

A Review of Research on Truss Structure Optimization Algorithms

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Abstract. Although algorithm selection is still a practical difficulty when working with high-dimensional mixed variables, truss structure optimization is a basic topic in structural engineering. This work examines the development of truss optimization algorithms in a methodical manner, contrasting and evaluating metaheuristic algorithms, developing machine learning paradigms, and conventional mathematical programming. The study methodically assesses several algorithms from three perspectives: computing resource consumption, optimization mechanism, and adaptation to discrete-continuous mixed variables. Analysis reveals that while metaheuristic algorithms, like genetic algorithms and particle swarm optimization, can successfully handle mixed variable problems, their application in large trusses is frequently limited by the high computational cost of high-frequency finite element analysis. Traditional gradient algorithms are limited when dealing with discrete sections. Thus, this paper investigates the introduction of Multi-Agent Reinforcement Learning (MARL) and Deep Neural Network (DNN) surrogate models, pointing out that they offer an efficient means of reducing the previously mentioned computational bottleneck through graph structure collaborative search and quick mechanical response prediction. The objective of this study is to offer data-driven design technical references and methodical algorithm selection criteria for the intelligent optimization of intricate spatial trusses.

Keywords: Truss optimization, traditional optimization algorithms, metaheuristic algorithms, machine learning, surrogate models

1. Introduction

The scale and complexity of modern spatial structures are growing at an unprecedented rate. In cutting-edge fields such as large deployable supports in aerospace, metamaterial lattices, and long-span bridges, space shells and space reticulated shells in civil engineering, truss structures, with their excellent axial force characteristics and extremely high material utilization, constitute the core skeleton of large load-bearing systems. As engineering constraints evolve towards multiple working conditions and multiple physical fields, the traditional trial-and-error method that relies on human intuition and experience can no longer reach the true upper limit of the performance of structural materials. Therefore, by using rigorous algorithmic logic, under the premise of meeting the strict

mechanical constraints such as displacement, stress, frequency and local buckling, the truss system can be transformed from manual design to automatic optimization. This has strategic research value and engineering guidance significance for promoting the intelligent transformation of modern structural engineering, greatly improving system safety and macroscopic mechanical efficiency [1].

In response to this core issue in structural engineering, many influential review articles have been published in the past two decades. Early reviews mainly focused on traditional mathematical programming methods and optimality criteria [2], and discussed in detail their local rapid convergence mechanism and mathematical rigor in continuous convex space. With the increase in the complexity of engineering problems, recent review studies have largely shifted to the meta-heuristic field represented by genetic algorithms [3] and particle swarm intelligence algorithms [4], and systematically affirmed their robustness in breaking through the bottleneck of discrete engineering variables and multi-peak space search [5]. However, most current review articles are limited to the improvement of single algorithms and have failed to fully combine the development of "data-driven" and "artificial intelligence" technologies in recent years. Especially when facing complex spatial grids with thousands of degrees of freedom, high-frequency finite element calls will generate significant computational burden. Existing literature rarely conducts systematic comparative studies of traditional gradient methods, swarm intelligence algorithms and the latest emerging deep neural networks (DNN) and reinforcement learning (RL), resulting in engineers lacking clear algorithm selection criteria when facing large-scale intelligent optimization tasks [6].

To fill the research gaps mentioned above, this paper breaks down the barriers of perspectives from a single algorithmic school of thought, aiming to provide a panoramic evaluation framework for intelligent optimization algorithms for truss structures. The two main contributions of this study are: This paper first builds a multi-dimensional cross-comparison system based on the underlying operational mechanisms, which include machine learning architectures, optimization mechanisms, adaptability, and time or computational consumption, as well as conventional mathematical programming and metaheuristic evolutionary mechanisms. When dealing with complex engineering constraints and extremely non-convex solution spaces, the system establishes the performance bounds of different algorithms. Second, by introducing and examining the uses of DNN surrogate models and multi-agent reinforcement learning (MARL) in overcoming computational bottlenecks and dynamic optimization of graph topology, this paper overcomes the limitations of earlier reviews that solely concentrated on "search logic." Using a cross-disciplinary fusion approach, this work seeks to offer a reference paradigm for algorithm selection in the lightweight design of next large-scale space trusses [7-9].

2. An overview of truss structure optimization problems

2.1. Classification of optimization problems

Trusses are a common sort of structure in many different fields, such as aeronautical engineering, metamaterial design and civil engineering, such as grid shell and truss bridge. Because of their effectiveness and low weight, the final designs are favored in practice. A truss design issue is the identification of truss configurations, such as truss shape, member size, and topology, to successfully achieve the given performance, as shown in Table 1. For instance, truss constructions were designed to minimize material consumption or overall cost while safely and effectively transferring loads. To date, a number of approaches have been put out to solve various truss design issues and create high-performance truss structures. Among the various design strategies, the truss optimization method is crucial. Over the last thirty years, several truss optimization techniques have been presented. It is

possible to create trusses with various characteristics by establishing appropriate goals and limitations. Node coordinates, member sizes, and member connections are examples of design parameters that can be used to classify truss optimization problems. Size optimization, shape optimization, topology optimization and integrated optimization are typical examples [10, 11].

Table 1. Comparison of characteristics of truss optimization types

optimization type	design variables	common objectives	features	limitations
size optimization	cross-sectional area of the member (A_i)	minimize structural weight or total material cost	the most fundamental approach, with the widest range of engineering applications; it involves discrete material selection from standard steel shape libraries	it is necessary to strictly satisfy nonlinear mechanical constraints, such as element stresses, nodal displacements, and local buckling of members
shape optimization	spatial coordinates of the Node (X)	minimization of compliance or stress equalization	involves the optimization of continuous variables and significantly alters the overall macroscopic load-transfer path	constrained by nodal displacement boundaries; requires measures to prevent geometric components from colliding or overlapping
topology optimization	member retention or density (ρ_i)	minimize structural compliance for a given material volume.	the underlying mathematics are extremely complex, necessitating the use of the base structure method or the SIMP method for processing	constrained by volume, equilibrium, and kinematic stability; the direct removal of structural members is prone to rendering the stiffness matrix singular
integrated optimization	synchronous combination of size, shape, and topology	optimal overall performance	it belongs to the highly challenging class of Mixed-Integer Nonlinear Programming (MINLP) problems	by superimposing all mechanical and geometric constraints, solving typically requires reliance on advanced algorithms or multi-layered nested strategies

2.2. Mathematical models for the optimization of truss structures

Before beginning the optimization of a truss structure, the optimization objectives must be translated into a basic mathematical model, upon which the optimization algorithm can then be refined. The following is an introduction to the basic mathematical model of the target algorithm.

2.2.1. Weight minimization model

In discrete sizing optimization problems of truss structures, the objective usually is to minimize the weight of the structure while satisfying nonlinear constraints on element stresses, nodal displacements, critical loads, etc. The classic mathematical formulation for the weight minimization problem is widely established in structural optimization literature [1], which can be expressed as follows:

$$\min W(X), X = \{x_1; x_2; x_3; \dots; x_N\}^T \quad (1)$$

$$S.t. g_j(X) \leq 0, j = 1, 2, \dots, k \quad (2)$$

$$X \in S_d = \{X_1; X_2; \dots; X_p\} \quad (3)$$

where $W(X)$ is the cost function corresponding to the structural weight, and X is the vector of N design variables, such as cross-sectional areas; $g_j(X)$ represents the j -th inequality constraint function, which typically represents mechanical constraints such as stress, displacement, or critical load; k represents the total number of inequality constraint functions; S_d represents a set containing P available discrete cross-sectional areas, provided according to production standards.

2.2.2. Shape optimization model

The classic mathematical formulation for the shape optimization problem is widely established in structural optimization literature [1], which can be expressed as follows:

$$X = \{x_1, y_1, \dots, x_n, y_n\}^T \quad (4)$$

$$\min_x W(X) = \rho_0 \sum_{i=1}^m A_i(X) L_i(X) \quad (5)$$

$$s.t \quad \mathbf{K}(A, X) \mathbf{u} = \mathbf{f} \quad (6)$$

$$g_j(X, \mathbf{u}) \leq 0, j = 1, \dots, n_g \quad (7)$$

$$X_{min} \leq X \leq X_{max} \quad (8)$$

Where ρ_0 represents the material density, $A_i(X)$ and $L_i(X)$ represent the cross-sectional area and length of the i -th member element; in this model, they are affected by the nodal coordinates ' X '; ' m ' represents the total number of member elements in the structure; $\mathbf{K}(A, X)$ is the global stiffness matrix affected by both cross-sectional area and coordinates; ' \mathbf{u} ' and ' \mathbf{f} ' are the global nodal displacement vector and the applied load vector, respectively.

2.2.3. Topology optimization model

This section consists of two main parts: the SIMP density method and discrete topology optimization. The mathematical expression of the SIMP density method is as follows [12].

$$\rho = \{\rho_1, \rho_2, \dots, \rho_m\}^T, 0 \leq \rho_i \leq 1 \quad (9)$$

$$\rho \min_{\rho} C(\rho) = \frac{1}{2} \mathbf{f}^T \mathbf{u} \quad (10)$$

$$s.t \quad \mathbf{K}(\rho) \mathbf{u} = \mathbf{f} \quad (11)$$

$$\mathbf{K}(\rho) = \sum_{i=1}^m [E_{min} + \rho_i^P (E_0 - E_{min})] (\mathbf{k})_i^0 \quad (12)$$

$$\frac{1}{V_0} \sum_{i=1}^m \rho_i v_i \leq \bar{V} \quad (13)$$

Where $C(\rho)$ is the compliance of the structure, characterizing the degree of deformation of the structure, and its value is equal to half of the work done by the external force; E_0 is the initial elastic modulus of the solid material, E_{min} is the elastic modulus of the weak material set to prevent the stiffness matrix from being singular, and its value is usually $10^{-9}E_0$; P is the penalty factor, and the value of P is usually greater than or equal to 3, mainly used to force the intermediate density ρ_i to converge to 0 or 1, to achieve a clear topological boundary; k_i^0 refers to the local stiffness matrix of the i -th element under unit elastic modulus; V_0 , v_i and \bar{V} are the initial total volume of the design domain, the volume of the i -th element, and the set upper limit score for retaining the target volume, respectively.

Although the SIMP variable density method has high solution efficiency when dealing with large-scale structures, it essentially uses a continuous relaxation strategy to handle discrete variables. This often results in grayscale cells between 0 and 1 in the optimization results, which lack clear physical meaning in discrete systems and are difficult to directly implement in manufacturing [13].

Therefore, in order to obtain a truss topology configuration with a clear force transmission path and that fully conforms to engineering reality, discrete topology optimization, namely Ground Structure Method, provides another more direct solution paradigm. Discrete topology optimization directly introduces pure 0-1 logical variables with very clear physical meaning, where "1" represents the retention of members and "0" represents the deletion of members, thus solving the grayscale problem [13]. Its mathematical expression is as follows:

$$\min \sum_{i=1}^m \rho_i L_i, \quad \rho_i \in \{0,1\} \quad (14)$$

and this mathematical framework has been extensively integrated with modern computational algorithms for robust topological exploration in spatial domains [14].

2.2.4. Integrated optimization model

When simultaneously optimizing size, shape and topology, such as in three nested layers or simultaneous solutions, the objective function can be expressed as follows [11].

$$\min W(A, X, T) = \sum_{i=1}^n t_i \rho_i A_i L_i(X) \quad (15)$$

$$S.TK(A, X, T)U = F \quad (16)$$

$$g_j(\sigma) = \frac{|\sigma_j|}{\sigma_{allow}^j} - 1 \leq 0, \quad (j = 1, 2, \dots, n) \quad (17)$$

$$h_k(u) = \left(\frac{|u_k|}{u_{allow}^k} \right) - 1 \leq 0, \quad (k = 1, 2, \dots, m) \quad (18)$$

$$\sum_{i=1}^m \rho_i A_i L_i \leq V_{max} \quad (19)$$

$$g_j(A, X, \rho, u) \leq 0, \quad j = 1, \dots, n_g \quad (20)$$

$$A_{min} \leq A_i \leq A_{max} \quad (21)$$

$$X_{min} \leq X \leq X_{max}, \quad 0 \leq \rho_i \leq 1 \quad (22)$$

Where W represents the corresponding structural weight; ρ_0 is the material density; $Li(X)$ is the length of the member element affected by the node coordinate ' X '; ' X ' is the shape variable vector, representing the spatial geometric coordinates of the node, $X^L \leq X \leq X^U$; \mathbf{K} is the global stiffness matrix affected by the cross-sectional area, coordinates and topology; U and F are the global node displacement vector and the external load vector, respectively. σ_j and u_k represent the actual stress of the j-th member and the actual displacement of the k-th node, respectively, with σ_{allow}^j and u_{allow}^k being their corresponding allowable upper limits. V_{max} represents the maximum allowable volume upper limit when considering changes in cross-sectional size, node shape, and topology simultaneously. The structure's natural frequency is denoted by ω_r , while its lower limit is denoted by ω_{allow}^r . To prevent resonance failure, $\omega_r \geq \omega_{allow}^r$. The member's cross-sectional area is represented by the dimension variable vector A . Owing to actual manufacturing constraints, its variable domain is often a set of discrete cross-sections, meaning that $A \in \{S_1, S_2, \dots, S_q\}$. T is a topological variable vector that represents the member's retention state and corresponds to ρ in the classical model. Additionally, $T \in \{0,1\}^n$ indicates that the member is eliminated when $t_i = 0$ and retained when $t_i = 1$.

2.3. Sensitivity analysis

Sensitivity analysis is an essential link between the "structural mechanical response" and the "optimization search direction" in structural optimization algorithms, particularly in conventional gradient-based algorithms like the Optimality Criterion Method (OC) and Sequential Quadratic Programming (SQP). Quantifying the partial derivatives of the goal function and constraints with regard to design variables is its primary purpose. The technique can precisely determine which variables are most important for enhancing structural performance by using sensitivity information. This allows the optimization process to converge along the shortest path. Using structural compliance as an example, its static sensitivity mostly falls into three categories:

- Size sensitivity can be expressed mathematically as follows:

$$\frac{\partial C}{\partial A_i} = -\frac{1}{2} \mathbf{u}^T \frac{\partial \mathbf{K}}{\partial A_i} \mathbf{u} \quad (23)$$

the derivative of structural compliance C in relation to the i-th member's cross-sectional area A_i . Physically speaking, it illustrates how increasing a particular member's cross-sectional area can enhance the overall stiffness of the structure [5]. The negative sign on the right side of the formula indicates that an increase in cross-sectional area will inevitably lead to a decrease in compliance, that is, an increase in stiffness. In the optimization iteration, members with larger absolute values of sensitivity should be given priority in material allocation.

- Topological sensitivity based on the SIMP model: Its mathematical expression is as follows:

$$\frac{\partial C}{\partial \rho_i} = -P \rho_i^{P-1} (E_0 - E_{min}) \left(\frac{1}{2} \mathbf{u}_e^T \mathbf{k}_i^0 \mathbf{u}_e \right) \quad (24)$$

The derivative of structural compliance C with regard to the elements' relative density ρ_i is represented by the formula. The local strain energy of the i-th element is actually represented by the term in parenthesis. In addition to reflecting how a change in element density affects macroscopic stiffness, the formula uses the penalty factor P to quantitatively accelerate the deterioration of low-

density elements, causing the topological configuration to advance toward a distinct 0 or 1 boundary [12].

- Shape sensitivity can be expressed mathematically as follows:

$$\frac{\partial \mathbf{K}}{\partial x_i} = \sum_{j \in N_i} \frac{\partial \mathbf{k}_j}{\partial x_i} \quad (25)$$

According to the formula, a change in node location x_i during truss node shape optimization will immediately alter the length and angle of the connecting members, which will alter the global stiffness matrix \mathbf{K} . It demonstrates that the sum of the partial derivatives of the stiffness matrices \mathbf{k}_j of all local components connected to that node with respect to that coordinate equals the form sensitivity of the global stiffness matrix to a particular node coordinate. For the purpose of determining the ideal node spatial geometry, this mathematically offers a gradient direction.

2.4. Finite element method and its synergistic mechanism with optimization algorithms

The finite element method (FEM) is the main numerical computational framework used in modern structural engineering to assess complex mechanical responses [15]. FEM is a numerical method for resolving engineering and physical issues. Its main idea is to discretize complex continuums into basic parts connected by nodes and use global system equations to calculate unknown quantities like displacement and stress. FEM is the fundamental building block of contemporary complex structure optimization, despite not being an optimization process in and of itself. FEM is necessary to provide precise mechanical feedback, especially for intricate, massive, or non-standard trusses that cannot be represented by straightforward linear models. Thus, FEM and optimization algorithms form a highly integrated "Exploration-Evaluation" collaboration process in the closed-loop framework of complicated truss structure optimization [10].

Specifically, the optimization algorithm is responsible for generating and updating structural parameters, such as updating element density based on the SIMP method or adjusting the cross-section or node coordinates of members [16]. The FEM is responsible for performing mechanical simulations and returning response results to determine the search direction. In modern engineering software, such as ANSYS, OptiStruct and Abaqus, the solver and optimizer are typically deeply integrated based on this logic.

However, this close collaboration also brings significant engineering challenges. When dealing with statically indeterminate or large space trusses, each population iteration of the optimization algorithm is often accompanied by dozens or even hundreds of FEM calls. This high frequency of finite element calls constitutes a computational bottleneck, which is also the core motivation for introducing machine learning to build surrogate models [6].

3. Classification and research progress of optimization algorithms

3.1. Traditional optimization algorithm

Traditional structural optimization methods are mainly based on analytical mechanics criteria and mathematical programming theory, and their evolution forms the basis of modern optimization techniques. These methods mainly include mathematical programming, Optimality Criteria (OC), and Fully Stressed Design (FSD). A. G. M. Michell's minimum weight truss theory, which first examined the issue of optimal material distribution under ideal circumstances, is where structural optimization theory got its start in 1904. The Optimality Criteria (OC) method was further expanded

and refined by academics like William Prager and G. I. N. Rozvany, offering an effective mechanical criterion framework for resolving challenging structural optimization problems with constraints [2].

From a mathematical standpoint, conventional optimization techniques usually express structural design problems as constrained minimization problems, where the constraints are stress, displacement, and equilibrium equations and the objective function is the least structural weight. These techniques, which use the gradient partial derivatives of variables with respect to the response to steer the search direction and carry out iterative updates, mainly rely on the sensitivity analysis discussed in Section 2.3. Therefore, conventional approaches typically provide excellent computing efficiency and quick convergence speed in problems with continuous variables and well-defined restrictions. An overview of a few important algorithms is provided below.

- Fully stressed design: In the early days of industry, this heuristic criterion was frequently employed. Each part should exceed the material strength limit under at least one working situation in the ideal state, according to its basic logic. The approach uses the iterative formula to update the cross-section.

$$A_{\{i+1\}} = A_i \left(\frac{\sigma_i}{\sigma_{allow}} \right) \quad (26)$$

When dealing with statically determinate trusses that are primarily controlled by stress, FSD has very high computational efficiency. However, when dealing with complex statically indeterminate structures that are strictly controlled by displacement, it is limited because global stiffness is not taken into account.

- Optimality criteria methods: The OC technique determines the mechanical requirements that must be met for a structure to reach its optimal state by using the Lagrange multiplier approach and variational principles [2]. Establishing a Lagrange function and solving for stationary points is its fundamental logic.

$$L(x, \lambda) = f(x) + \sum \lambda_i g_i(x) \quad (27)$$

The OC approach is more capable of managing several global restrictions, including displacement, than FSD. The OC approach exhibits a quicker rate of convergence in situations with many variables but few active restrictions, like decreasing the compliance of complex space trusses [17]. However, it is more challenging to derive the analytical updating process for the multiplier in multi-condition and multi-physics linked issues.

- Sequential linear programming (SLP) and sequential quadratic programming (SQP) are the two basic categories of mathematical programming [2]. This approach constructs a Taylor expansion through sensitivity analysis, treats truss optimization as a rigorous constrained nonlinear optimization problem, and directs the variables to iterate in the gradient's direction. The rigorous proof of convergence is its advantage. However, because it frequently uses a continuous relaxation strategy, this approach is prone to get stuck in local optima when dealing with mixed truss optimization issues containing a high number of discrete variables, such as the discrete selection of standard steel section libraries.

- The typical, standard paradigm for optimizing truss topology is the basis structure method. This method, which involves the methodical removal of inefficient members from a predetermined dense 3D network, offers a rigorous mathematical modeling framework for discrete spatial truss topology optimization [14]. The reasoning is to convert the topology problem into a cross-sectional area

optimization problem; a member is deemed to need to be eliminated when the area gets close to zero. The optimal force-transmission path of the structure can be intuitively explored using this method. However, the computational scale grows exponentially as the number of pre-set connections between nodes increases, and the optimization results frequently include a large number of extremely fine micro-members, which presents significant challenges for subsequent engineering manufacturing and node-splicing design.

Traditional approaches are generally limited when handling discrete variables or highly nonlinear mixed problems, are essentially local optimizations, and mainly rely on gradients and starting designs. In order to satisfy the demands of more complicated engineering applications, these constraints have led researchers to continue developing contemporary optimization techniques based on intelligent algorithms and uncertainty analysis.

3.2. Evolutionary and population-based metaheuristic optimization methods

Since the cross-sectional dimension variables and geometric node coordinates and topological variables are fundamentally different in physical properties and order of magnitude, the search space of such mixed variables exhibits high non-convexity and nonlinearity, which easily leads to traditional algorithms getting trapped in local extrema [5]. As described in the comprehensive mixed variable mathematical model covering size, shape and topology constructed in Section 2.2.4, metaheuristic algorithms have become a research hotspot due to their characteristics of not relying on gradient information and strong robustness in the face of such extremely complex engineering solution domains. Currently, the most widely used metaheuristic frameworks in truss engineering mainly include the following categories.

3.2.1. Evolutionary algorithms (EA)

These algorithms are based on the principle of simulating the natural selection, hybridization, and mutation processes of biological evolution. Genetic algorithms (GA) are the most typical example. In truss optimization, this set of biological concepts is rigorously mapped into the language of engineering mechanics. For example, Chromosome represents a specific truss design scheme used for the section numbering sequence of all members; Gene represents a single design variable used to represent the area A_i of the i -th member; Fitness corresponds to the total weight of the structure [3]. The survival of the fittest mechanism in engineering is manifested in that truss designs that do not meet stress or displacement constraints will be subject to a "penalty function", which greatly reduces their probability of being retained and participating in cross-variation during iteration.

In deeper mechanical mapping, GA has good engineering applicability in handling mixed integer nonlinear programming (MINLP). First, in topology optimization, directly deleting members can easily lead to singular stiffness matrices. GA often uses the small area method, such as letting $A_i = 10^{-6}$, to maintain the positive definiteness of the matrix. Second, in terms of constraint handling, modern GA often uses the Deb feasibility rule to replace the static penalty function [18]. This rule prioritizes retaining feasible solutions that satisfy all stress and displacement constraints; if both are infeasible solutions, the evolution is carried out to minimize the total degree of constraint violation, and its objective function can be expressed as:

$$CV = \sum \max(0, g_j) + \sum \max(0, h_k) \quad (28)$$

This mechanism greatly improves the global convergence success rate of GA in complex statically indeterminate trusses [18].

3.2.2. Swarm intelligence algorithm

Unlike evolutionary algorithms that focus on natural selection, swarm intelligence algorithms optimize by simulating the social cooperative behavior of biological groups. Particle swarm optimization (PSO) is a classic example, which utilizes the "momentum" dynamics similar to gradients and is highly efficient in optimizing continuous geometric variables of trusses [4]. Its velocity and position update formulas are as follows:

$$v_{id}^{t+1} = \omega v_{id}^t + c_1 r_1 (pbest_{id} - x_{id}^t) + c_2 r_2 (gbest_d - x_{id}^t) \quad (29)$$

$$x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} \quad (30)$$

Where ω is the inertia weight, and c_1 and c_2 are learning factors.

Although classic PSO excels in continuous space, it performs poorly in discrete space design cases, especially when dealing with section selection. Therefore, binary particle swarm optimization (BPSO) is often introduced, which maps velocities to probabilities through the Sigmoid function. Its mathematical expression is as follows:

$$S(v_{id}) = \frac{1}{1 + \exp(-v_{id})} \quad (31)$$

This indicates that if $rand < S(v_{id})$, $x_{id}^{t+1} = 1$, implying that the section is selected or the member is retained; otherwise, it equals 0. Compared with PSO, which requires complex probability mapping when dealing with discrete variables, the ant colony algorithm (ACO), which also belongs to swarm intelligence, shows more direct engineering applicability in the force transmission path optimization of truss discrete topology due to its natural discrete graph network search logic.

3.2.3. Other algorithms inspired by physics or nature

In addition to the two main algorithm frameworks mentioned above, the Simulated Annealing (SA) algorithm has also been applied in specific cross-section optimization tasks due to its unique local escape mechanism. However, whether it is a single evolutionary, swarm or other algorithm, it is difficult to achieve absolute universality when dealing with actual truss engineering. Faced with the complex problem of needing to deal with discrete cross-sections, continuous coordinate fine-tuning and topological connectivity at the same time, the current trend of engineering applications has shifted to hybrid optimization strategies. For example, GA with strong discrete mapping ability is used as the main framework to determine the macroscopic topology and cross-section model, while local nested PSO or traditional gradient method is used to fine-tune the node geometric coordinates. Studies have shown that the final performance of metaheuristic algorithms under complex constraints is highly dependent on their balance strategy between global exploration and local exploitation [5].

3.3. Machine learning-based algorithms

When dealing with large spatial trusses containing high-dimensional variables and multiple constraints, traditional algorithms and metaheuristic algorithms are often limited by the computational cost of frequent finite element method calls. To address this, machine learning (ML) techniques have been introduced into the field of structural optimization, using surrogate models or intelligent search strategies to alleviate the computational bottleneck. Currently, the most representative cutting-edge applications in truss structure optimization are deep neural networks (DNNs) and multi-agent reinforcement learning (MARL).

3.3.1. Deep neural networks (DNNs)

DNN is a feedforward artificial neural network based on a multilayer perceptron (MLP). Its theoretical basis is the universal approximation theorem, which can effectively fit the nonlinear mapping between structural design variables and mechanical response [19]. Its forward propagation process performs linear weighted fusion and nonlinear activation on high-dimensional input features layer by layer. The mathematical expression is as follows:

$$h^l = \sigma(W^l * h^{\{l-1\}} + b^l) \quad (32)$$

Where W^l and b^l are the weight matrix and bias vector of the l -th layer, respectively [19]. By calculating the loss function between the output layer prediction and the real physical label, the model updates the weights using backpropagation and gradient descent until the fully connected network converges to the neighborhood of the global optimum.

In truss optimization, DNN is mainly used to reduce or replace the cost of finite element method (FEM) solutions. Typical engineering applications include:

Proxy model and fast reanalysis: In the optimization of large-span reticulated shells involving thousands of members, frequent assembly of the global stiffness matrix leads to high computational costs. The time required to forecast the mechanical response can be reduced to milliseconds once the DNN has been trained using a sample set. High-frequency iteration can be supported and substantial computational resource release can be achieved by directly integrating this proxy model into the optimizer [6, 20].

Discrete section probability prediction: The DNN can be built as a classification network to solve the data combination issue brought on by discrete material selection. The model reduces the search boundary of the heuristic method by directly outputting the probability distribution of each member's best section based on the geometry and load characteristics of the structure [7].

DNN improves the paradigm of optimization: DNN has been thoroughly incorporated into the optimization mechanism in a few recent works. In addition to lowering the assessment cost, adaptively training and updating the DNN model throughout the evolution process allows for direct population optimization in parallel, preserving the variety of structural options while guaranteeing lightweight design [8].

3.3.2. Multi-agent reinforcement learning (MARL)

Multi-agent reinforcement learning (MARL) provides a new theoretical framework for describing the dynamic evolution of complex structural topologies. In order to precisely map the mechanical physical entity of the truss into a mathematical graph network, researchers have proposed graph

representation in truss optimization [9]. This involves converting nodes into vertices and physical members into graph edges. This approach assigns separate agent identities to each component of the structure. In iterative optimization, each agent works together to jointly optimize the global stiffness matrix by watching the stress and displacement states of its neighboring components. Examples of such activities include coordinating the change of cross-sectional dimensions or node coordinates.

MARL presents a differential reward system that is very consistent with the logic of structural mechanics in order to steer the algorithm toward convergence. By measuring the effect of particular agent activities on the overall structural compliance, this method assesses the actual contribution of a single member to the global macroscopic force transmission path. This method, which integrates multi-agent game theory and graph topology, has demonstrated good applicability in the joint optimization job of high-dimensional topological variables.

4. Comparative analysis of algorithms in truss optimization applications

4.1. Analysis of the advantages and disadvantages of the three main algorithm frameworks

Traditional optimization algorithms, metaheuristic algorithms, and machine learning-based algorithms exhibit notable theoretical differences in optimization performance, computational efficiency, and adaptability to complex variables for the multidimensional constraints and mixed variable characteristics of truss structure optimization. Overall, as Table 2 illustrates, machine learning paradigms offer state-of-the-art solutions for computing bottlenecks, metaheuristic algorithms excel in robust global search, and conventional methods guarantee exact mathematical rigor.

As mentioned in Section 3, traditional mathematical programming is prone to local optima when dealing with discrete variables [2]. While metaheuristic algorithms adapt well to mixed variable spaces [3, 4, 18], their high-frequency iterations lead to significant computational costs [5]. Consequently, emerging DNN surrogate models and MARL provide effective paths to break through these bottlenecks [7-9, 20].

Table 2. Characteristics comparison of genetic algorithm-based optimization methods

algorithm categories	representative algorithm	core advantages	limitation	applicable scenarios
evolutionary algorithm	genetic algorithm (GA), differential evolution (DE)	it does not rely on gradient information; its discrete coding is well-suited to engineering practice, and it excels at handling stringent constraints	finite element methods are frequently used, resulting in high computational costs; their ability to perform local refinement searches is relatively weak	discrete size optimization. multi-objective comprehensive optimization. discrete basis structure topology optimization
swarm intelligence algorithm	particle swarm optimization (PSO), ant colony optimization (ACO)	it has high efficiency for global search in continuous space; requires few adjustment parameters and has strong robustness	when dealing with purely discrete variables, mapping is required, which limits the performance, under complex stress conditions, it is prone to premature convergence	finding continuous geometric variables. optimizing force transmission paths in discrete topologies

Table 3 compares the core mechanisms, advantages, disadvantages and best applicable scenarios of the emerging machine learning paradigm in truss engineering selection.

Table 3. Feature comparison of truss optimization algorithms based on machine learning

algorithm type	core mechanism	application scenarios	advantages	limitations
deep neural networks (DNNs)	high-dimensional nonlinear mapping to construct a data-driven proxy model	rapid reanalysis and discrete section probability prediction of large space trusses	millisecond-level prediction, greatly reducing the cost of finite element method calls	it heavily relies on the generation of high-quality training data
multi-agent reinforcement learning (MARL)	decentralized action execution, with individual contributions evaluated based on differential rewards	local component section adjustment and joint fine-tuning of node coordinates	it can accurately quantify the contribution of a single member to the global stiffness, and is suitable for multi-variable collaboration	as the number of agents increases, training convergence becomes more difficult

4.2. Degree of optimization of truss structure

The depth of intervention of optimization algorithms in truss design directly determines the extent of improvement in the macroscopic mechanical properties of the structure, and the degree of optimization exhibits a rigorous hierarchical progression. Dimensional optimization at the basic level mainly involves adjusting the cross-sectional area of tension or compression members within a given initial topology network. Extensive engineering practice and literature show that using metaheuristic algorithms for purely discrete dimensional optimization of conventional planar or spatial trusses, under the premise of strictly satisfying axial stress and global displacement constraints, can typically achieve a reduction in self-weight. This level of optimization does not fundamentally change the force transmission path; it mainly involves the balanced adjustment of redundant structural materials.

Advanced shape optimization opens up more critical design freedom, allowing for continuous adjustment of the spatial geometric coordinates of truss nodes. By guiding nodes to move in the opposite direction of high stress areas through algorithms, the axial force distribution inside the truss can be highly homogenized, effectively reducing local stress peaks. Joint optimization of shape and size can often reduce the structural self-weight by an additional 15% to 20% compared to single size optimization, and multiply the global deformation stiffness of the truss system [10].

The highest level is reflected in the highly challenging field of integrated optimization. When topology reduction strategies such as eliminating zero and inefficient members, the spatial continuous evolution mechanism of nodes, and the precise material selection method for discrete steel sections are deeply integrated into the same mixed integer nonlinear solution framework, the algorithm achieves global reconstruction of the truss topology [11]. At this stage, the macroscopic force transmission path is greatly optimized, and the final self-weight of the structure can usually be reduced by 30% or even more than 40% [11, 21]. Combining the latest DNN agent enhancement and MARL mechanism, the current integrated optimization strategy can ensure extreme lightweight while taking into account highly nonlinear mechanical constraints such as the lower limit of the natural frequency of the truss and the Euler buckling critical load [21].

5. Conclusion

This paper systematically reviews the development of truss structure optimization algorithms and compares and analyzes the applicability differences of traditional mathematical programming, metaheuristic algorithms, and machine learning in multidimensional constraint and mixed variable optimization. Research shows that the core challenge of truss optimization lies in the coordination between discrete section selection and continuous geometric fine-tuning. Traditional algorithms are prone to getting trapped in local optima when dealing with discrete variables; metaheuristic algorithms effectively improve the engineering adaptability to mixed variable spaces through probabilistic search, but suffer from limitations such as high frequency of finite element method calls and high computational cost; while deep neural network (DNN) surrogate models and multi-agent reinforcement learning (MARL) provide effective technical paths to overcome computational bottlenecks and achieve high-dimensional topology optimization through rapid mechanical response prediction and dynamic graph structure interaction.

Based on the above conclusions, this paper proposes the following three strategic recommendations for the intelligent optimization design and engineering practice of complex spatial trusses. First, it is advised to encourage the combination of mechanistic models with data-driven methods. Deep neural networks are proposed to replace high-frequency classical finite element static/dynamic solutions with lightweight mechanical proxy models in truss evaluations requiring large-scale degrees of freedom, thereby lowering processing costs. Second, strengthening the constructability constraints prior to intervention is advised. To ensure that the optimization results can be well integrated with subsequent node assembly and actual construction, the discrete basis structure method with clear physical logic should be given priority during the topology generation stage. Additionally, slenderness ratio constraints and node anti-crossing constraints should be explicitly introduced in the penalty function to reduce the generation of components without physical meaning. Finally, it is recommended to adopt a heterogeneous collaborative hybrid solution architecture. In the macro-topology evolution and discrete section initial selection stages, a genetic algorithm with strong discrete variable processing capabilities should be used; in the node geometry fine-tuning stage, a particle swarm optimization algorithm or mathematical programming method with continuous space optimization advantages should be introduced to balance the algorithm's global exploration and local development capabilities.

This paper primarily conducts a qualitative evaluation from the perspectives of the algorithm mechanism and theoretical applicability. The convergence accuracy and hyperparameter sensitivity of various algorithms have not yet been uniformly benchmarked and rigorously cross-validated. Looking to the future, the current optimization paradigm is mostly limited to static and deterministic mechanical assumptions, and the consideration of the semi-rigid characteristics of truss nodes and alternating dynamic loads is relatively limited. Future research on truss optimization should gradually transition toward robust design. It is recommended to introduce random parameters such as initial geometric defects of members and load uncertainty in the evaluation closed loop. At the same time, further improving the topology generalization ability of the surrogate model so that it can adapt to the drastic changes in the degree of freedom during the evolution process will be the key direction for promoting the safe and lightweight design of large and complex space trusses in a real service environment.

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