

Compressed Sensing in MRI: Sampling Masks and their Comparison

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Abstract. In order to improve the MRI imaging speed, a method of compressed sensing is used. This method utilizes sparsity and compressibility on images, breaking the Nyquist sampling theorem, being able to reconstruct images from fewer data. This paper replaces the patients doing scanning with a high-resolution image of the scanning results, to simulate compressed sensing. Apart from basic normal distribution, four other distributions are tried, they are standard linear, modified linear, Weibull, and cosine distributions. Images are sampled around 35% and reconstruct by wavelet transform. In the end, the modified linear distribution shows the highest image match. Besides, by increasing the sampled points in center, the difference from reconstructed image and complete image become smaller.

Keywords: MRI compressed sensing, sampling strategy, wavelet transform image reconstruction

1. Introduction

MAGNETIC Resonance Imaging (MRI) is a non-invasive imaging technique widely used for disease diagnosis and medical research. However, its scanning speed is slow, which means patients are required to stay still in the machine for a long time [1-3]. That's difficult for children and someone who have disease such as claustrophobia and hyperactivity [4]. Besides, long scanning time leads to higher cost, made it a heavy burden for family and government [5]. If the scanning speed improved, more people can have the ability, and be more comfortable to do this scan. Additionally, for some emergency rescue, every minute speed up is crucial for lifesaving.

In this case, many researchers dedicated to the development of acceleration technology. The concept of compressed sensing is proposed as one of the solutions. Researchers find that if the signal is sparse and incoherence, then it can be reconstructed from a sampling point much lower than the Nyquist sampling theorem require [6,7]. One commonly used compressed sensing method in MRI is sampling under normal distribution in k-space, and remove undersampling part from reconstruction, such as wavelet transform. After several times of transformation, pictures will become more approaching to the full sampling image, but only requires part of the time.

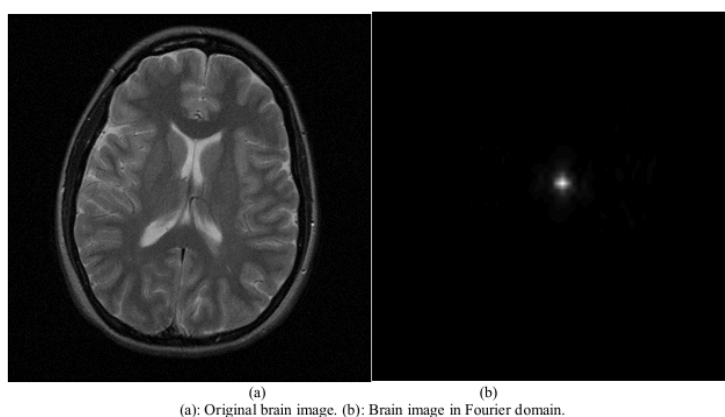
In this paper, some other sampling patterns were tried and compared with normal distribution. They are standard linear distribution, modified linear distribution, Weibull distribution, and cosine distribution. Because of space and equipment constraints, a high-definition MRI scan image was taken as real human brain in this experiment, and different sampling methods were applied on this image. All the sampling rates are controlled around 35%. This approach first sampled the pictures in k-space under different distributions, then reconstructed them by wavelet transform, and compared the difference in the end. In addition, we have studied the same approach in more depth, by changing the number of center samples.

The following sections, section II describes the sampling methods for five distributions, and introduces how to reconstruct them and compare them. Section III shows the difference between results and first picture, discusses the reason. A conclusion of this experiment and description of future work are made in session IV.

2. Method

Brain scanning is one of the most common clinical applications of MRI. Most brain images show sparsity in the appropriate transformation domain. This means compress sensing can be well integrated in this field. Before we can do that, we need to understand how compress sensing works and how this technology is used in conjunction with MRI.

In order to successfully apply compress sensing, the desired image is first required to show transformation sparsity in the transform domain used, and MR image usually satisfies this condition in the Fourier domain. The combination of MRI and CS is based on the researchers' desire to speed up the imaging as much as possible without compromising the quality of the image, that is, taking fewer samples and reconstructing them in a suitable way.



(a): Original brain image. (b): Brain image in Fourier domain.

Figure 1. Image in Fourier domain

2.1. Undersampling

When undersampling this image in the Fourier domain, it should be noted that the sampling method should take into account that the energy distribution of MR image in k-space is not uniform. As shown in Figure. (1a,1b), low frequency points appear white and are concentrated in the center, and these points have higher energy. Therefore, it can be concluded that a higher sampling density should be adopted in the central region of k-space when designing an undersampled mask.

In the actual experiment, we adopted five different probability distributions to design our sampling strategy, and compared which strategy had the best performance in the process of image

reconstruction when the same number of samples were taken.

You can clearly see the variation in sampling density between the different masks in Figure 2. What they have in common is that they all have the highest sampling density in the center of the Fourier domain. A 1-D Probability Density Function graph are provided to help you understand the difference between 2-D masks better.

In addition, the total sample size of all masks is 35%.

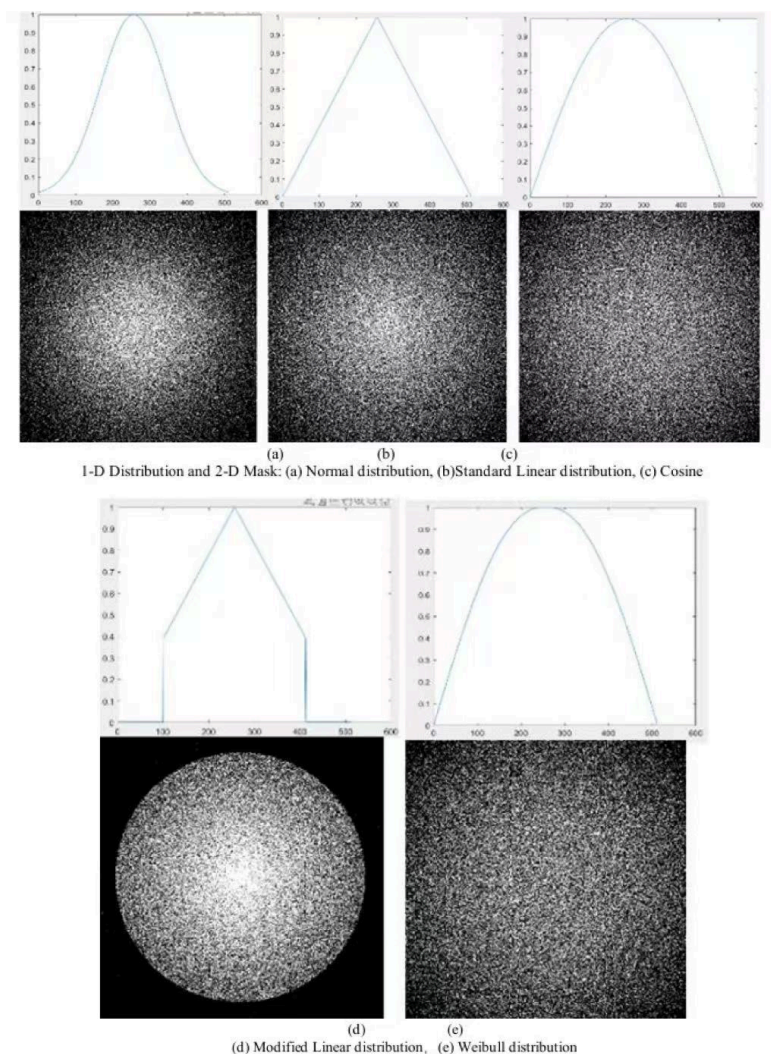


Figure 2. 1-D Distribution and 2-D Mask

2.2. Reconstruction

After the missing value is returned to zero and the Fourier transform is reversed, discontinuous image artifacts appear in the sampled images, which are similar to noise. (See Figure (3)) The image then needs to be reconstructed to make the simulated image as similar as possible to the original image.

At this stage, the wavelet transform is applied to simulate the missing value of the image. In other words, we apply wavelet transform on the image formed by the samples collected in Fourier domain to simulate the missing value returned to zero and remove the noise. When the image is returned to

the Fourier domain by reversing the wavelet transform and the Fourier transform, the corresponding pixel points are covered with the samples collected before to further correcting the image.

Usually based on different sampling strategies, 30-80 cycles of iteration will be carried out to achieve the reconstruction result that is closest to the original image.

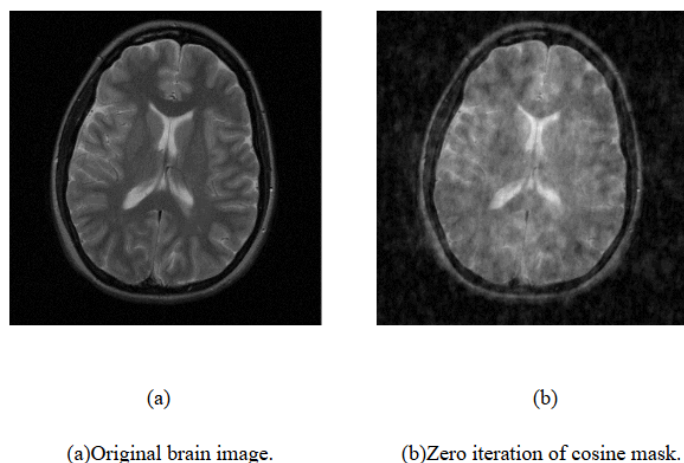


Figure 3. Noise on zero iteration

Each mask will need different iterations according to its sampling strategy to achieve the optimal effect, but the more iterations in the impression is not the better. We chose five different sampling strategies:

- (a) normal distribution needs 50 times.
- (b) Standard Linear distribution needs 80 times.
- (c) Cosine needs 80 times.
- (d) Modified Linear distribution needs 24 times.
- (e) Weibull distribution needs 80 times.

3. Results

3.1. Simulation results

We did 10 simulations to each mask and all `im_diff_energy` shown in this section are the average of the 10 simulation values, rounded to the closest whole number.

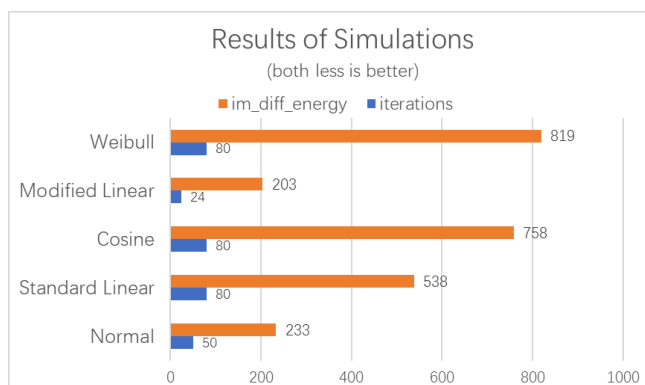


Figure 4. This is the result of the simulations

Figure (4) presents the simulation results. The Normal (Gaussian) mask, which nowadays is widely used in MRI Compressed Sensing, yielded an `im_diff_energy` of 233 within 50 iterations. The application of the Standard Linear method with 80 iterations produced an `im_diff_energy` of 538 and utilizing the Cosine mask for 80 iterations generated a value of 758. Surprisingly, the Modified Linear mask, executed across only 24 iterations, resulted in a value of 203. Applying the Weibull method over 80 iterations led to a value of 819.

The Normal mask gave a satisfying result comparatively with the second lowest `im_diff_energy` and taking 50 iterations, which means the reconstructed image is of good quality and takes less time to produce. The Weibull, Cosine and Standard Linear mask all took 80 iterations, but is not that effective with high `im_diff_energy`. The Modified Linear mask performed best among all 5 masks. It not only achieved the lowest `im_diff_energy` of 203, but also only took 24 iterations, half of the second least Normal mask. The Modified Linear mask exhibited not only increased speed but also proved to be as, maybe even more, effective than the Normal mask.

3.2. Refinement to Weibull and Cosine mask

As the outcomes of the Weibull mask and the Cosine mask are not as satisfying, we made the refined versions of them by making the must-be-sampled area in the middle bigger, from 255-258 to 240-273, 30 pixels larger in side length. This caused very minor increase in the sampling rate. The results of simulations are presented in Figure (5). The `im_diff_energy` considerably decreased, meaning the refined versions outplayed the original masks. From the results, we can confirm that in the k-space, the more pixels in the middle are sampled, the better the quality of the reconstructed image will be.

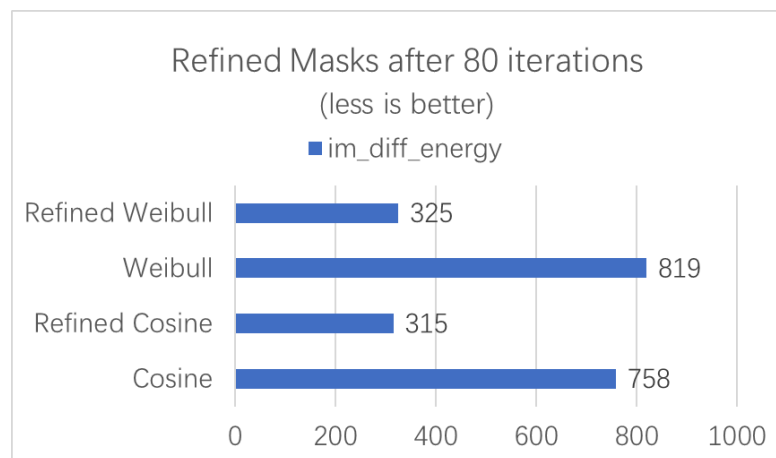


Figure 5. This shows the contrast between the simulations results of refined masks and the original masks

4. Discussion

4.1. The poisson distribution

The Poisson Distribution is also a probability distribution function worth researching when considering the pdfs of sampling masks. Overall, the Poisson mask we tried showed great potential. It reconstructed the image successfully with acceptable details, but its `im_diff_energy` is high. However, it showed its value and potential when the sampling rate becomes very low. It often needs

a sampling rate of only 15%, but can also get a clear image. Breaking the limits of sampling rates might be its further research topics.

4.2. Improvement to the Modified Linear mask

Our Modified Linear mask has shown room for further improvement. The Modified Linear mask is sometimes unstable. In several occasions, it reached the `im_diff_energy` of 165 and also many times around 180. Maybe through further optimization, the Modified Linear mask can have even better performance.

5. Conclusion

We chose to compare the difference between normal distribution and four other distributions. Under 35% sampling rate, all the distributions can make the picture reconstruct. The modified linear distribution shows the best result and use less iterations, and the cosine distribution has the highest image lost, reconstructing the worst image. With the comparison of different size of center, it was found that relative more pixels chosen in center can make the reconstruction better.

For future work, there is still a lot of places can be improved. More detailed comparison and more accurate control on variables can polish this experiment and make the results better. Also, in every distribution, we plan to find a suitable proportion for central sampling and edge sampling. If images obtain a clear result, the sampling ratio can be further reduced and save more time.

Acknowledgement

Yizhou Meng, Xiang Fei, and Congzhong Wang contributed equally to this work and should be considered co-first authors.

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