

Physics-Guided Multi-Source Fusion Geometric Error Compensation and Knowledge Distillation Verification Platform for Large CNC Machine Tools

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Abstract. When verifying geometric-thermal coupling error compensation algorithms for large CNC machine tools on real equipment, there exist problems such as high experimental costs, difficulty in unifying working conditions, and unfair comparison of different compensation strategies. To solve these problems, this paper builds a physics-guided digital verification platform for multi-source fusion geometric error compensation and knowledge distillation. The platform first establishes a physical error model covering 21 geometric errors, Abbe/Bryan coupling effects and component-level dynamic thermal responses, which is used to generate geometric-thermal coupling error data with ground-truth labels under different working conditions. On this basis, multi-source sensor observations of encoders, gyroscopes and laser interferometers are simulated, and adaptive Kalman filtering based on innovation statistics is adopted to realize error state fusion and uncertainty estimation. For the model part, an LSTM teacher network with physical consistency constraints is trained, and then a lightweight student network is obtained through output and feature knowledge distillation. Closed-loop evaluation is carried out under unified execution constraints, combined with uncertainty gating and virtual CNC execution module. Test results show that the distilled student network has an RMSE of 65.851 μm , which is 74.5% lower than that without compensation, and close to 65.140 μm of PID; its P95 error is 120.819 μm ; it has 2182 parameters, about 96.0% less than the physics-guided LSTM. This platform can be used for digital verification and performance reproduction of intelligent compensation algorithms for large CNC machine tools before engineering deployment. The involved methods of multi-source data fusion, deep learning modeling and knowledge distillation lightweighting are also specific applications of data science and intelligent computing in industrial scenarios.

Keywords: large CNC machine tool, digital verification platform, geometric-thermal coupling error, multi-source information fusion, knowledge distillation

1. Introduction

Large CNC machine tools are core equipment for precision machining of large and complex components in aerospace, energy equipment, shipbuilding and other fields. Their spatial positioning accuracy directly determines the machining quality of key components and the service performance of the equipment. As the machine tool travel increases to several meters or even more than ten meters, 21 geometric errors accumulate and amplify along the kinematic chain. Thermal deformations of lead screws, guide rails, columns and other components evolve dynamically with working conditions and couple with geometric errors, and the comprehensive error amplitude can reach hundreds of micrometers. Existing studies have shown that thermally induced errors usually account for 40% to 70% of the total machining errors of machine tools [1, 2]. Compared with the refinement and transformation of mechanical structure, error compensation can significantly improve machining accuracy without changing the machine tool body structure, and is regarded as one of the most economical ways to improve machine tool accuracy in engineering [3, 4]. Therefore, research on geometric-thermal coupling error modeling and intelligent compensation for large CNC machine tools has important theoretical significance and engineering application value.

In terms of geometric error modeling, spatial error modeling and identification of machine tools have formed a relatively systematic research foundation. Sun Guangming et al. [5] systematically reviewed the research progress of spatial error modeling and identification methods for CNC machine tools oriented to error compensation. Ramesh et al. earlier systematically sorted out compensation methods for machine tool geometric errors, cutting force errors and fixture-related errors. Modeling methods based on multi-body system theory and homogeneous transformation matrix can describe the propagation of position, straightness, angle and perpendicularity errors in the kinematic chain [6, 7]. Schwenke et al. summarized the development status of machine tool geometric error measurement and compensation technology. Huang et al. [8] further established a volumetric error model of three-axis machine tools based on Abbe/Bryan principle, revealing the mechanism that angle errors are converted into spatial displacement errors through offset force arms. In terms of thermal errors, Mayr et al. systematically summarized modeling, measurement and compensation methods for thermal problems of machine tools. Li et al. reviewed spindle thermal error compensation technologies. Related research has gradually developed from static temperature mapping to dynamic thermal response modeling and digital twin-driven thermal error compensation [9].

In terms of state estimation and intelligent compensation, adaptive Kalman filtering improves state estimation ability under time-varying working conditions by online adjusting noise statistical characteristics [10, 11]. Recurrent neural networks represented by LSTM [12] are widely used in machine tool thermal error prediction and compensation due to their long sequence memory ability. For example, Zhao et al. [13] realized machine tool thermal error compensation based on optimized LSTM model, and Yuan et al. [14] proposed ITSA-GRU-A model and constructed a cloud-edge-physical collaborative thermal error control framework. Physics-informed machine learning [15, 16] significantly improves generalization ability under data sparsity and distribution shift conditions by embedding physical priors into network training. In terms of model lightweighting, knowledge distillation [17, 18] provides an effective technical path to balance model accuracy and real-time deployment. In terms of verification methods, digital twin technology [19] provides a new paradigm for virtual mapping of equipment operation status and algorithm verification. The DT-CEPA method proposed by Ji et al. [20] uses digital twin to realize machine tool contour error prediction.

However, there is still an obvious gap between the theoretical design of compensation algorithms and real machine deployment. First, multi-condition experiments on real large machine tools have high cost and long cycle, and it is difficult to obtain batch error data with ground-truth labels. Second, most existing studies focus on a single link of error modeling, state estimation or compensation prediction [20], and lack a unified verification environment covering the complete closed loop of "error generation – multi-source observation – state fusion – intelligent compensation – virtual execution". It is difficult to fairly compare different algorithms under consistent sensing conditions and execution constraints. Third, there is a lack of systematic pre-deployment quantitative evaluation means for the contradiction between the parameter scale of deep compensation models and the millisecond-level real-time constraints of numerical control systems.

To this end, this paper constructs a physics-guided digital verification platform for intelligent compensation algorithms of large CNC machine tools. The main contributions include: (1) A full-link digital closed-loop verification platform covering error modeling, multi-source observation, adaptive fusion, physical constraint learning, knowledge distillation and virtual CNC execution is proposed and implemented. It can generate geometric-thermal coupling error data with ground-truth labels, and provide a unified and reproducible closed-loop evaluation environment for different compensation algorithms. (2) A compensation framework combining physical consistency constrained LSTM teacher network and innovation statistics-based adaptive Kalman fusion is proposed, which reduces the compensation RMSE under complex error field from 115.911 μm of ordinary LSTM to 67.767 μm . (3) Through output and feature joint knowledge distillation and uncertainty gating, a lightweight student network with only 2182 parameters is obtained. Under the condition that the closed-loop accuracy is equivalent to PID, the model scale is compressed by about 96.0%, taking into account both compensation accuracy and real-time deployment requirements.

2. Overall system architecture

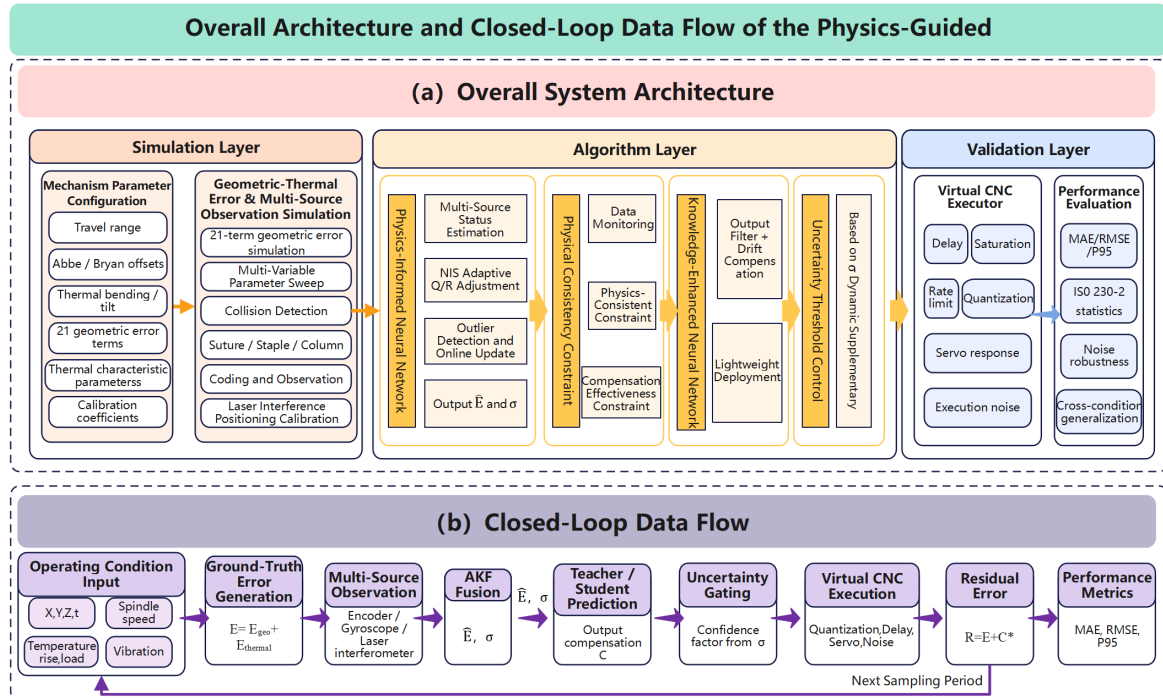


Figure 1. Overall architecture of the full-link digital verification platform for physics-guided intelligent compensation

The overall system architecture is shown in Figure 1, including physical parameter configuration, error and multi-source observation simulation, adaptive Kalman fusion, teacher-student distillation, virtual CNC execution and performance evaluation. The fusion results and posterior uncertainty are input into the compensation network, and the network output enters the virtual actuator with unified consideration of delay, amplitude limiting, change rate, quantization, servo response and execution noise after gating, and finally obtains the closed-loop residual error.

3. Parametric modeling of geometric-thermal coupling errors

In this paper, μm is uniformly used for error and compensation, mm for geometric dimension, rad for angle error, $^{\circ}\text{C}$ for temperature, and min for time.

3.1. Parameterization of 21 geometric errors and abbe/bryan linearization

For digital verification, a parametric method is used to construct a reproducible spatial error field: 18 rigid body geometric errors are generated by superposition of double harmonic functions and random disturbances, and 3 inter-axis perpendicularity errors are added to form a set of 21 geometric errors. The angle error produces additional linear displacement through the offset force arm, and its first-order linearization relationship is:

$$\Delta \mathbf{r}_A = \boldsymbol{\varepsilon} \times \mathbf{p} \quad (1)$$

where $\boldsymbol{\varepsilon}$ is the angle error vector, $\mathbf{p}$ is the offset vector of the measurement point relative to the rotation center, and $\Delta \mathbf{r}_A$ is the additional linear displacement error caused by the angle error. The errors of each axis propagate along the kinematic chain through the homogeneous transformation matrix, and the three-dimensional spatial error is obtained by difference with the ideal motion matrix.

3.2. Component-level dynamic thermal error channel modeling

Considering that lead screws, guide rails and columns have different thermal response speeds, first-order thermal inertia models are established respectively:

$$\Delta T_i(t) = g_i(1 - e^{-t/\tau_i}), \quad i \in \{\text{screw, rail, column}\} \quad (2)$$

where $\Delta T_i(t)$ is the temperature rise response of the i -th component, g_i is the thermal gain coefficient, and τ_i is the thermal time constant. The thermal positioning error of the lead screw is calculated by $\delta_{\text{thermal}} = \alpha l \Delta T$. The temperature rise of the guide rail is used to describe the change of thermal straightness, and the asymmetric temperature rise of the column is used to describe the thermal angle error. The three types of thermal errors evolve respectively to simulate cold start, continuous machining and thermal stability stages.

3.3. Simulation superposition of geometric-thermal errors

The platform is mainly used to compare compensation strategies, and first-order superposition is adopted under small angle and small displacement conditions:

$$E(t) = E_{\text{geo}} + E_{\text{thermal}}(t) \quad (3)$$

In the simulation, the travels of X, Y and Z axes are 5000, 3000 and 2000 mm respectively, and the thermal time constants of lead screw, guide rail and column are 8, 25 and 50 min respectively. The above parameters are used to construct a representative digital verification environment, so the results of this paper are mainly used for relative performance evaluation among different compensation algorithms.

4. Multi-source fusion and physics-guided knowledge distillation intelligent compensation

4.1. Multi-source observation simulation and adaptive Kalman fusion

The noise levels of various sensors are set with reference to the nominal accuracy ranges of common industrial encoders, gyroscopes and laser interferometers (for example, the measurement uncertainty of laser interferometer is usually $0.1 \sim 1 \mu\text{m}$), and provide parameter reference for subsequent real machine tool data access. Three types of sensor observations are uniformly expressed as:

$$z_i = E + b_i + w_i \quad (4)$$

where z_i is the observation value of the i -th type of sensor, E is the real error state, b_i is the sensor bias drift, and w_i is the random measurement noise. Aiming at the problem that fixed noise covariance is difficult to adapt to temperature, vibration and load changes, the normalized innovation statistic is used to online adjust the noise covariance, and the correction is smoothed by forgetting factor. The state update is:

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k(z_k - H\hat{x}_{k|k-1}) \quad (5)$$

where $\hat{x}_k$ is the fused error state estimate, $\hat{x}_{k|k-1}$ is the predicted state, K_k is the Kalman gain, z_k is the multi-source sensor observation value, and H is the observation matrix. The filter outputs three-axis fusion error and posterior standard deviation, which are used for intelligent compensation prediction and uncertainty gating respectively.

4.2. Physics-guided LSTM teacher network

The teacher network adopts LSTM structure, which takes position, time, temperature, spindle speed, load, vibration, fusion error and historical state as sequential inputs, and predicts compensation amount and residual error after compensation at the same time. To avoid the network relying only on data fitting, physical consistency constraints are added in addition to the supervised loss:

$$\mathbf{L} = \mathbf{L}_{\text{data}} + \lambda_{\text{phy}} \mathbf{L}_{\text{phy}} \quad (6)$$

$$\mathbf{L}_{\text{phy}} = ||\mathbf{E} + \mathbf{C} - \mathbf{R}||^2 \quad (7)$$

where $\mathbf{L}_{\text{data}}$ is the network prediction error, λ_{phy} is the physical constraint weight, and $\mathbf{E}$, $\mathbf{C}$, $\mathbf{R}$ are error state, compensation amount and residual error respectively. At the same time, penalty is imposed on the situation that the error increases after compensation, so as to improve the stability of model output.

4.3. Knowledge distillation student network and uncertainty adjustment

Considering the large scale of the teacher network, a lightweight student network is further constructed. During training, joint distillation is carried out by using real compensation labels, teacher outputs and teacher intermediate features at the same time:

$$\mathbf{C}_s = (1 - \lambda_{\text{kd}}) \mathbf{C}_{\text{hard}} + \lambda_{\text{kd}} \mathbf{C}_t \quad (8)$$

where $\mathbf{C}_s$ is the training target of the student network, $\mathbf{C}_{\text{hard}}$ is the real compensation label generated based on control constraints, $\mathbf{C}_t$ is the prediction result of the teacher network, and λ_{kd} is the distillation weight. The posterior standard deviation of Kalman filter is further used to adjust the compensation output: when the estimated uncertainty increases, the compensation intensity is reduced, and a dead zone is set to reduce execution jitter caused by tiny errors.

5. Closed-loop verification results and discussion

5.1. Compensation labels and virtual CNC execution

Compensation labels comprehensively consider control cost, amplitude, change rate and command resolution constraints. The outputs of all methods uniformly go through amplitude limiting, change rate limiting, quantization, single-cycle delay, first-order servo response and execution noise to ensure fair comparison.

5.2. Comparison of multiple methods under unified constraints

Under the same test set and virtual CNC execution constraints, no compensation, PID, LSTM, GRU, physics-guided LSTM and distilled student network are compared uniformly. The results are shown in Table 1.

Table 1. Comparison of closed-loop performance of different compensation methods

Method	RMSE/ μm	P95/ μm	Number of parameters
No compensation	258.279	422.106	—
PID	65.140	121.528	—

Table 1. (continued)

LSTM	115.911	226.813	6342
GRU	116.107	227.935	4806
Physics-guided LSTM	67.767	125.410	54150
Distilled student network	65.851	120.819	2182

The results show that the RMSE after compensation by ordinary LSTM and GRU still exceeds 115 μm , while it drops to 67.767 μm after adding physical constraints. The RMSE of the distilled student network reaches 65.851 μm , which is about 74.5% lower than that without compensation, close to 65.140 μm of PID, and the P95 error is slightly lower than that of PID. The number of parameters of the student network is reduced from 54150 of physics-guided LSTM to 2182, a reduction of about 96.0%, and the single sample inference time is about 0.0038 ms, indicating that knowledge distillation can significantly reduce model deployment cost while maintaining high compensation accuracy.

5.3. Analysis of positioning accuracy and closed-loop residual error before and after compensation

In terms of positioning accuracy, the RMSE of three-axis comprehensive positioning error without compensation is 258.279 μm , and P95 reaches 422.106 μm . After enabling the closed-loop compensation of distilled student network, the RMSE drops to 65.851 μm and P95 drops to 120.819 μm , with improvements of 74.5% and 71.4% respectively. The significant decrease of P95 statistic shows that compensation not only reduces the average error level, but also compresses the tail extreme positioning deviation to about 29% of the uncompensated condition, which is particularly important for suppressing occasional large errors that most affect accuracy consistency in large-stroke machining.

From the time dimension, the evolution of closed-loop residual error is closely related to the thermal state of the machine tool (see Table 2 in Section 5.5 for details): in the cold start stage (0~20 min), the residual RMSE is 180.250 μm , with a decrease of only 17.6%. The reason is that in this stage, the thermal transient gradient is large, the adaptive Kalman filter is still in the convergence process, and the high posterior uncertainty makes the gating mechanism actively reduce the compensation intensity to avoid miscompensation. After entering the continuous heating stage, the decrease rate increases to 70.6%. In the thermal stability stage (after 60 min), the residual RMSE further drops to 45.964 μm , with a decrease of 82.2%. The closed-loop residual error presents an evolution trend of "large at first, then small, and gradually converging", which is consistent with the filter convergence process and the thermal state stabilization process.

From the composition of residual error, the residual error after thermal stability mainly comes from the components that are difficult to be eliminated by compensation algorithms, such as command quantization, single-cycle delay and first-order servo lag in the execution link, as well as the sensing noise floor. The closed-loop RMSE of distilled student network (65.851 μm) is very close to that of PID (65.140 μm), which also indicates that under the current virtual CNC execution constraints, both methods have approached the compensable limit determined by the execution conditions. Further accuracy improvement depends on the improvement of resolution and response characteristics of the execution link.

5.4. Parameter sensitivity and module ablation

Parameter sensitivity analysis shows that when λ_{kd} changes in the range of $0.1 \sim 1.0$, the RMSE is $65.590 \sim 66.771 \mu\text{m}$; when λ_{phy} changes in the range of $0 \sim 0.2$, the RMSE is $65.539 \sim 65.826 \mu\text{m}$, indicating that the model is generally stable to parameter changes. When the uncertainty thresholds are 0.4, 0.8 and $1.2 \mu\text{m}$, the RMSE is 67.089, 65.851 and $65.851 \mu\text{m}$ respectively, indicating that too low threshold may produce too strong gating, and the performance is basically stable after $0.8 \mu\text{m}$. Ablation experiments show that the RMSE of Baseline LSTM is $121.011 \mu\text{m}$, which drops to $115.911 \mu\text{m}$ after introducing AKF; it further drops significantly to $67.767 \mu\text{m}$ after adding physical constraints, and is $65.851 \mu\text{m}$ after adding knowledge distillation. The results show that physical consistency constraint contributes the most to reducing errors, and knowledge distillation greatly compresses the model scale while maintaining compensation performance.

5.5. Compensation performance at different thermal stages

To avoid only using global error indicators, the test process is divided into three stages according to thermal state: cold start $0 \sim 20$ min, continuous heating $20 \sim 60$ min and thermal stability after 60 min. The results are shown in Table 2.

Table 2. Compensation performance at different thermal stages

Thermal stage	RMSE before compensation/ μm	RMSE after compensation/ μm	Decrease rate/%
Cold start	218.831	180.250	17.6
Continuous heating	267.375	78.622	70.6
Thermal stability	258.787	45.964	82.2

5.6. Discussion

Compared with existing studies, the differences and contributions of this work are mainly reflected in the following points.

First, the verification paradigm is different. The thermal error studies of Zhao et al. based on optimized LSTM and Yuan et al. based on ITSA-GRU-A model mainly focus on the error prediction link, and the accuracy evaluation mostly relies on the open-loop prediction results on specific experimental benches. Although the DT-CEPA proposed by Ji et al. introduces digital twin for contour error prediction, it does not consider compensation execution constraints and model lightweighting. This paper incorporates geometric-thermal error generation, adaptive state estimation, physical constraint learning, knowledge distillation and virtual CNC execution into the same closed-loop digital verification environment, and evaluates the closed-loop residual error after "prediction + gating + execution" rather than open-loop prediction accuracy. This setting is closer to engineering deployment conditions, and also enables quantitative comparison of different compensation strategies under completely consistent sensing and execution constraints.

Second, the difference in modeling methods. In the experiment, the closed-loop RMSE of purely data-driven LSTM and GRU both exceed $115 \mu\text{m}$, and drop to $67.767 \mu\text{m}$ after adding physical consistency constraints, with a decrease of about 41.5%. This result shows that in the geometric-

thermal strong coupling error field, physical constraints can suppress the overfitting and wrong compensation direction of purely data-driven models, which is consistent with the judgment in physics-informed machine learning research that physical priors help improve generalization ability under data sparsity and distribution shift conditions.

Third, in terms of deployment feasibility. Knowledge distillation compresses the number of model parameters from 54150 to 2182, with a compression rate of about 96.0%, and the single sample inference time is about 0.0038 ms, which is significantly less than the millisecond-level interpolation cycle of numerical control system, indicating that the distilled model meets the time requirements of industrial real-time control. Compared with PID, the student network has equivalent closed-loop accuracy and slightly better P95 index. PID needs independent tuning per axis and lacks explicit modeling ability for multi-axis coupling and non-stationary thermal drift. Learning-based compensators can directly use multi-source fusion state and uncertainty information, and have better scalability in working condition expansion and multivariable coupling scenarios.

It should be noted that this paper still uses parametric simulation data for verification, which cannot replace real large CNC machine tool experiments. Although the platform considers geometric errors, dynamic thermal responses, multi-source sensor uncertainty and CNC execution constraints, the cutting loads, structural deformations and environmental disturbances in actual machining are more complex. Therefore, the results of this paper are mainly used for algorithm comparison and pre-deployment verification under unified conditions. In the follow-up, real machine tool multi-source sensing data will be introduced for parameter calibration and real machine compensation experiments.

6. Conclusion

Aiming at the problems of high cost, long cycle and difficulty in unified comparison of different methods for real machine verification of intelligent compensation algorithms for large CNC machine tools, this paper constructs a physics-guided digital verification platform for multi-source fusion geometric error compensation and knowledge distillation. The platform integrates 21 geometric error and dynamic thermal error modeling, multi-source observation, adaptive Kalman fusion, physics-guided teacher network, knowledge distillation and virtual CNC execution, covering the complete process from error generation, state estimation to closed-loop compensation evaluation.

Experimental results show that the distilled student network has an RMSE of 65.851 μm , which is 74.5% lower than that without compensation, and close to 65.140 μm of PID; the P95 error is 120.819 μm , and the number of parameters is 2182, which is about 96.0% less than that of physics-guided LSTM. The analysis of positioning accuracy and closed-loop residual error before and after compensation shows that the tail extreme error is compressed to about 29% of the uncompensated condition, the residual error gradually converges with filter convergence and thermal state stabilization, and the RMSE in thermal stability stage decreases by 82.2%. The above results show that under the constructed digital environment and unified execution constraints, the platform can quantitatively compare different compensation algorithms, and take into account both compensation accuracy and model lightweighting requirements.

References

- [1] Mayr, J., Jedrzejewski, J., Uhlmann, E., et al. (2012). Thermal issues in machine tools. *CIRP Annals*, 61(2), 771-791. <https://doi.org/10.1016/j.cirp.2012.05.008>
- [2] Li, Y., Zhao, W., Lan, S., et al. (2015). A review on spindle thermal error compensation in machine tools. *International Journal of Machine Tools and Manufacture*, 95, 20-38.

<https://doi.org/10.1016/j.ijmachtools.2015.04.008>

- [3] Ramesh, R., Mannan, M. A., & Poo, A. N. (2000). Error compensation in machine tools — a review: Part I: geometric, cutting-force induced and fixture-dependent errors. *International Journal of Machine Tools and Manufacture*, 40(9), 1235-1256. [https://doi.org/10.1016/S0890-6955\(00\)00009-2](https://doi.org/10.1016/S0890-6955(00)00009-2)
- [4] Schwenke, H., Knapp, W., Haitjema, H., et al. (2008). Geometric error measurement and compensation of machines — an update. *CIRP Annals*, 57(2), 660-675. <https://doi.org/10.1016/j.cirp.2008.09.008>
- [5] Sun, G., Han, B., Zhang, D., et al. (2025). Research progress on spatial error modeling and identification methods for CNC machine tools oriented to error compensation. *Journal of Mechanical Engineering*, 61(19), 202-228. <https://doi.org/10.3901/JME.2025.19.202>
- [6] Huang, M., Niu, X., Niu, J., et al. (2024). Geometric error analysis and modeling of three-axis CNC machine tools. *Journal of Tianjin University of Technology*, 40(1), 22-29.
- [7] Zhang, C. (2022). *Research on on-line measurement and active compensation technology of geometric error for large-scale machine tools* [Doctoral dissertation]. Tianjin University.
- [8] Huang, Y. B., Fan, K. C., Lou, Z. F., et al. (2020). A novel modeling of volumetric errors of three-axis machine tools based on Abbe and Bryan principles. *International Journal of Machine Tools and Manufacture*, 151, 103527. <https://doi.org/10.1016/j.ijmachtools.2020.103527>
- [9] Ma, C., Li, M., Liu, J., et al. (2025). Application research of thermal error compensation in digital twin system. *Journal of Harbin University of Science and Technology*, 30(1), 129-137. <https://doi.org/10.15938/j.jhust.2025.01.013>
- [10] Mohamed, A. H., & Schwarz, K. P. (1999). Adaptive Kalman filtering for INS/GPS. *Journal of Geodesy*, 73(4), 193-203. <https://doi.org/10.1007/s001900050236>
- [11] Limaverde Filho, J. O. A., Fortaleza, E. L. F., & Silva, J. G., et al. (2022). Adaptive Kalman filtering for closed-loop systems based on the observation vector covariance. *International Journal of Control*, 95(7), 1731-1746.
- [12] Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. *Neural Computation*, 9(8), 1735-1780. <https://doi.org/10.1162/neco.1997.9.8.1735>
- [13] Zhao, X. R., Hu, Z. W., Tang, J., et al. (2025). Research on machine tool thermal error compensation based on an optimized LSTM model. *Actuators*, 14(12), 567. <https://doi.org/10.3390/act14120567>
- [14] Yuan, Q., Ma, C., Liu, J., et al. (2022). Correlation analysis-based thermal error control with ITSA-GRU-A model and cloud-edge-physical collaboration framework. *Advanced Engineering Informatics*, 54, 101759. <https://doi.org/10.1016/j.aei.2022.101759>
- [15] Raissi, M., Perdikaris, P., & Karniadakis, G. E. (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378, 686-707. <https://doi.org/10.1016/j.jcp.2018.10.045>
- [16] Cuomo, S., Schiano Di Cola, V., Giampaolo, F., et al. (2022). Scientific machine learning through physics-informed neural networks: Where we are and what's next. *Journal of Scientific Computing*, 92, 88. <https://doi.org/10.1007/s10915-022-01939-z>
- [17] Hinton, G., Vinyals, O., & Dean, J. (2015). Distilling the knowledge in a neural network [Preprint]. *arXiv*. <https://doi.org/10.48550/arXiv.1503.02531>
- [18] Gou, J., Yu, B., Maybank, S. J., et al. (2021). Knowledge distillation: A survey. *International Journal of Computer Vision*, 129(6), 1789-1819. <https://doi.org/10.1007/s11263-021-01453-z>
- [19] Tao, F., Zhang, H., Liu, A., et al. (2019). Digital twin in industry: State-of-the-art. *IEEE Transactions on Industrial Informatics*, 15(4), 2405-2415. <https://doi.org/10.1109/TII.2018.2873186>
- [20] Ji, S., Ni, H., Hu, T., et al. (2024). DT-CEPA: A digital twin-driven contour error prediction approach for machine tools based on hybrid modeling and sparse time series. *Robotics and Computer-Integrated Manufacturing*, 88, 102738. <https://doi.org/10.1016/j.rcim.2024.102738>