

Terramechanics-Based Traversability of Mobile Robots on Complex Terrain

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Abstract. The traversability of wheeled and tracked mobile robots on deformable terrains such as soft sand, muddy ground, and gravel depends on the mechanical interaction between the wheel/track and the ground. Over the past six decades, four modeling paradigms have been developed to address the traversability of mobile robots: the semi-empirical, continuum mechanics, granular mechanics, and data-driven paradigms. This paper systematically reviews the applications of these four paradigms to wheeled robots, tracked robots, and planetary rovers, and compares them in terms of physical fidelity, calibration convenience, computational efficiency, and prediction capability, thereby clarifying the applicability and limitations of each paradigm at different research stages. In view of the lack of a common benchmark, the insufficient modeling of extreme terrains, and the gap in multi-robot formation terramechanics, this paper proposes future research directions, including the construction of benchmark datasets, the coupling of constitutive models for extreme media with the wheel boundary, and the terramechanics modeling of repeated passages in robot formations.

Keywords: terramechanics, mobile robots, traversability on complex terrain, modeling paradigms

1. Introduction

Mobile robots are moving from structured factory floors to unstructured complex terrains such as planetary surfaces, post-earthquake ruins, muddy battlefields, and polar ice sheets, which poses severe challenges to their autonomous traversability. According to their locomotion structure, mobile robots can be divided into legged, wheeled, and tracked types. Unlike legged robots that rely on discrete footholds, wheeled and tracked robots acquire traction through continuous compaction and shearing between their ground-contact components and the soil; the two therefore share strong commonalities in the terramechanics framework. Consequently, for wheeled and tracked robots, which dominate deep-space exploration and field operations, traversability on complex terrain is essentially a continuous wheel/track–soil interaction mechanics problem, and the accuracy of the mechanical description of this interaction directly determines the upper bound of traversability prediction.

Considerable progress has been made in the traversability of wheeled/tracked mobile robots on various complex terrains. Based on the differences in their underlying mechanical methods, existing

studies can be divided into four modeling paradigms, namely the semi-empirical, continuum mechanics, granular mechanics, and data-driven paradigms. However, researchers in this field currently adopt modeling routes based on different physical ideas, and these paradigms have not yet been systematically compared and evaluated against one another. There is therefore a need for a review that systematically organizes the terramechanics research on mobile robots over complex terrain from a mechanical perspective.

This paper reviews and compares the existing literature on each paradigm, identifies the shortcomings of each paradigm, and points out the gaps in the current research field. Finally, it proposes several prospects for future research, with the aim of providing references for subsequent terramechanics-based traversability studies of wheeled/tracked mobile robots.

2. Research status of the four paradigms

2.1. Research based on the semi-empirical terramechanics paradigm

The theoretical cornerstone of semi-empirical terramechanics is the pressure–sinkage equation proposed by Bekker in 1956, which was established on the basis of plate penetration tests and takes the following basic form [1]:

$$p = \left(\frac{k_c}{b} + k_\varphi \right) z^n \quad (1)$$

where p is the ground pressure, b is the smaller dimension of the plate (or wheel rim), z is the sinkage, k_c is the cohesive modulus of soil deformation, k_φ is the frictional modulus of soil deformation, and n is the sinkage exponent. These parameters are collectively called the Bekker parameters and must be measured soil by soil using a bevameter. The parameters k_c and k_φ represent the contributions of the cohesive and frictional components of the soil to the bearing capacity, respectively, while n controls the shape of the pressure–sinkage curve.

In a 1967 paper, Wong and Reece first applied Bekker's plate sinkage formula in its complete form to the wheel rolling problem; this framework was later called the Bekker–Wong model [2]. At an arbitrary angle θ along the contact arc, the normal stress is obtained by projecting the Bekker equation through geometric relations:

$$\sigma(\theta) = \left(\frac{k_c}{b} + k_\varphi \right) [r(\cos\theta - \cos\theta_f)]^n \quad (2)$$

where r is the wheel radius and θ_f is the entry angle of the contact arc. The tangential stress $\tau(\theta)$ is provided by the Janosi–Hanamoto shear model mentioned later in the text [3]. Integrating the normal and tangential stresses along the contact arc yields the vehicle-level parameters such as the vertical load, traction force, and driving torque of the wheel [2].

In 1961, Janosi and Hanamoto proposed the relationship describing the development of soil shear stress with shear displacement [3]:

$$\tau = (c + \sigma \tan\varphi) [1 - e^{-\frac{j}{K}}] \quad (3)$$

where j is the shear displacement, c and φ are the cohesion and internal friction angle of the soil, respectively, and K is the shear deformation modulus.

The Bekker–Wong model and the Janosi–Hanamoto shear equation enable the semi-empirical paradigm to compute both the sinkage and the traction force simultaneously using only a set of soil parameters measured by a bevameter and the wheel geometry, without the need for full-vehicle tests.

The Bekker–Wong semi-empirical framework can solve many engineering problems in robot design practice. For wheeled robots, Jia et al. applied the Bekker–Wong model to the traction estimation and control of off-road wheeled robots, validated on platforms in the 50–150 kg range; their results showed that when the ratio of the wheel diameter to the mean soil particle diameter exceeds 30, the continuum assumption essentially holds and the traction prediction accuracy of the Bekker model is within 15–20% [4]. However, the traditional Bekker model does not consider the slip-sinkage effect, and its prediction error increases significantly under high-slip conditions. Ding et al. pointed out that the Bekker model regards the sinkage and compaction resistance under a given load as constants, whereas experiments show that both increase markedly with the slip ratio, and the actual compaction resistance can reach twice the value predicted by the traditional method [5].

For tracked robots, Wong derived the shear stress–displacement relationship beneath the track plate, which, together with the Janosi–Hanamoto equation, constitutes the theoretical model for track traction prediction. In their review, Bai et al. categorized the terramechanics research routes of tracked robots into four types—purely empirical, semi-empirical, model tests, and numerical analysis—and pointed out that the semi-empirical route achieves the best compromise between engineering operability and prediction accuracy [6].

For planetary rovers, the Bekker–Wong framework has been refined to be applicable to steering and slope scenarios. Some studies based on the semi-empirical framework have considered the influence of lateral forces on wheel–soil sinkage; for robot traversability on high-risk terrain such as craters and slopes, this modification improves the credibility of traversability prediction.

However, the semi-empirical paradigm seeks to approximate engineering accuracy with a minimum number of parameters, which causes it to be notably distorted under some unconventional conditions (e. g., high slip ratios). With the maturation of soil constitutive theory over recent decades, new theoretical methods such as the continuum mechanics paradigm attempt to describe wheel–soil interaction from the physical mechanisms of soil deformation and failure, providing a physically more realistic framework for describing wheel–soil interaction [7, 8].

2.2. Research based on the continuum mechanics paradigm

The theoretical foundation of the continuum mechanics paradigm is composed by the Mohr–Coulomb criterion, the Drucker–Prager criterion, and the Cam-clay critical state theory. As related research has advanced, these criteria have been gradually refined and successfully applied.

The Mohr–Coulomb yield criterion is the most widely used soil failure criterion, and its expression is:

$$|\tau| = c + \sigma_n \tan \varphi \quad (4)$$

where τ is the limiting shear stress, σ_n is the normal stress on the failure plane, c is the cohesion, and φ is the internal friction angle. This formula is still widely used in various textbooks on soil mechanics and rock mechanics.

The Drucker–Prager criterion can be regarded as a more refined version of the Mohr–Coulomb criterion, appearing as a cone in stress space, and its core formula is:

$$\sqrt{J_2} = A + BI_1 \quad (5)$$

where J_2 is the second invariant of the deviatoric stress, I_1 is the first invariant of the stress, and A and B are material parameters. In their comparative review, Zeng et al. pointed out that the Drucker–Prager model is more widely used than Mohr–Coulomb in wheel–soil interaction simulations [8].

Cam-clay and its practical version, Modified Cam-clay, constitute the most systematic framework in soil constitutive theory to date [8, 9]. The model establishes a three-dimensional state space of the specific volume v , the mean effective stress p' , and the deviatoric stress q . However, it is still rarely used in practice, since it requires the measurement of many preliminary parameters, and calibrating these parameters requires complete consolidation tests and triaxial compression curves, far beyond the workload that a bevameter can accomplish in the field. Therefore, the application of Cam-clay in terramechanics is still at the stage of methodological exploration.

Xie et al. made further progress in applying the Drucker–Prager route to terramechanics [9]. Based on the rate sensitivity of the strength and stiffness of soil, especially cohesive soil, when vehicles travel at different speeds, they additionally embedded the effects of strain rate on the cohesion, elastic modulus, and hardening/softening parameters into the traditional D–P framework, establishing a rate-dependent enhanced Drucker–Prager model.

The Drucker–Prager model has also been applied to tire–ground finite element analysis in ABAQUS to investigate the effects of tire pressure, axle load, and soil strength on the contact pressure distribution and rut depth. The simulation results successfully reproduced the stress concentration in the tire shoulder region observed in experiments, a detail that semi-empirical models cannot describe.

The application of the continuum paradigm in mobile robot terramechanics is still limited, mainly because the continuum mechanics paradigm relies on simulations that take tens of minutes to hours to run and cannot be used for the online traversability judgment of mobile robots within milliseconds. For example, simulations by Hu et al. on the Chrono platform showed that using the smoothed particle hydrodynamics (SPH) method, a continuum approach, a 10-second full-vehicle driving simulation of the Yutu lunar rover takes about 2.8 hours, whereas the equivalent semi-empirical model requires only 20 seconds [10].

2.3. Research based on the granular mechanics paradigm

A fundamental difference between the granular mechanics paradigm and the previous two paradigms is that it does not treat the soil as a continuum; instead, it uses the discrete element method (DEM) to discretize the soil into a large number of independent particles and reproduces the mechanical behavior of the soil by tracking the motion of each particle and the contact forces between particles.

In the application of DEM to mobile robot terramechanics, DEM plays two main roles: a validation tool and a calibration data source. As a validation tool, Nakashima et al. used DEM to analyze the wheel performance of a lunar microrover; compared with soil bin experiments, the prediction errors of the sinkage and traction force can be controlled within 10–15% [11]. Because bevameter tests cannot be performed on the Martian and lunar surfaces, using DEM as a calibration data source is particularly important in the field of planetary rovers. Related literature proposed a complete procedure for building a virtual bevameter with DEM and then mapping the DEM data to semi-empirical model parameters through Bayesian inference; after calibration, the model maintains near-DEM-level accuracy while its computational speed increases from 80 minutes to only about 30 seconds [10]. Kim et al. further combined DEM with an artificial neural network (ANN) to establish a hybrid terramechanics model, in which DEM provides high-fidelity single-condition data as the

training set and the neural network learns the generalization rules across conditions, thereby reducing the computational cost required by pure DEM while maintaining accuracy [12].

In summary, the granular mechanics paradigm currently has unique advantages in mobile robot terramechanics. When facing extreme conditions such as high slip, deep sinkage, and particle flow that both semi-empirical and continuum models struggle to fit, the granular mechanics paradigm is an important high-confidence reference source.

2.4. Research based on the data-driven paradigm

The data-driven paradigm is built upon the rapid development of machine learning and statistical inference methods over the past decade. The data-driven approach does not presuppose an equation form; instead, it takes known quantities at the input—such as wheel geometry, load, and slip ratio—and outputs predictions such as sinkage and traction force that are learned from data.

Online parameter estimation is the earliest direction in which the data-driven paradigm has been applied to mobile robot terramechanics. Existing studies use the onboard sensing information of planetary rovers, such as the wheel torque, slip ratio, and sinkage, to invert the soil cohesion and internal friction angle in real time, realizing for the first time an online estimation process in which the robot senses the mechanical parameters of the soil beneath its wheels while driving.

In the application to parameter calibration, the role of the data-driven paradigm is to replace or reduce traditional bevameter tests. Karpman et al. proposed a hybrid modeling concept, in which a fully calibrated semi-empirical model serves as the main body and a neural network is attached to it; this network learns the residual between the output of the main model and the experimental values, thereby assessing the accuracy of the model [13]. However, the core bottleneck of such data-driven methods lies in the acquisition of training data. High-fidelity labeled data in terramechanics must be generated condition by condition through DEM simulations or physical soil bin experiments; a single DEM wheel–soil simulation can take tens of minutes to hours, and such models are trained only on a limited combination of loads and speeds. If the model is directly extrapolated to condition ranges not covered by training, the prediction accuracy drops significantly [13, 14], which fundamentally limits the condition coverage of the training samples.

3. Cross-comparison of the four paradigms

The four paradigms are now placed under a unified set of evaluation dimensions and, based on the review above and related surveys, are compared in terms of physical fidelity, calibration convenience, computational speed, and prediction capability. The comparison results are shown in Table 1.

Table 1. Cross-comparison of the four modeling paradigms

Paradigm	Physical fidelity	Calibration convenience	Computational speed	Prediction capability
Semi-empirical	Relatively low; parameters are not intrinsic soil properties, and errors grow when deviating from calibration conditions	Convenient; measurable in the field with a bevameter	Millisecond level; can run online	Acceptable at low slip; larger error at high slip
Continuum mechanics	Medium; can distinguish elastic/plastic/dilatancy/contraction states	Difficult; requires triaxial or consolidation tests, hard to implement in the field	Hour level; offline only	Better than semi-empirical but inferior to DEM

Table 1. (continued)

Granular mechanics (DEM)	Highest; no presupposed equations, simulates natural particle contact	Moderate; a virtual bevameter can replace physical calibration	Hour level; offline only	Relatively accurate, nearly coincides with experiments
Data-driven	Depends on training data; unreliable when generalization is insufficient	Depends on data; requires generating a training dataset	Millisecond level after training	Assists semi-empirical equations; inaccurate when training is insufficient

Based on the above analysis, the four paradigms are not in an absolute competitive relationship of elimination and victory; instead, they should be applied to different stages of mobile robot traversability research on complex terrain. At present, no single paradigm can simultaneously satisfy all requirements of accuracy, speed, and scenario generalization.

4. Research gaps and future prospects

After reviewing and comparing the four paradigms, several gaps in this field that have not yet been filled become increasingly clear.

First, there is a lack of quantitative comparison among the paradigms under a common benchmark. Each paradigm uses data measured by its own calibration method as the reference; such self-referential comparative validation cannot answer which paradigm is advantageous within different condition ranges.

Second, there are still many gaps in the research on extreme media and special terrains. The main body of existing literature focuses on dry sand, loam, and clay, whereas the media of mud, snow, and gravel—which most easily cause mobile robots to become unstable in actual operations—rarely appear in the analytical theory of terramechanics, and literature combining these models with mobile robot traversability research remains limited to date.

Third, research on multi-robot formation operations is still in a blank stage. All existing wheel–soil mechanics models take a single vehicle as the analysis unit. When multiple wheeled robots pass successively along the same path over a soft soil section, the wheels of the trailing robot contact not the original soil but the soil already disturbed by the leading vehicle; for the mechanical behavior of repeated wheel/track–soil contact, no relevant research data exist in any of the four paradigms.

In response to the above gaps, several meaningful future research prospects can be proposed.

First, a unified benchmark dataset across paradigms can be established. Based on the approach of open dataset construction in existing studies, the prediction results of the four paradigms can be obtained under identical conditions and compared with physical experimental data as the common benchmark. Such a dataset can provide references for paradigm selection in subsequent research.

Regarding terramechanics modeling of extreme media and special terrains, mature constitutive models already exist in the respective disciplines of mud, snow, and gravel (e. g., the Herschel–Bulkley rheological equation for mud). These models can be coupled with the wheel as a specific loading boundary.

Regarding the terramechanics modeling of multi-robot formation operations, the newly added variables compared with a single robot include at least the spacing between two robots and the time interval between successive passages. To model these variables, the construction of multi-robot scenarios can be combined with existing studies on the mechanics of repeated wheel passages, and

the traversability can be discussed case by case according to different variables; this will provide references for the traversability analysis of multiple mobile robots [15].

5. Conclusion

This paper systematically reviews the terramechanics research on mobile robots over deformable terrain, classifies the existing modeling methods into four paradigms—semi-empirical, continuum mechanics, granular mechanics, and data-driven—and compares them across four dimensions: physical fidelity, calibration convenience, computational speed, and prediction capability. The semi-empirical paradigm has the best computational efficiency and is the only route that can be embedded in real-time control loops, but its error grows systematically when deviating from the calibration conditions. The continuum mechanics paradigm, based on the constitutive system from Mohr–Coulomb to Cam-clay, provides the most complete description of physical processes, but its calibration threshold is high, and its simulations are time-consuming, making it suitable only for offline analysis. The granular mechanics paradigm, based on the discrete element method and particle contact mechanics, has the highest fidelity and nearly coincides with experiments under high-slip conditions, but its computational cost is high, and it cannot run online. The data-driven paradigm bridges offline simulation and online prediction through machine learning and Bayesian inference, but is currently limited by the scarcity of labeled data and insufficient generalization. The four paradigms do not replace one another; instead, they should be applied cooperatively to different stages of research in this field. The paper also points out the gaps in this field, including the lack of a multi-paradigm benchmark, the insufficient modeling of extreme media, and the gap in formation terramechanics, and proposes several prospects for future research based on these gaps, with the aim of providing references for subsequent terramechanics-based traversability studies of wheeled/tracked mobile robots.

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