

Redundancy Utilization and Motion Planning of Redundant Manipulators

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Abstract. Redundant manipulators provide extra degrees of freedom that can be exploited to improve motion flexibility, obstacle avoidance, singularity avoidance, and joint-limit handling. However, redundancy also introduces multiple inverse kinematics solutions and increases the complexity of motion planning and control. This paper reviews methods for utilizing redundant degrees of freedom and solving inverse kinematics, with emphasis on the development from null-space and task-priority methods to optimization-based approaches. Techniques including gradient projection, flexible task priorities, task-transition control, set-based control, and performance-based multi-objective optimization are discussed. The review also examines motion planning under multiple constraints, including environmental obstacles, self-collision, joint limits, singularities, path length, and motion smoothness. Particular attention is given to the interaction between inverse kinematics and path planning, including methods that jointly optimize target configurations and motion paths. Particle swarm optimization and the PASO framework are introduced as examples of intelligent optimization for high-dimensional redundant manipulators. Overall, research is shifting from finding feasible inverse kinematics solutions toward integrated optimization of safety, motion performance, and the complete motion process. Future work should improve computational efficiency and the coordination of multiple constraints and objectives for high-degree-of-freedom robots.

Keywords: Redundant manipulators, inverse kinematics, motion planning, redundancy resolution, optimization

1. Introduction

A robotic manipulator is a robotic system composed of multiple joints and links. By controlling joint motion, it can move the end effector to a desired position and orientation. The degrees of freedom (DOF) of a manipulator are commonly defined as the number of independent variables that describe its motion capability. In three-dimensional space, a general end-effector pose has six DOF: three translational and three rotational. A redundant manipulator has more degrees of freedom than are required to complete a particular task. For a full six-DOF pose task, a seven-DOF manipulator provides one redundant degree of freedom and is therefore one of the most commonly studied redundant structures [1].

Redundant manipulators are more flexible in complex environments than non-redundant manipulators because the same end-effector pose can be reached through multiple joint configurations. The extra degrees of freedom can be used not only to adjust the manipulator configuration but also to improve obstacle avoidance, singularity avoidance, and joint-limit handling. However, redundancy also creates multiple inverse kinematics solutions and increases the complexity of motion planning and control [2]. Therefore, determining how redundant degrees of freedom should be allocated and utilized to satisfy task requirements while improving motion performance has become a central topic in redundant manipulator research.

To address these problems, inverse kinematics methods for redundant manipulators have gradually progressed from classical solution techniques toward optimization-based approaches. Early work focused mainly on kinematic modeling and inverse kinematics, whereas later studies increasingly incorporated practical objectives such as obstacle avoidance, joint constraints, and motion stability [3]. More recently, flexible task-priority strategies [4] and particle swarm optimization (PSO) [5] have been applied to redundant manipulators to improve their adaptability to increasingly complex task conditions.

This paper focuses on motion planning and the utilization of redundant degrees of freedom in redundant manipulators. It first examines methods for allocating redundant degrees of freedom and solving inverse kinematics, and then reviews motion planning and motion optimization under complex constraints. It also compares the characteristics of different approaches for resolving multiple solutions and improving motion performance. Through this review, the paper further investigates how structural redundancy can be converted into practical motion capabilities and provides a reference for future research on the motion planning and control of redundant manipulators.

2. Utilization of redundant degrees of freedom and inverse kinematics methods

2.1. Utilization of redundant degrees of freedom under multiple task constraints

In manipulator tasks, the primary objective is often to reach a desired end-effector position and orientation. At the same time, secondary objectives such as obstacle avoidance, joint-limit avoidance, singularity avoidance, and motion smoothness must also be considered. Null-space motion is one of the most established approaches for handling such secondary tasks [6]. One or more redundant degrees of freedom allow a manipulator to change its joint configuration without changing the end-effector pose. For example, a seven-DOF manipulator performing a six-DOF pose task has one independent redundant direction, whereas an eight-DOF manipulator has two. Coordinated joint motion along these directions leaves the primary end-effector task unchanged and is therefore referred to as null-space motion.

Null-space methods identify joint motions that leave the end-effector pose unchanged, while the Gradient Projection Method (GPM) extends this principle by using null-space motion to accomplish secondary objectives. GPM defines an objective function for a secondary task, uses its gradient to determine an improvement direction, and then applies null-space projection so that the primary task is not affected. When several secondary tasks are present simultaneously, however, conflicts can arise among objectives such as joint-limit avoidance, singularity avoidance, and obstacle avoidance. A fixed task-priority scheme may also increase computation time or fail to produce a feasible solution. To address this problem, Kelemen et al. proposed the Flexible Priority Solution (FPS), which dynamically resets task priorities according to the current computational situation. Priorities can be reduced when a task makes the problem difficult to solve and restored when conditions

improve. Numerical simulations and experiments on a five-link manipulator showed that FPS can substantially reduce computation time in complex cases [4].

After task priorities can be adjusted dynamically, another issue is that task definitions themselves may change during motion. Traditional task-priority schemes are usually fixed, whereas practical applications may require tasks to be temporarily added, removed, or switched. An and Lee therefore proposed Prioritized Inverse Kinematics with Multiple Task Definitions, which enables smooth transitions between tasks through task-transition control while ensuring bounded joint motion. Simulations with a KUKA LWR and a humanoid robot showed that the method can reduce discontinuities caused by task switching [7].

Conventional redundancy-control methods for joint-limit and singularity avoidance often formulate these constraints as secondary objectives that are continuously optimized, driving the manipulator as far as possible from the constraint boundaries. However, when a variable is already within a safe range, continuous optimization can unnecessarily occupy the available redundant degrees of freedom. For example, if a joint limit is 100° , a conventional method may continue to drive the joint away from the limit even after it reaches a safe value such as 75° . Antonelli et al. addressed this issue through set-based control, in which some secondary tasks are represented as set-based constraints rather than fixed optimization objectives. The method requires the corresponding variables only to remain within allowable ranges, reducing competition from tasks that have already satisfied their safety requirements and allowing integration with multi-task prioritized inverse kinematics. Numerical experiments showed that the method can satisfy multiple constraints while keeping joint variables within their allowable ranges [8].

2.2. Optimization-based inverse kinematics methods

For the same end-effector task, redundant manipulators usually have multiple inverse kinematics solutions. The problem is therefore not simply to find a feasible solution, but to select a suitable one according to specific performance requirements. A pseudoinverse solution can minimize the norm of joint velocity, but in practice other performance criteria may be more important. For example, manipulability indicates motion flexibility in a particular configuration, and a low manipulability value suggests proximity to a singularity. Smoothness indicates whether motion is continuous and whether joint velocity changes abruptly. Other important criteria include distance from joint limits and energy or torque requirements. Appropriate performance measures are therefore needed to select solutions for particular objectives. Because several criteria often need to be considered simultaneously, multi-objective optimization is required, and weighted combinations of individual criteria remain a practical approach [9].

The optimization problem can become difficult and highly nonlinear when many criteria must be optimized simultaneously, especially for high-DOF manipulators. Evaluating candidate solutions individually may become computationally expensive, making population-based search methods attractive. These methods consider a population of candidate solutions and continually update their search directions using information acquired in previous iterations. Particle Swarm Optimization (PSO) is one such algorithm. In PSO, particles represent candidate solutions and their fitness is evaluated by an objective function. Each particle updates its position according to both its own historical best solution and the best solution found by the swarm. For redundant manipulators, a particle can represent a joint configuration, allowing the algorithm to search a high-dimensional joint space for an inverse kinematics solution that achieves the desired end-effector position and orientation. Collins and Shen extended PSO to hyper-redundant manipulators and proposed PSOIK

as the inverse kinematics component of PASO. Tests on manipulators with 30 to 180 DOF showed that the method could converge to solutions satisfying the task requirements [5].

A single inverse kinematics solution guarantees only that the manipulator reaches one target configuration. Because redundant manipulators can adopt multiple joint configurations at the same end-effector pose, solving inverse kinematics independently at each time step can produce large configuration changes between successive steps. In extreme cases, a manipulator can return to the same end-effector position after completing a closed path but end in a different joint configuration. Hauser and Emmons therefore proposed a global redundancy-resolution method based on continuous pseudoinversion. The method defines a continuous mapping between workspace and joint space, helping the manipulator preserve configuration continuity during continuous motion and reducing discontinuities in inverse kinematics solutions. It also considers consistency along closed workspace paths so that the manipulator returns as closely as possible to its initial configuration, thereby improving the stability of continuous motion [10].

Building on this development, research has progressed from finding continuous inverse kinematics solutions toward optimizing the entire motion process. Reiter et al. investigated higher-order inverse kinematics for redundant manipulators and applied it to time-optimal trajectory planning. Their method minimizes motion time while satisfying constraints on position, velocity, acceleration, torque, and other variables. Using a seven-DOF space manipulator as an example, the approach considered end-effector trajectory tracking and higher-order continuity and solved the resulting problem through nonlinear optimization. Experimental results showed a substantial computational-time advantage in some tests. However, because the optimization problem is non-convex, the resulting solution is not guaranteed to be globally optimal, and non-convergence or singularity issues can occur in some cases [11].

3. Motion planning and integrated optimization of redundant manipulators

3.1. Multi-constraint motion planning methods

Motion trajectories for redundant manipulators are often complex, and the joints cannot simply move from their initial positions to a target configuration obtained from inverse kinematics because multiple constraints must be satisfied throughout the motion [12]. Collision avoidance is a primary concern. In addition to avoiding environmental obstacles, collisions between different links of the manipulator, known as self-collisions, must be prevented, particularly for high-DOF manipulators. Experiments by Collins and Shen showed that, among randomly generated configurations, approximately 42% of 60-DOF configurations and 60% of 90-DOF configurations involved self-collision, indicating that this problem becomes more significant as the number of degrees of freedom increases [5].

Joint limits must also be considered. Each joint has a finite range of motion, and some apparently short paths may be infeasible because one or more joints would exceed their limits. Although redundant manipulators provide additional motion options, joint motions remain coupled and must therefore be coordinated appropriately during path planning [13].

Singularities are another important consideration in motion planning. An optimized inverse kinematics solution may provide good performance at the final pose, but it does not guarantee that the manipulator will avoid singular configurations along the entire path from the initial pose to the target pose. Singularity avoidance must therefore be incorporated into trajectory planning [14].

After these constraints are satisfied, path length and motion smoothness should also be considered. Shorter and smoother paths are generally desirable because they can reduce unnecessary

joint motion, improve efficiency, and avoid abrupt changes in velocity and acceleration. Motion planning for redundant manipulators must therefore address collision avoidance, joint limits, singularity avoidance, and trajectory performance simultaneously while ensuring the feasibility of the complete trajectory.

3.2. Integrated optimization of inverse kinematics and path planning

For redundant manipulators, a single end-effector target pose can usually correspond to multiple joint configurations. Therefore, after a target pose is specified, an appropriate configuration must be selected from the available inverse kinematics solutions [2]. Traditionally, inverse kinematics is first used to calculate a target joint configuration, after which path planning determines a motion path from the initial configuration to that target. Although this approach treats inverse kinematics and path planning separately, the two processes are closely related.

Different target configurations can lead to substantially different motion paths. For example, two inverse kinematics solutions may both achieve the same end-effector pose, but one may be much farther from the initial configuration and require larger joint motions, whereas another may not be optimal according to a local inverse kinematics criterion but may produce a shorter path and make obstacle avoidance easier. Consequently, a target configuration that is locally optimal does not necessarily produce the best overall motion from the initial state. Conversely, path planning that does not consider the choice of target configuration may produce unnecessarily long or complex trajectories.

Some studies therefore treat inverse kinematics and path planning jointly rather than fully decoupling the two processes [5]. In this approach, selection of the target configuration can account for the ease of subsequent path planning, while the path planner can exploit multiple feasible configurations of a redundant manipulator to choose a target configuration that is more conducive to motion. In addition to ensuring that the end effector reaches the target pose, the integrated process can consider collision avoidance, path length, motion smoothness, and joint displacement, producing solutions that are more suitable for practical manipulator operation [12].

For manipulators with many degrees of freedom, integrated optimization requires searches over larger variable spaces while satisfying more constraints, which substantially increases computational complexity. Such high-dimensional problems are often difficult to solve directly using conventional analytical methods. Intelligent optimization methods such as particle swarm optimization (PSO) have therefore been applied. PSO can explore a broad search space of candidate manipulator configurations, and the quality of each candidate can be evaluated using end-effector position error, orientation error, collision conditions, and other objectives.

The PASO method proposed by Collins and Shen is an example for high-DOF manipulators. It first uses PSO to solve the inverse kinematics problem. After identifying a joint configuration that achieves the desired end-effector position and orientation, it plans a path through configuration space while considering both environmental collisions and self-collisions. PASO therefore integrates inverse kinematics and path planning within a single framework rather than treating them as isolated problems. Experiments showed that the method can be applied to manipulators with very large numbers of degrees of freedom and can address both inverse kinematics and path planning in high-dimensional spaces.

Overall, the key idea of jointly optimizing inverse kinematics and path planning is that the two problems should not be treated as independent. For redundant manipulators, the selected target configuration influences the resulting motion path, while path-planning requirements can also influence the choice of inverse kinematics solution. Integrating the two processes can therefore

make better use of redundant degrees of freedom. As the number of degrees of freedom and constraints increases, developing safer and smoother motion paths while maintaining computational efficiency remains an important topic for future research.

4. Conclusion

Compared with conventional manipulators, redundant manipulators have additional degrees of freedom and can therefore change their joint configurations while maintaining the primary end-effector task. This provides additional options for obstacle avoidance, singularity avoidance, and joint-limit handling. Early studies focused mainly on using these extra degrees of freedom to accomplish secondary tasks without compromising the primary task. As the field developed, however, simply finding a feasible inverse kinematics solution became insufficient. Multiple performance objectives were introduced, and optimization methods were used to select better solutions from the feasible set. The emphasis gradually shifted from optimizing only the final end-effector pose or joint configuration toward optimizing the complete trajectory, including motion continuity and executability. Collision avoidance, joint limits, singularities, path length, smoothness, and other considerations must therefore be addressed simultaneously, transforming motion planning from a single-objective problem into a multi-constraint, multi-objective problem. Because the selected target joint configuration can influence the resulting path and path-planning requirements can influence the choice of inverse kinematics solution, some studies have begun to integrate inverse kinematics and path planning. Intelligent optimization methods such as PSO have also been used to address the increasingly complex search spaces associated with high-DOF manipulators.

Overall, research on redundant manipulators is shifting toward integrated optimization that considers not only inverse kinematics but also safety, motion performance, and the complete motion process. Future work should improve the computational efficiency of methods for high-DOF manipulators and strengthen the coordination of multiple objectives, including collision avoidance, singularity avoidance, path length, and motion smoothness, so that these methods can be deployed more reliably and robustly in practical robotic manipulation.

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