

Ultralow-Frequency Flexural Wave Control Based on Gradient Metamaterial Beams

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Abstract. Ultralow-frequency (ULF) vibrations (<30 Hz) can travel a long way and are poorly absorbed. They are relatively easy to learn from the foundation, so there is a lack of accuracy in the instrument foundation, degradation of instrument accuracy, and fatigue damage to structural components. Due to the long wavelength and good penetration of these vibrations, they are difficult to control. The first kind of passive control method is a tuned mass damper; The second is to add damping; Both are affected by narrow operating ranges and large added mass. Metamaterials with local resonance have a good bandgap and can be used to address the problem of broadband ultralow-frequency vibration reduction. A beam with a periodic arrangement of inclined resonators is taken as in this paper. Dispersion relations can be obtained by means of Bloch-periodic eigenvalue analysis of finite-element discretisation. The performance of wave attenuation is then checked by calculating the transmission coefficient. Change the length of the inclined beam to achieve a spatial grade, and there are both forward and reverse types. According to the numerical results, both grading schemes have the same bandgap expansion. The first stopband has a widening of 10.53% in terms of absolute bandwidth, and the second is about 126.32% higher than that of a uniform baseline. Therefore, the spatiotemporal modulation of metamaterials is not limited by the narrow bandwidth of conventional resonator-based metamaterials.

Keywords: Ultralow-frequency vibration, Locally resonant metamaterial, Gradient design, Bandgap broadening

1. Introduction

Ultralow-frequency (ULF) vibrations (<30 Hz) can travel long distances and have weak attenuation, thus loosening the foundation of precision equipment, affecting instrument accuracy, etc. The old way of passive control for tuned mass dampers (TMDs) and added damping layers is a narrow band, has a large added mass, or is not effective at low frequencies. Sadek and others have shown in [1] that TMDs are very sensitive to frequency detuning, and Hoang and others have also found that they are not suitable for structures with closely spaced modes [2].

Elastic materials with periodic microstructures have regions that are poor conductors of mechanical waves at a certain frequency. Kushwaha and others [3] were the first to provide a general expression for the acoustic dispersion in periodic elastic media. Liu and others [4] later put forward the idea of locally resonant phononic crystals in 2000 and showed that the bandgap

frequency can be separated from structural periodicity at subwavelength scales. Hussein and others [5] later conducted a general review of the above area. Although there have been some progress, uniformly arranged resonator arrays still have the problem of having narrow-band attenuation windows near the individual resonance frequencies.

Therefore, gradient metamaterials have been developed to widen the bandgap by altering the unit cell in space. Zhu and others [6] first put forward the concept of "rainbow effect" in acoustic metamaterials. Hu and others [7] have expanded the attenuation band by about 172.8% by changing the stiffness of the local resonator in the beam. De Ponti and others [8] have shown that graded elastic metasurfaces can realise rainbow trapping by having different resonator lengths at different places. Wang and others [9] have developed a metamaterial plate based on magnetorheological elastomers, and at 92 Hz, the relative bandwidth is about 79.3%. Colombi and others [10] have put forward a resonant metawedge for earthquakes. Xiao and others [11] have shown that some periodic arrays of resonators can achieve wideband attenuation. Stein and others [12] have shown that increasing the degrees of freedom in resonant units can broaden the band gap. Zhang and others have recently realised that there are more low-frequency wide-band gaps in graded optimisation schemes [13].

A Locally Resonant Metamaterial Beam with Inclined Beam Resonators is Proposed in this paper. The Host Beam is made of aluminium (Al). The inclined support beam is made of polymethyl methacrylate (PMMA), and because it has a small elastic modulus, it can be used to form a "soft spring" that connects the lead (Pb) mass block to the host beam and create a mass-spring type local resonator. Design for forward and reverse gradient is to change the length of the PMMA inclined beam. The Goal of improving the flexural-wave bandgap at very low frequencies is to expand it.

2. Band structure

As shown in Figure 1, the three parts of the unit cell of the proposed metamaterial beam are a substrate beam, an oblique supporting strut and a cuboid inertial mass. Aluminum (Al) is the base material that will carry the weight and runs along the x-axis. The dimension in the y-direction of the out-of-plane direction is 20mm. The cells are arranged at regular intervals along the length, and the particular material properties and forms of these cells can be seen in Table 1.

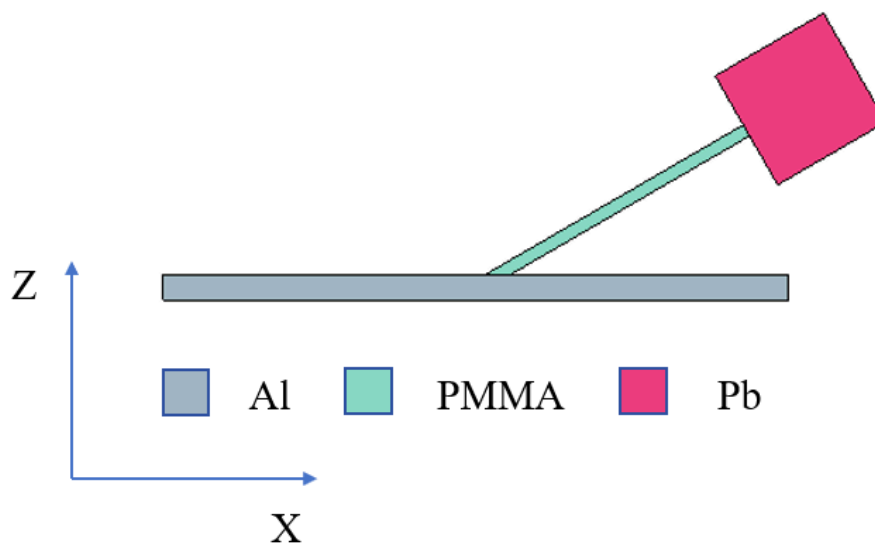


Figure 1. Schematic of the unit cell

Table 1. Material and geometric parameters of the unit cell

Component	Material	Elastic modulus (GPa)	Density (kg/m ³)	Poisson's ratio	Width (mm)	Height (mm)
Host beam	Al	70.0	2700	0.33	25	2
Inclined beam	PMMA	3.1	1165	0.38	35	1
Mass block	Pb	40.8	11600	0.369	10	10

Dispersion characteristics of the periodic cell are obtained by means of finite-element discretization in the Bloch-periodic condition. For a one-dimensional beam structure that is infinitely long in the x-direction, the mechanical field in each constituent cell can be expressed according to the Bloch theorem:

$$u(r + R) = u(r)e^{ik \cdot R} \quad (1)$$

As shown in Eq. (1), $u(r)$ is the displacement field of the structure, r is the position vector within the unit cell, $R = na$ is the lattice translation vector with lattice constant a and integer n , k is the Bloch wave number that describes the phase variation of the propagating wave, i is the imaginary unit, and e is Euler's number. The dot product $k \cdot R$ is the phase change of the wave after propagating one lattice period.

$$u_{right} = u_{left}e^{ik_x a} \quad (2)$$

In Eq. (2), u_{left} and u_{right} denote the displacements at the left and right boundaries of the unit cell, respectively; k_x is the x-component of the Bloch wavenumber; a is the lattice constant; i is the imaginary unit; and e is Euler's number. The term $e^{ik_x a}$ represents the phase shift acquired by the elastic wave as it propagates through one lattice period along the x-direction.

By solving the eigenvalue problem along the Gamma-X path of the first Brillouin zone boundary, i.e., sweeping the normalized wave number $\bar{k}$ from 0 to 1 with $k_x = \frac{\pi \bar{k}}{a_1}$ ($a_1=25$ mm), the eigenfrequencies corresponding to each wavenumber are obtained, and the dispersion relation curves (band structure) are plotted.

At the time of calculation, the host beam, inclined beam and mass block are all divided into two-dimensional plane stress elements. It is assumed that the model is in plane-stress conditions, and its thickness in the out-of-plane direction is 20 mm. The Analysis Type is Eigenfrequency Analysis. The first few eigenfrequencies are obtained during the wave number sweep, and then the band structure diagram is finally obtained.

The band structure calculated is shown in Fig. 2. As shown in the figure, there are obvious bandgaps in the target frequency range. The first bandgap is from point A 29.181Hz to point B 44.397Hz; the second bandgap is from point C 339.99Hz to point D 352.05Hz.

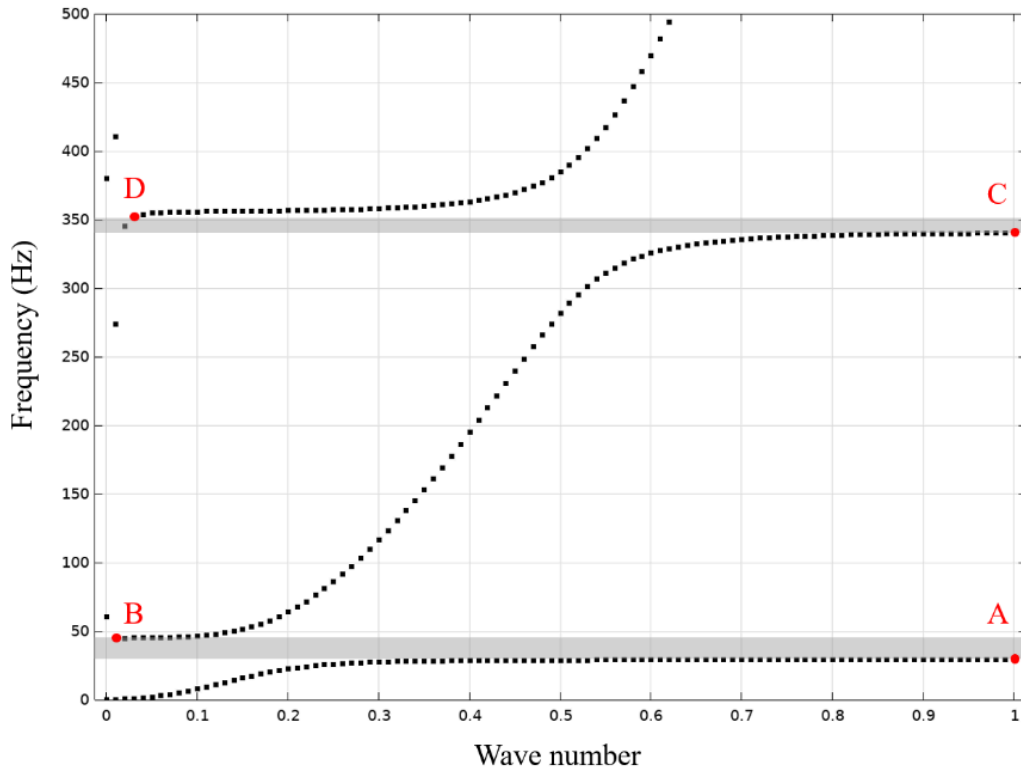


Figure 2. Band structure of unit cell

Fig. 3 is the deformation pattern at the top and bottom edges of the first two flexural stopbands shown in Fig. 2. A relatively large amplitude of localisation in the inclined resonator assemblies indicates that they are local resonators.

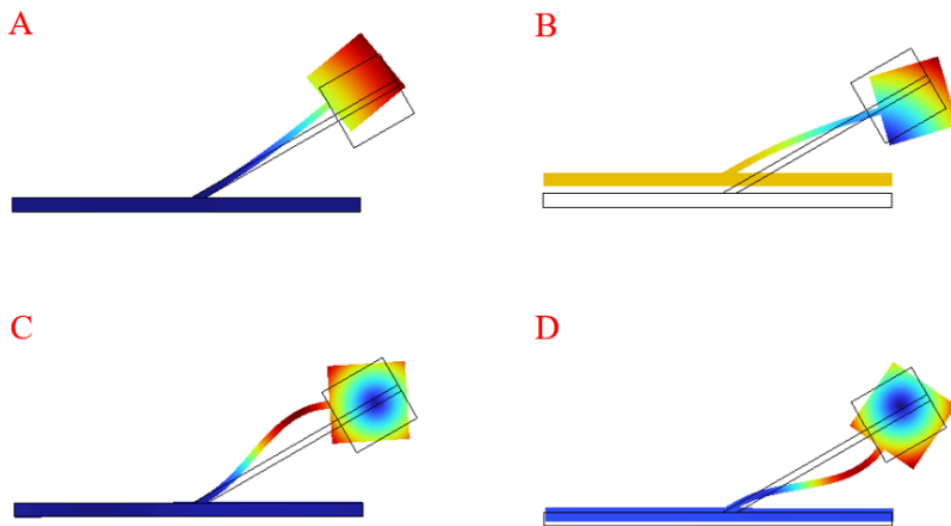


Figure 3. The mode shapes at the first two flexural wave band gap boundaries

To verify the band structure, the finite element method is used to study the propagation of flexural waves in the metamaterial beam. The corresponding simulation is shown in Figure 4.

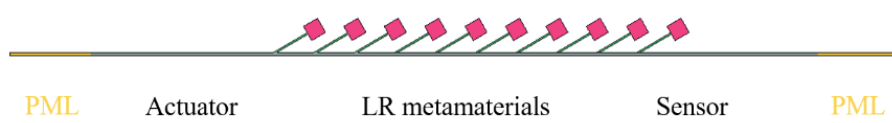


Figure 4. Schematic of the finite element simulation

PML layers have been added to both ends so that the dispersion relation will not be reflected at the boundary. Flexural excitation is added at the left end, then damped and modified by the middle resonant array, and finally collected at the right end. The transmission coefficient is obtained from the square of the ratio of out-of-plane displacement at the output and input terminals, as shown in Figure 5.

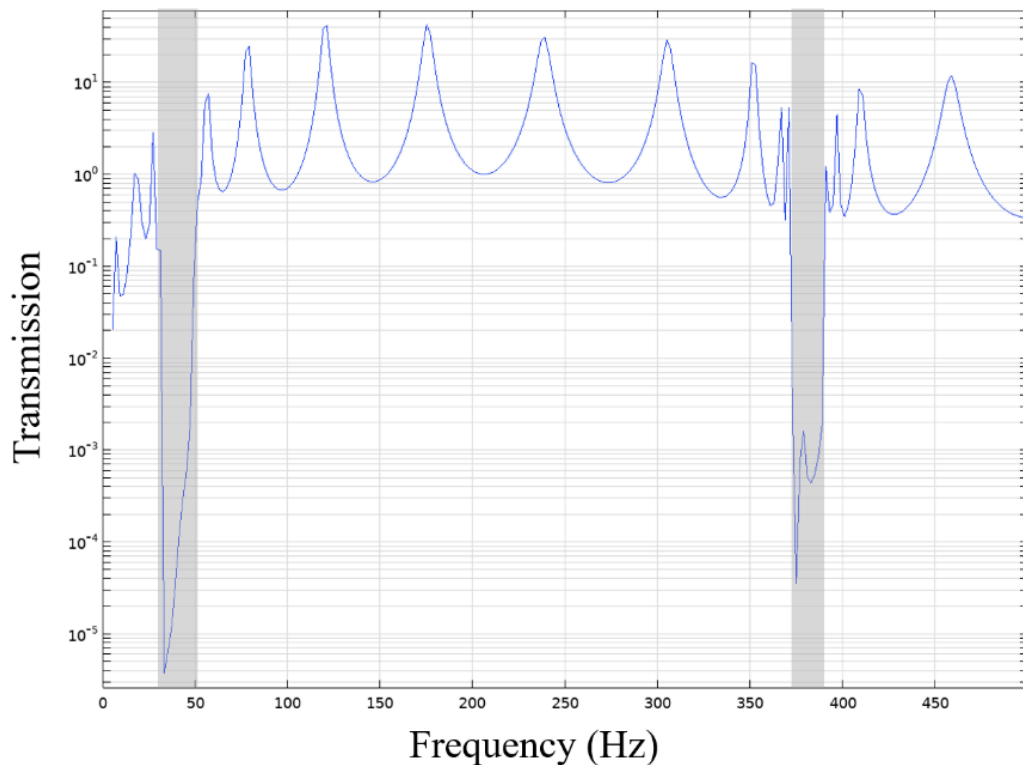


Figure 5. Transmission spectrum of the homogeneous metamaterial beam

Fig. 5 is the transmission spectrum of the homogeneous metamaterial beam under flexural wave excitation. The shaded Areas are the first two stopbands. The first attenuation zone is 31-50 Hz; it has an absolute bandwidth of 19 Hz and a relative bandwidth of 46.91%, so it can suppress broadband noise in the ultralow-frequency region well. The second zone is at 372-391Hz, has the same absolute bandwidth as the first zone, but its relative bandwidth is only about 4.98%. The stopband locations obtained from the transmission calculation are in good agreement with the Bloch-Floquet dispersion predictions, so the calculation method is reliable.

3. Gradient design

To address the small bandgap of the uniform locally resonant metamaterial beam, a gradient Design is used in this paper. Gradient metamaterials can be used to change the shape or material properties of local resonators at different locations in the structure to obtain position-dependent natural

frequencies and merge several discrete narrow-bandgaps into a continuous broadband attenuation band.

The length of the inclined beam is taken as the gradient design variable in this paper. A forward gradient is a positive change of 0.2 mm in beam length along the direction of wave propagation (x-direction), and a reverse gradient is a decrease in the same direction. By changing the length of the inclined beam, it is possible to adjust the stiffness and natural frequency of the local resonator in each unit cell, and thus various cells will resonate sequentially at different frequencies. As a result, several bandgaps are formed and a wideband attenuation region appears. The transmittance is also calculated in the same finite element scheme, and the results are shown in Fig. 6.

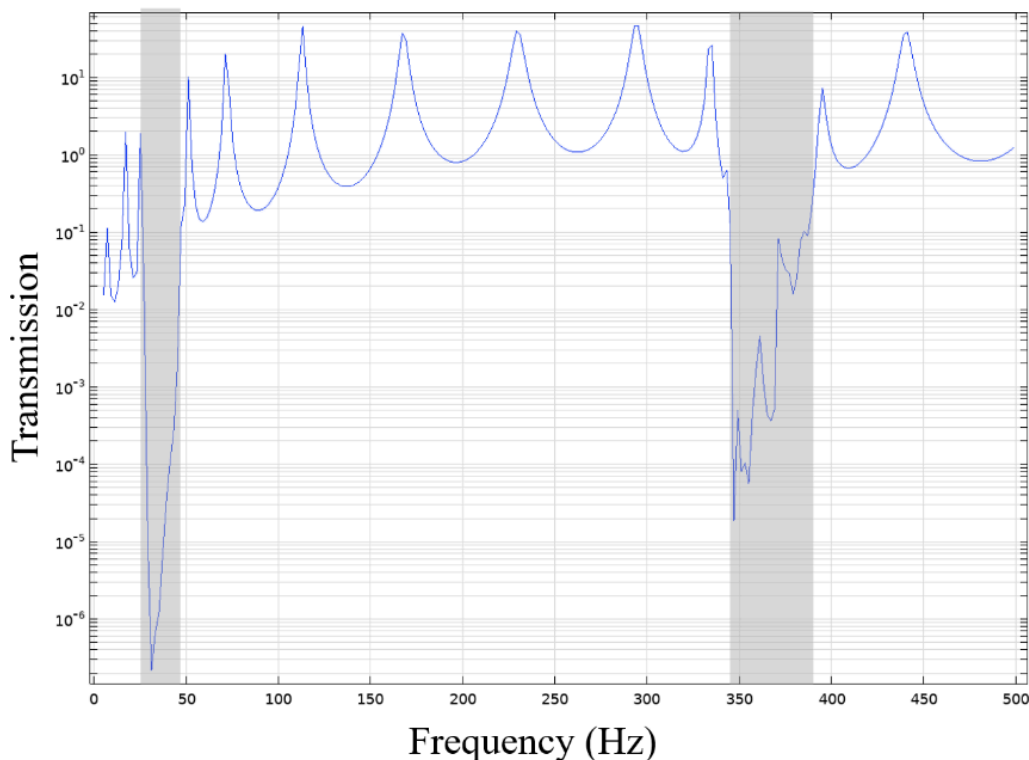


Figure 6. Transmission spectrum of the forward gradient metamaterial beam

A Graded-Architecture beam is somewhat attenuating compared with a uniform one. As shown in Fig. 6, the shaded areas of the forward-graded transmission spectrum are the first two flexural stopbands. The first attenuation zone is from 26-47 Hz, and its absolute bandwidth is 21 Hz; it accounts for about 57.53 per cent of the total. The second region is from 345-388Hz, its absolute width is 43Hz, and the relative width is 11.73%. The forward-graded arrangement has increased the frequency of the first stopband from 19 Hz to 21 Hz by about 10.53%, and the range of the second stopband has been expanded to about 43 Hz (an increase of about 126.32%). The number of low-frequency resonators has risen significantly, and to broaden the original narrow-band characteristic of uniform local resonators, spatial modulation of resonator parameters can be used. Therefore, we can extend the length of the inclined beam to change its position-dependent resonance frequency, and then a small spectral gap will merge with the extended attenuation continuum. It is worth noting that the broadband performance does not have a large mass penalty or structural complexity and is thus light.

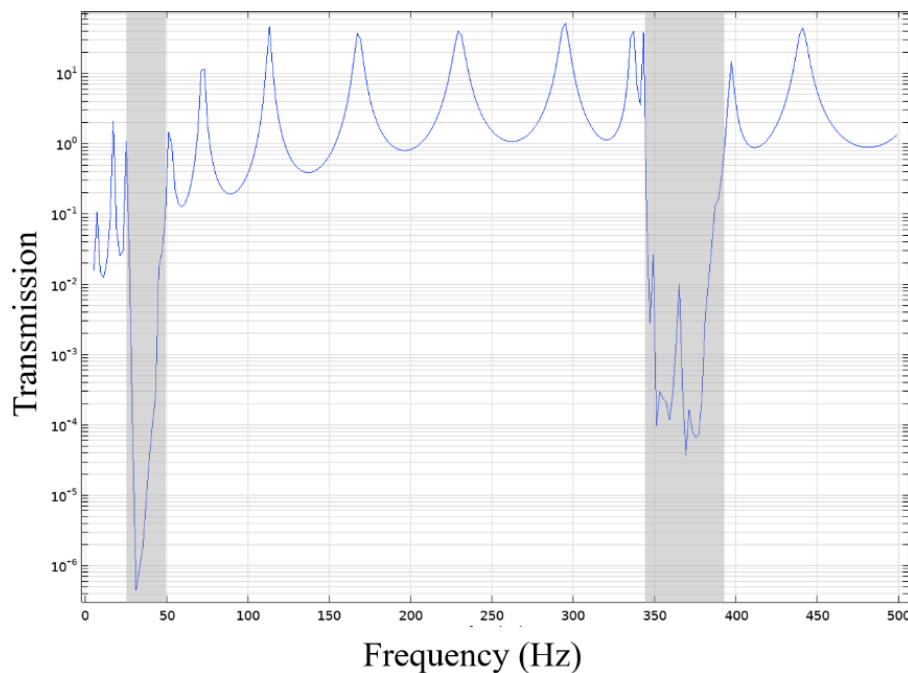


Figure 7. Transmission spectrum of the reverse gradient metamaterial beam

The reverse gradient design is shown in Fig. 7; that is to say, the length of the inclined beam decreases linearly along the wave propagation direction (x-direction) by steps of 0.2 mm, and it has the same symmetry as the forward gradient. As shown in the transmittance curve of Fig. 7, the band gap range of the reverse gradient metamaterial beam is essentially the same as that of the forward gradient design; both the first and second band gaps have highly coincident start/end frequencies and absolute bandwidths. Therefore, it can be concluded that the band gap broadening effect in the proposed inclined-beam gradient metamaterial beam is primarily due to the gradient variation itself, not its direction (whether it is an increase or decrease); that is, the design strategy of gradients has good symmetry and robustness and is suitable for flexible arrangement under different spatial constraints in practical engineering.

4. Conclusion

To sum up, the architecture of the beam is an array of oblique resonators. A single unit cell is an aluminium substrate, an oblique-strut PMMA spring element and a lead (Pb) inertial mass; thus, it can be considered a mass-spring system. Dispersion curves can be obtained by means of Bloch-periodic eigenvalue theory and finite-element discretization. Then, based on the calculation of transmission coefficient, the wave attenuation performance is tested by exciting flexural waves at the input terminal and receiving them at the output terminal via the central resonant array. The results of the two methods are in good agreement. Change the length of the inclined beam in steps of 0.2 mm for forward and reverse gradient design. Both Gradient Designs have the same bandgap broadening. The bandgap of a gradient metamaterial beam is much larger than that of a uniform metamaterial beam. The absolute bandwidth of the first bandgap is increased by 10.53%, and that of the second bandgap is increased by 126.32%. It can solve the problem of narrow bandgaps in conventional locally resonant metamaterials. The reason is that there are different values of effective stiffness and resonance frequency at various places, so many narrow bandgaps merge to form a

continuous broadband attenuation region. A good controller of ultralow-frequency flexural waves over a wide range of frequencies is the Gradient Metamaterial Beam.

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