

Key Influencing Factors of Bayesian Update in Structural Model Revision—Taking the Uncertainty Quantification of Truss Structures as an Example

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Abstract. Bayesian update, as a probabilistic inversion method, can dynamically correct model parameters by integrating observation data. However, its effectiveness is affected by various factors, and existing research mainly focuses on the algorithm itself, with insufficient exploration of the impact mechanisms of observation configuration and information quality. This paper addresses the problem of the accuracy of truss structure models being affected by model precision and performance degradation, and studies the application of the Bayesian update method in structural model correction. Systematic numerical simulation experiments were carried out with a specific truss model. The cross-sectional parameters of the elements were set and a Monte Carlo sampling method was used. Using the control variable method, the effects of three key factors on the Bayesian updating effect—information quality, number of measurement points and updating method—were systematically analyzed. The mechanisms through which these key factors affect the performance of Bayesian updating were clarified, providing a theoretical basis and reference for optimizing the placement of measurement points and selecting updating methods in practical engineering.

Keywords: Bayesian update, uncertainty quantification, model update

1. Introduction

Truss structures are widely used in construction, bridges, and other fields. The accuracy of truss structure models directly affects their safety assessment, lifespan prediction, and operation and maintenance decision-making. However, initial models often deviate from reality due to uncertainties in material properties, boundary conditions, external loads, and performance degradation over time. Bayesian updating is a statistical method that can revise probabilities based on new evidence. This method is based on Bayes' theorem and provides a mathematical framework for updating hypothesis probabilities as more data becomes available.

Compared to traditional statistical methods, deterministic optimization methods often ignore parameter uncertainty, which can result in revised results that deviate from the physically true solution. When sample sizes are small, Bayesian updating can be used to adjust model parameters

based on new samples. Dynamic updating can also improve prediction accuracy. When frequency and mode data points are limited, traditional statistical methods may not provide reliable estimates or may result in ill-conditioned solutions. Bayesian methods introduce experience gained from past data into the analysis in the form of prior distributions. When the amount of data is small, this method can bias the posterior distribution to a physically plausible range, significantly improving the stability and reliability of small-scale sample identification. The state of a truss structure changes dynamically throughout its lifecycle, and Bayesian updating provides a natural framework for sequence assimilation. Therefore, it has wide applications in updating truss structure models. Beck et al. proposed and verified the feasibility of applying Bayesian theory to the field of civil engineering in 1998 [1].

After Beck proposed the theoretical basis, the relevant research mainly went in two directions. One is the research on the Bayesian update framework. For example, Miao Ji et al. proposed a method for updating the structural simulation model based on the improved Markov Chain Monte Carlo Method (MCMC) algorithm and the surrogate model, and built a three-story steel frame structure model for verification [2]. Song Jingwen et al. proposed the variable observation domain adaptive Bayesian update method, which combines the BUS algorithm with the Gaussian process model, effectively reducing the computational cost of the model update process [3]. Wang Haohao et al. proposed the Bayesian model update method under inaccurate probability input and its application [4]. Second, the optimization of sampling algorithms. The quality of prior information is the biggest source of error. By improving the sampling method, the error can be reduced to a certain extent. These include MCMC [5], Hamiltonian Monte Carlo (HMC) [6] and Elliptical Slice Sampling (ESS) [7]. There has been a lot of research and improvement on the sampling method. For example, Cabezas et al. proposed a new framework that uses the combination of normalized flow and ESS sampling to efficiently sample from the complex probability distribution [8]. Robnik et al. tuned the Hamiltonian function and developed Microcanonical Hamiltonian Monte Carlo (MCHMC) sampling based on HMC [9]. Kook et al. incorporated constraints into HMC sampling [10].

However, existing research on Bayesian updates largely focuses on the algorithm itself and its applications, with insufficient discussion of its impact on observation configuration and information quality. More specifically, analysis of crucial issues such as observation data quality and representativeness, optimization strategies for the number and layout of measurement points, and differences in information utilization between different update methods is lacking. Therefore, this paper systematically analyzes the key factors affecting the performance of Bayesian updates, including observation data quality, measurement point configuration, and differences in update methods. The aim is to deepen the understanding of uncertainty quantification and probabilistic inversion mechanisms, and ultimately provide suggestions for algorithm improvement and optimization to improve the accuracy and efficiency of updates.

2. Research on key influencing factors of Bayesian updates

This study selects a specific truss structure as the research object and uses OpenSees as the finite element simulation software. The actual cross-sectional area of element 10 is 0.7, the actual cross-sectional area of element 18 is 0.6, and the actual cross-sectional area of other elements is 1.0. Then, the cross-sectional areas of elements 10 and 18 are set to 1.0 as initial values for 10,000 Monte Carlo samplings. A load is applied in the y-direction of node 11 to calculate the reference displacements of nodes 4, 5, 6, 10, 11, and 12. A simple rejection sampling method is used, with the cross-sectional areas of elements 10 and 18 set to 1.0 as initial values for 10,000 Monte Carlo

samplings. The tolerance error between the calculated displacement and the reference displacement is 5%. Selective acceptance of parameter samples is performed, followed by approximate Bayesian parameter updates.

Approximate Bayesian Computation (ABC) is a Bayesian update method that uses the approximation idea and has an advantage in computational efficiency compared with the complete Bayesian update [11]. Using grid approximation, the parameter space is divided into 20 grids, the prior distribution is defined as a uniform distribution, and the prior values are the same, all being 1.0. The empirical likelihood function is calculated using the Monte Carlo success frequency to obtain the posterior distribution. The grid search method is used to find the maximum a posteriori estimate. The grid with the maximum a posteriori probability is found, and the midpoint value of the interval is taken as the maximum a posteriori estimate. This part studies the influence of various factors on the Bayesian update results by using the control variable method. Matlab is used to visualize the results derived from OpenSees calculations under different conditions, and the influence mechanism of each factor is analyzed and discussed. Figure 1 shows the truss structure view.

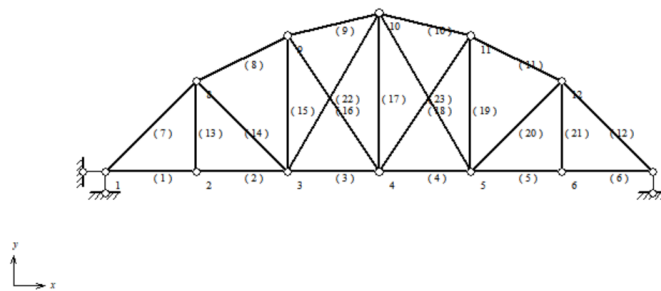


Figure 1. Truss structure view (picture credit : original)

2.1. Impact of information quality

Keeping other conditions unchanged, the information quality was changed, and the allowable error of the received data was set to 10%, 5%, and 2%, respectively. The posterior probability distribution of the cross-sectional area of the calculation unit 18 and the maximum a posteriori estimate were visualized using Matlab. The results calculated and exported by OpenSees under different conditions were visualized.

As can be seen from the figure 2, when the tolerance error is large, the posterior probability distribution is flat and the maximum a posteriori estimate differs significantly from the true value. As the information quality improves, the range of the posterior probability distribution becomes more concentrated, and the difference between the maximum a posteriori estimate and the true value decreases. That is, the uncertainty of Bayesian updates decreases as the information quality improves.

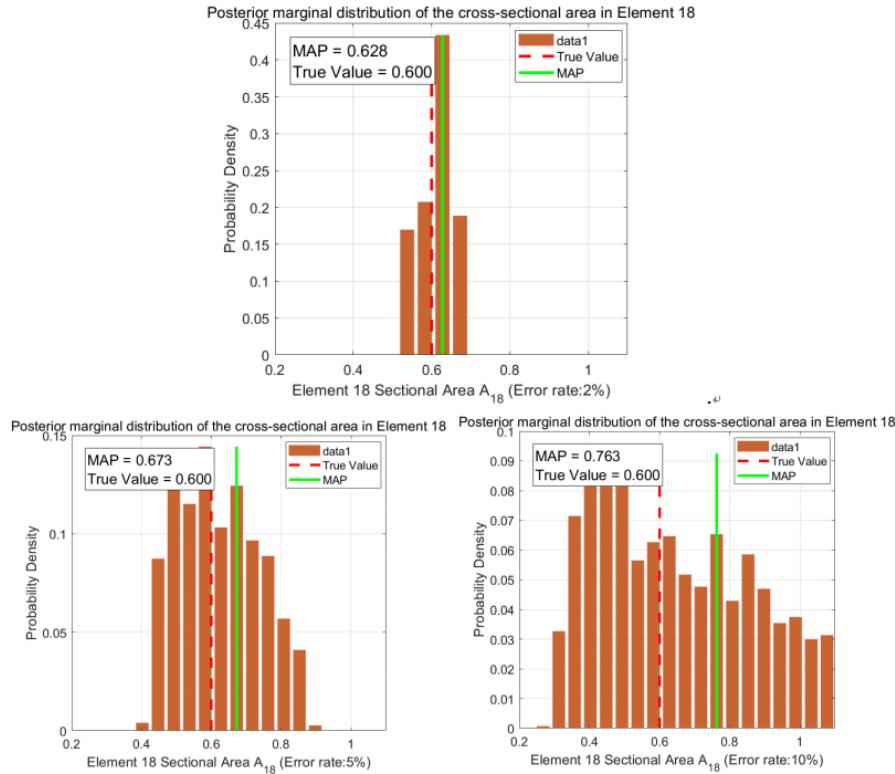


Figure 2. True values and maximum a posteriori estimates of cell 18 when the errors are 10%, 5%, and 2%. (picture credit : original)

2.2. Influence of the number of measuring points

Keeping other conditions unchanged, change the number of measuring points as follows: 1 measuring point: node 11, 2 measuring points: nodes 4 and 5, 3 measuring points: nodes 4, 5, and 6, 4 measuring points: nodes 4, 5, 6, 11, and 6 measuring points: nodes 4, 5, 6, 10, 11, and 12. Calculate the maximum posteriori estimate and the true value of the cross-sectional area of unit 10 under different numbers of measuring points and compare them.

As shown in figure 3, when the number of measurement points is small, the error between the maximum a posteriori estimate of the cross-sectional area of unit 10 and the true value is large, at 47.14%. As the number of measurement points increases, the error between the maximum a posteriori estimate of the cross-sectional area of unit 10 and the true value decreases, and the magnitude of change also decreases, showing a convergence trend. This indicates that as the number of measurement points increases, the recognition accuracy improves, and the recognition accuracy exhibits diminishing marginal returns.

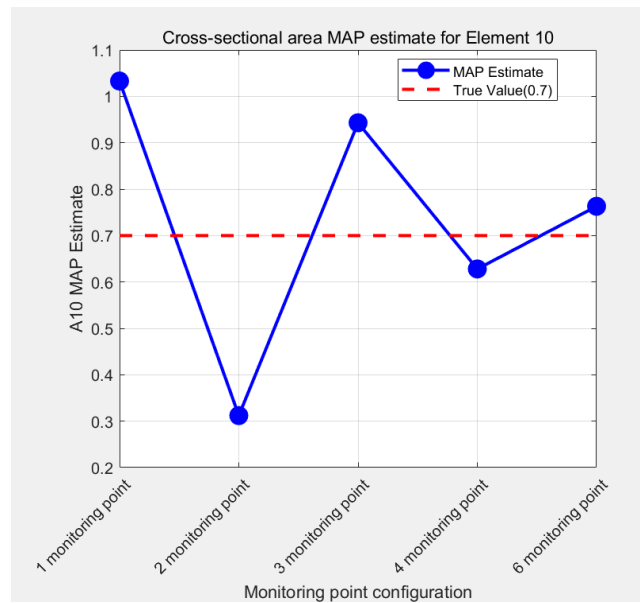


Figure 3. True values and maximum a posteriori estimates of unit 10 under different measurement point conditions. (picture credit : original)

2.3. Impact of Bayesian update method

Under the same structure, error, and measurement points, the simple rejection approximation method and the full Bayesian update based on the Gaussian likelihood function are compared. The simple rejection approximation method selects the acceptance parameter samples by comparing the difference between simulated and observed data, setting a hard threshold of 5%, and performing 10,000 Monte Carlo samplings to calculate the acceptance rate, posterior probability distribution, and maximum a posteriori estimate. The likelihood function method uses the Gaussian likelihood function, with the standard deviation of the measurement error set to 0.01. The continuous probability distribution of the cross-sectional area of cell 10 and the maximum a posteriori estimate are obtained, and the two sets of data are fitted in Matlab, as shown in Figure 4.

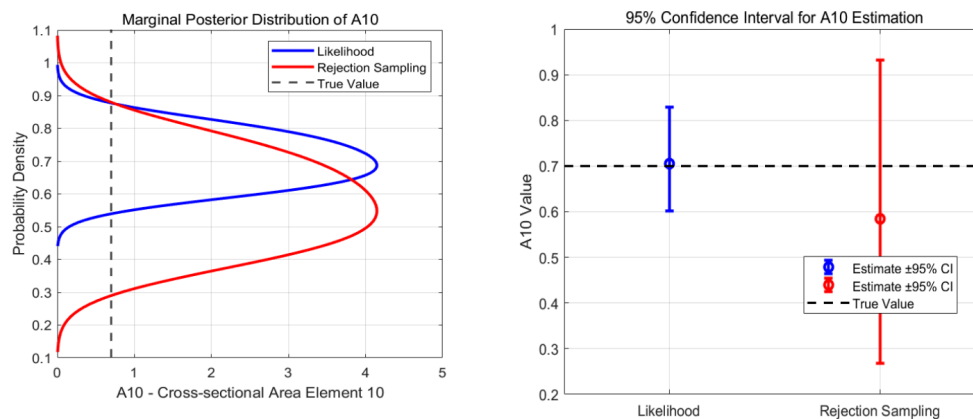


Figure 4. The posterior frequency distribution and 95% confidence interval of unit 10 are calculated using two methods. (picture credit : original)

The simple rejection sampling method has an acceptance rate of 9.95%, a maximum a posteriori estimate of 0.584, and an error of 16.5%. The likelihood function method yields a MAP estimate of 0.708 with an error of 0.7%. Both methods can obtain MAP estimates close to the true value, but the likelihood function method is more accurate. In terms of computational efficiency, the simple rejection approximation method is efficient when the acceptance rate is high, while the likelihood function method calculates all data, making it computationally slow.

3. Analysis and discussion

3.1. Mechanism of influence

The core of Bayesian updating is to update the prior probability using observed data to obtain the posterior distribution. When information quality is low, the constraints provided by the observed data are weak, and the maximum a posteriori estimate is prone to deviating from the true value. The number of measurement points directly determines the amount of information in the observed data. When there are few measurement points, the data is insufficient to uniquely determine all parameters, and it is greatly affected by measurement noise.

The accuracy of the simple rejection sampling method heavily depends on the size of the allowable error. Since it only calculates the accepted data, its information utilization efficiency is low. The likelihood function method can finely evaluate the probability of each parameter value generating observed data, and its accuracy is higher. However, calculating the likelihood value of all sampling points results in a higher computational cost.

3.2. Implications of experimental results for practical engineering applications

The more measurement points there are, the smaller the error between the calculated and actual values, but the marginal benefit diminishes. Therefore, in practical engineering, it is unnecessary to use too many measurement points. Considering the impact on project costs, a balance should be struck between cost and accuracy, and the quality of measurement points should be improved.

When computational resources permit, the likelihood method should be the preferred method for Bayesian updates to obtain more reliable and accurate parameter estimates. Simple discard selection methods are easy to implement and have certain advantages in scenarios requiring rapid initial screening.

4. Conclusion

This paper explores the performance influencing factors of the Bayesian update method in the correction of truss structure models. By selecting a typical truss model and using numerical simulation and the controlled variable method, the influence of various factors on the update effect of the Bayesian structure model is systematically studied, including three important aspects: the quality of the validation information, the number of measurement points and the update method. Through quantitative analysis of the influence of key factors on the Bayesian update process, the mechanisms of the quality and quantity of the observation data are revealed, as well as the basis for selecting the update method in the probabilistic inversion of the structural model. This provides a theoretical foundation and reference for sensor optimization, observation scheme design and the selection of update algorithms in practical engineering.

This study is based on a simple truss model. Future work could be extended to more complex truss structures, or surrogate models could be introduced to explore their impact on computational efficiency and error. It could also explore the use of methods such as Kriging and artificial neural networks to improve the computational efficiency of forward models, and in combination with case studies, consider structural reliability analysis under extreme loads, such as applying a large flood load to the truss to study its displacement and determine structural reliability. This research has some potential value for future research.

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