} \quad (6)$$

A smaller ESS indicates that only a few samples carry most of the weight, and the resulting posterior contraction may become unstable.

2.3. Sensitivity matrix and numerical estimation

In structural parameter identification, the effectiveness of a given loading case and sensor layout largely depends on the sensitivity of the measured responses to parameter perturbations [4]. If a small change in a parameter leads to a noticeable change in the response, the corresponding observations are more likely to provide informative constraints for Bayesian updating [4]

For small perturbations around a nominal parameter vector θ , the response increment can be approximated by a first-order linearization [4]

$$\Delta y \approx J(\theta) \Delta \theta \quad (7)$$

Here $J(\theta)$ is the sensitivity matrix, whose entries are defined as

$$J_{ij}(\theta) = \frac{\partial y_i}{\partial \theta_j} \quad (8)$$

Here, i indexes the response component and j indexes the parameter. The sensitivity matrix quantifies how perturbations in each parameter are transmitted to each response component. It provides a direct indication of the informativeness of a candidate scheme and forms the basis for constructing the Fisher information matrix in the subsequent analysis [4]

In most engineering problems, the forward model is implemented using numerical software, where the internal solution process is a black box. Hence, a closed-form expression for the sensitivities is not available. Hence, a finite difference approach is taken in the proposed research [5,6]. A small change is introduced to the j -th parameter while keeping all other parameters constant. A unit vector is defined as e_j with a 1 in its j -th component. A forward difference approximation for the sensitivity is expressed by [6].

$$J_{ij}(\theta) \approx \frac{f_i(\theta + \Delta \theta_j e_j) - f_i(\theta)}{\Delta \theta_j} \quad (9)$$

The choice of the step size $\Delta \theta_j$ is required to be done in a way that ensures a compromise between numerical stability and the level of approximation. A large step size $\Delta \theta_j$ can lead to an amplification of numerical errors, whereas a small step size $\Delta \theta_j$ can cause the accumulation of rounding errors. A technique commonly adopted in engineering is to use a relative step size $\Delta \theta_j = \alpha \theta_j$ with a predetermined scaling factor α [5,6].

2.4. Fisher information matrix and ranking criteria

Using only $J(\theta)$ is not sufficient in choosing a scheme. The accuracy of measurement is not uniform in the components of the response, because of possible discrepancies in noise levels. If such uncertainty is not considered, some measurement and loading configurations could be considered highly sensitive, but their actual updating capabilities could be limited. These aspects have also been noted in various studies on sensor placement and information metrics [7,8].

$$F(\theta) = J(\theta)^T \Sigma^{-1} J(\theta) \quad (10)$$

With Gaussian error models of the measurements, let the covariance matrix of the observation errors be denoted by Σ . The Fisher information matrix with respect to parameters is given by [9].

It represents how sensitivity and accuracy of measurements together contribute to the parameter identification and is directly related to the decrease of uncertainties during Bayesian updates [1,8].

After obtaining $F(\theta)$, scalar criteria are required for quantitative ranking. Different criteria emphasize different information characteristics. In this study, three commonly used criteria are adopted [10]:

$$\phi_{\text{tr}} = \text{tr}(F(\theta)), \phi_{\text{det}} = \det(F(\theta)), \phi_{\text{min}} = \lambda_{\text{min}}(F(\theta)) \quad (11)$$

Here ϕ_{tr} reflects the overall level of information accumulation and emphasizes the aggregate constraint. ϕ_{det} measures information in a volume sense. ϕ_{min} focuses on the weakest identifiable direction through the minimum eigenvalue [10].

2.5. Cramér–Rao lower bound

To provide a more intuitive, parameter-level interpretation, the Cramér–Rao lower bound (CRLB) is further introduced as an auxiliary ranking metric. It helps explain why certain parameters remain difficult to reduce in uncertainty under some selected schemes [9].

Assuming that $F(\theta)$ is invertible, the CRLB associated with the j -th parameter is defined as [9]

$$\text{CRLB}_j = F(\theta)^{-1}_{jj} \quad (12)$$

A smaller CRLB_j indicates stronger identifiability, or a tighter achievable variance bound, for the j -th parameter under the corresponding loading and measurement design. However, when ϕ_{min} is very small or $F(\theta)$ is nearly singular, the inverse $F(\theta)^{-1}$ may be numerically unstable. Therefore, this study uses the degeneracy indicated by ϕ_{min} as a preliminary check, and reports CRLB_j only for non-degenerate designs [9,10].

3. Method

3.1. Numerical example and parameter settings

For this purpose, a two-dimensional Pratt truss structure is used as a test case to demonstrate whether a scheme evaluation approach based on sensitivities and the Fisher information matrix has a potential for screening candidate schemes of loading conditions and measurement arrangements effectively as a first ranking of schemes prior to Bayesian updating by using only a forward model and a model of measurement errors. Such a point of view of putting information metrics at a focal point of decisions about which schemes to update is in agreement with a reason mentioned by Cao et al. in sensitivity-based information-screening frameworks [4]. The forward analyses are implemented in OpenSees [11]. Structural members are modeled using linear-elastic truss elements to suppress nonlinear effects, so that the information content is primarily driven by the loading and measurement design.

3.1.1. Geometry and nodes

A two-dimensional Pratt truss is adopted as the benchmark structure. Its geometric configuration is shown in Figure 1. The total span is $8a=16\text{m}$, consisting of four equal panels of length $2a=4\text{m}$. In this study, $a=2\text{ m}$.

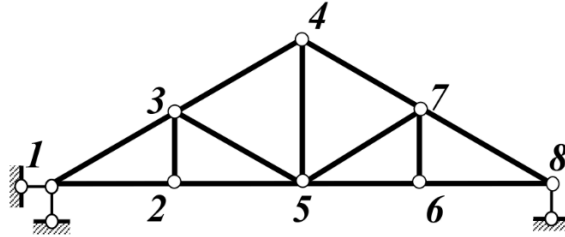


Figure 1. Geometry and node numbering of the 2D Pratt truss benchmark model

The lower-chord nodes are aligned horizontally: 1 $(0,0)$, 2 $(2a,0)$, 5 $(4a,0)$, 6 $(6a,0)$, 8 $(8a,0)$. The upper-chord nodes follow a gable-shaped profile: 3 $(2a,a)$, 4 $(4a,2a)$, 7 $(6a,a)$. The node numbering and node sets are consistent with the OpenSees input script, which facilitates direct extraction of response quantities from subsequent analyses [12].

3.1.2. Boundary conditions and loading cases

The truss is modeled as simply supported. Node 1 is assigned a pin support that restrains both the horizontal and vertical translations, whereas Node 8 is assigned a roller support that restrains only the vertical translation.

To construct a screenable set of candidate loading cases, four vertical point-load patterns are considered. Let PPP denote the magnitude of a downward point load:

- L1: apply PPP at Node 3 only.
- L2: apply PPP at Node 4 only.
- L3: apply PPP at Node 7 only.
- L4: apply PPP simultaneously at Nodes 3, 4, and 7.

These patterns are defined because the structure is assumed to behave linearly under small elastic deformations. Accordingly, the response under L4 can be interpreted as the superposition of the responses under L1, L2, and L3. This design also facilitates subsequent verification of the numerical implementation [1].

3.1.3. Parameters to be updated

The axial stiffness EA of truss members is selected as the set of parameters to be identified and updated. To reduce the parameter dimension, the members are grouped into two categories [2]. The equivalent axial stiffness of the lower- and upper-chord members is denoted by EA_{12} . The equivalent axial stiffness of the web members (e.g., verticals and diagonals) is denoted by EA_3 . Accordingly, the parameter vector is defined as:

$$\theta = [EA_{12}, EA_3]^T \quad (13)$$

To validate the numerical implementation, synthetic observations are generated by adding zero-mean Gaussian noise with a standard deviation of $\sigma=0.001\text{m}$ [1,13].

3.1.4. Candidate measurement responses and measurement vectors

To make the information ranking closer to practical decision-making, a candidate measurement pool is constructed to examine how different response types and sensor (measurement) combinations affect parameter identifiability [9,10]. Based on OpenSees outputs, the following response quantities are considered: the vertical displacements U_{y2} , U_{y3} , U_{y4} , U_{y5} , U_{y6} , U_{y7} , the horizontal displacements U_{x3} , U_{x7} , and a horizontal relative displacement measure [12].

$$D_{x37}=U_{x3}-U_{x7} \quad (14)$$

Based on the candidate response pool, different measurement vectors y can be constructed by selecting different subsets of response quantities. To examine whether the choice of measurement combination affects parameter identifiability and posterior contraction, this section focuses on the following measurement sets:

$$y_{M1}=[U_{y5},U_{y2}]^T,y_{M2}=[U_{y5},D_{x37}]^T \quad (15)$$

3.1.5. Finite-difference perturbation settings

A relative-perturbation strategy is adopted in this study. The scaling factor is set to $\alpha=10^{-3}$, which yields $\Delta EA_{12}=16$ and $\Delta EA_3=8$. This choice preserves the validity of the first-order approximation while avoiding excessively small steps that could be dominated by rounding and numerical errors. The sensitivity of the results to the step size, and the selection of an optimal α will be discussed later. Numerical stability is further assessed through order-of-magnitude checks [5,6].

3.2. OpenSees model and data-generation process and sensitivity matrix and fisher information matrix computation

This section describes the generation and organization of the numerical-example data. All datasets used in the subsequent analyses are directly derived from OpenSees outputs and the finite-difference computations [6,11,12].

Based on the finite-difference results obtained in Section 3.2, this section computes the sensitivity matrix and, under the Gaussian error assumption, the corresponding Fisher information matrix [9].

3.2.1. Sensitivity matrix computation

For the loading case L4L4L4 and the measurement set $M1=\{U_{y5},U_{y2}\}$, the corresponding sensitivity matrix $J_{L4,M1}$ is

$$J_{L4,M1} = \begin{bmatrix} 4.054625 \times 10^{-6} & 6.243750 \times 10^{-7} \\ 3.6183125 \times 10^{-6} & 1.184750 \times 10^{-6} \end{bmatrix} \quad (16)$$

For the measurement set $M2 = \{[U_{y5}, D_{x37}]\}$ the corresponding sensitivity matrix $J_{L4,M2}$ is

$$J_{L4,M2} = \begin{bmatrix} 4.054625 \times 10^{-6} & 6.243750 \times 10^{-7} \\ -4.36295731 \times 10^{-7} & -1.184779274 \times 10^{-6} \end{bmatrix} \quad (17)$$

3.2.2. Fisher information matrix calculation

For M1

$$F_{L4,M2} \approx \begin{bmatrix} 1.6630 \times 10^{-5} & 3.0485 \times 10^{-6} \\ 3.0485 \times 10^{-6} & 1.7935 \times 10^{-6} \end{bmatrix} \quad (18)$$

For M2

$$F_{L4,M2} \approx \begin{bmatrix} 1.6630 \times 10^{-5} & 3.0485 \times 10^{-6} \\ 3.0485 \times 10^{-6} & 1.7935 \times 10^{-6} \end{bmatrix}$$

3.2.3. Information indicators

After calculation, L_2 exhibits negligible sensitivity to EA_3 for the current response set. As a consequence, the Fisher information matrix becomes nearly singular, and the values of ϕ_D and ϕ_E are close to zero. Even when ϕ_A is not small; this type of loading primarily accumulates information along a single direction, and therefore provides very limited support for the joint identification of the two-parameter vector. This interpretation, namely that information-matrix degeneracy leads to weak identifiability along the poorly informed direction, is consistent with discussions in Bayesian finite element model updating regarding identifiability and posterior uncertainty reduction [7,8]. In addition, the Cramér–Rao lower bound of scheme M2 along the direction is significantly smaller than that of scheme M1, indicating that M2 is more informative for updating EA_3 [9].

3.3. Bayesian updating

3.3.1. Setup

To examine whether the ranking results obtained in Section 3.3 based on the sensitivity matrix and the Fisher information matrix are effective, Bayesian updating is performed for different candidate combinations under the same conditions [1,2]. The $\theta_{\text{true}} = \begin{bmatrix} EA_{12} \\ EA_3 \end{bmatrix} = \begin{bmatrix} 16000 \\ 8000 \end{bmatrix}$, Synthetic

observations are generated as $y_{obs}=f(\theta_{true})$ In the likelihood function, the measurement-noise standard deviation is set to $\sigma=1mm[2]$.

3.3.2. Comparison and analysis of updating results

Table 1. Posterior statistics and effective sample size (ESS) for different loading cases and measurement sets

Loading case	Measurement set	ESS	Posterior mean of EA12	Posterior standard deviation of EA12	Posterior mean of EA3	Posterior standard deviation of EA3
L1	M1	4193.3	16070.3	559.6	7987.0	563.2
L1	M2	5824.2	16085.6	714.4	7997.4	563.2
L2	M1	4172.9	16039.4	482.6	7997.5	576.9
L2	M2	5183.6	16060.7	611.8	7995.8	577.3
L3	M1	5026.2	16068.8	603.4	7989.8	575.7
L3	M2	5824.2	16085.6	714.4	7997.4	563.2
L4	M1	1647.8	16019.9	224.8	7985.9	565.6
L4	M2	2088.4	16013.6	257.8	8016.1	527.0

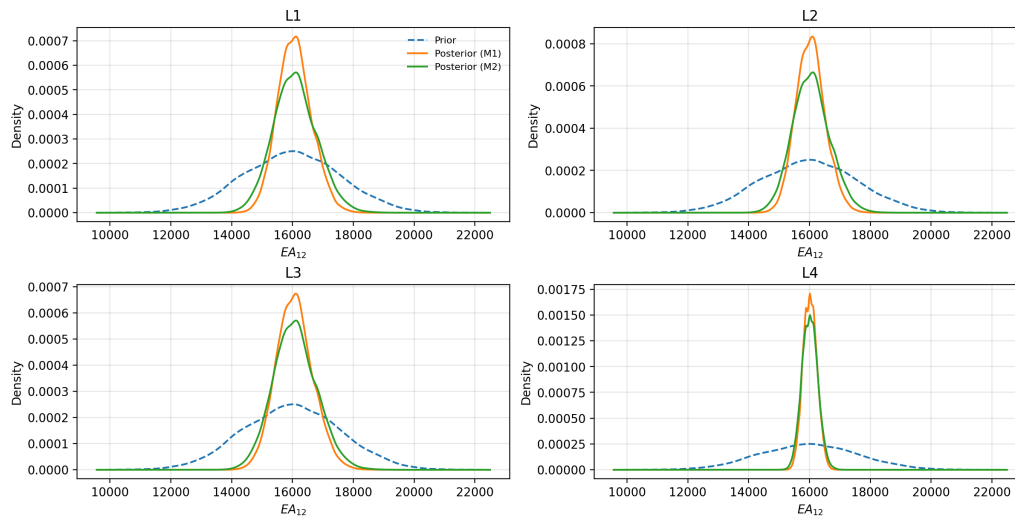


Figure 2. Prior and posterior marginal distributions of EA12EA_{12}EA12 under loading cases L1–L4 (M1 vs M2).(picture credit : original)

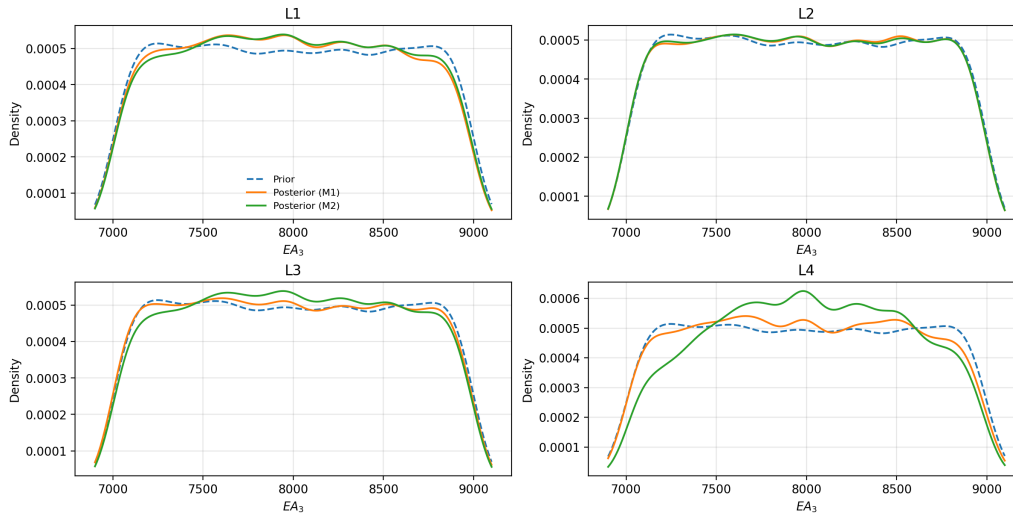


Figure 3. Prior and posterior marginal distributions of EA_3 under loading cases L1–L4 (M1 vs M2).(picture credit : original)

As shown in Table 1, Figure 2 and Figure 3, examining the posterior distribution of EA_{12} , the loading case L4 is clearly more informative than L1-L3, yielding a noticeable reduction in posterior variance. This is broadly consistent with the expectation in Section 4.3 that the overall sensitivity increases when multi-point loads are combined. In contrast, the posterior uncertainty reduction of EA_3 remains limited in most cases, indicating that the current loading. measurement combinations provide weak updating capability for EA_3 [1,2].

3.3.3. Consistency between information ranking and posterior contraction

By combining the results in Section 3.3 with the posterior standard-deviation contraction ratios obtained in Section 3.4, a relatively stable trend can be observed. Schemes with higher information content tend to yield more pronounced posterior contraction. For example, under the L4 loading case, most information indicators outperform those of L1-L3, Therefore, the sensitivity-based information matrix can at least provide a reliable prioritization direction for scheme selection [1,4]. In this example, the updating performance for EA_3 remains limited. This is primarily due to insufficient structural informativeness with respect to EA_3 under the current loading and measurement designs. To obtain a more substantial contraction for EA_3 , the scheme set should include measurement vectors, or response quantities, that can better excite and capture the response of the web-member group.

4. Analysis

4.1. Component validity assessment

To compare sensitivity magnitude and measurement accuracy on a consistent scale, the analysis begins at the level of an individual observation component. Let $f_i(\theta)$ denote the model response associated with the i -th observation component. The corresponding sensitivity row vector is defined as

$$\mathbf{j}_i = \frac{\partial \mathbf{f}_i(\theta)}{\partial \theta} \Big|_{\theta=\theta_0} \quad (19)$$

Let the noise standard deviation of this component be σ_i . The noise-standardized sensitivity index is defined as:

$$\eta_i = \frac{\|\mathbf{j}_i\|_2}{\sigma_i} \quad (20)$$

If η_i is small, the response change induced by parameter perturbations is likely to be dominated by measurement noise. Including such a component in the updating procedure may therefore provide little benefit, and it should be screened out with priority [4,7].

In addition, prior uncertainty is incorporated into the assessment, because η_i alone is insufficient. The index η_i does not account for the actual variability of the parameters. Introducing the prior covariance matrix Σ_θ , a signal-to-noise ratio (SNR)-type indicator is defined as:

$$\gamma_i = \frac{\sqrt{\mathbf{j}_i \Sigma_\theta \mathbf{j}_i^T}}{\sigma_i} \quad (21)$$

Set two thresholds $\tau_{\gamma,1} < \tau_{\gamma,2}$, $\tau_{\gamma,1} = \text{Quantile}_{0.3}(\{\gamma_i\})$, $\tau_{\gamma,2} = \text{Quantile}_{0.7}(\{\gamma_i\})$

$\gamma_i \geq \tau_{\gamma,2}$: Strongly recommended, $\tau_{\gamma,1} \leq \gamma_i < \tau_{\gamma,2}$: General information, $\gamma_i < \tau_{\gamma,1}$: Priority removal.

4.2. Degeneracy detection and removal

In the preceding numerical example, the Fisher information matrix has been computed for each loading case L and measurement set M , together with the three ranking criteria (A-, D-, and E-criteria). However, a high rank does not necessarily imply usability. For certain L, M combinations, $A_{L,M}$ may be non-negligible while the corresponding $F_{L,M}$ is nearly singular. In such cases, quantities involving F^{-1} can become ill-conditioned and numerically unreliable [9,10].

Let the degradation-detection thresholds be τ_D and τ_E , defined by

$$\tau_D = \alpha_D \max(D_{L,M}), \tau_E = \alpha_E \max(E_{L,M})$$

It is recommended to set α_D and α_E in the range 10^{-3} to 10^{-2} , which effectively uses a 2–3 order-of-magnitude gap from the best-performing candidates as the screening line. Combinations whose $D_{L,M}$ or $E_{L,M}$ values fall 2–3 orders of magnitude below the maximum in the candidate set typically contribute little to parameter identifiability [4,10].

$$D_{L,M} \leq \tau_D \text{ or } E_{L,M} \leq \tau_E \quad (22)$$

Accordingly, any (L,M) combination failing this criterion is removed and excluded from subsequent ranking and Bayesian updating [10].

4.3. Standardized screening workflow

Firstly, sensitivities are evaluated at the nominal point θ_0 , and candidate observation components are scored using the noise-standardized index η_i and the prior-aware signal-to-noise type index γ_i . Components are then tiered using the quantile thresholds $\tau_{\gamma,1}$ and $\tau_{\gamma,2}$, forming an effective observation pool. Second, candidate measurement sets M are constructed from the pool, and for each (L, M) the Fisher information matrix $\mathbf{F}_{L,M}$ is computed under the Gaussian error model. A degeneracy check is applied using the relative thresholds τ_D and τ_E . Near-singular combinations are removed to avoid numerical ill-conditioning in quantities involving \mathbf{F}^{-1} . Third, only within the non-degenerate set, the ranking criteria A , D , and E are computed, and parameter-level interpretation is provided using CRLB. If needed, Bayesian updating can be used as a cross-check, where ESS is monitored to diagnose weight degeneracy, and the thresholds or candidate sets are adjusted if instability is observed.

4.4. Stability and engineering considerations

Criteria for screening are evaluated on a nominal parameter value θ_0 . This is a common practice. However, it is noted that the obtained results may depend on the choice of the nominal parameter value θ_0 [14]. For a nonlinear structural response or a significant amount of prior knowledge on a problem, rankings obtained using A-, D-, and E-based methods may differ.

For practical engineering applications, it is therefore recommended to assess ranking stability using a small number of prior samples rather than relying on a single evaluation. Specifically, draw K samples $\theta^{(k)} \left\{ \theta^{(k)} \right\}_{k=1}^K \sim p(\theta)$, and recompute the criteria $A_{L,M}^{(k)}$, $D_{L,M}^{(k)}$, $E_{L,M}^{(k)}$ at each sample point. The sample-averaged criteria can then be used as the basis for ranking [14]

$$\bar{A}_{L,M} = \frac{1}{K} \sum_{k=1}^K A_{L,M}^{(k)}, \bar{D}_{L,M} = \frac{1}{K} \sum_{k=1}^K D_{L,M}^{(k)}, \bar{E}_{L,M} = \frac{1}{K} \sum_{k=1}^K E_{L,M}^{(k)} \quad (23)$$

However, it is important to note that there is a dependence of numerical stability in sensitivity evaluation on the size of the finite difference step. Therefore, evaluations of the perturbation factor α in terms of steps or orders of magnitude are recommended when working in a realistic framework. When a dependence of the ranking on α is observed, attention should be focused on optimizing finite difference parameters or improving solver accuracy [5,6]. Changes in the level of noise affect the scale of overall information content. Taking this observation into consideration, degeneracy thresholds can be expressed in a relative form in relation to absolute values [7,10].

5. Conclusion

This research examines information screening ahead of Bayesian update. It is clear that a scheme based solely on sensitivity-based ranking is not guaranteed to be usable because some combinations can appear informative due to noise scale and matrix degeneracy while ending up with limited or even unstable posterior contraction. With this motivation, the proposed workflow builds on the finite-difference sensitivities and Fisher information matrix to rank candidates based on A-, D- and E-type criteria with explicit degeneracy checks and tiered rules for enhanced interpretability and robustness. The numerical example generally coincides with what is expected. Programs whose priors are richer eliminate more posterior standard-deviation and loading on multiple points is helpful for some parameters. Updates to the web-member parameter group remain weak, suggesting that the limitation is structural informativeness, rather than the updating algorithm per se. In case a stronger contraction in such parameters is desired, the response components in the measurement vector should better excite the behaviour of interest, and catch it well, along with a basic stability check at the nominal point and the finite-difference step value .

Authors contribution

All the authors contributed equally and their names were listed in alphabetical order.

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