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Dynamic Characteristics of a Bistable Negative Poisson's Ratio Buckled-Beam Piezoelectric Vibration Energy Harvester

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Abstract. To address the low output power and narrow operating bandwidth of linear piezoelectric energy harvesters under low-frequency and low-amplitude environmental vibration, this paper proposes a piezoelectric vibration energy harvester with a bistable negative Poisson's ratio (Bistable Negative Poisson's Ratio, BNPR) based on a cosine curved beam configuration. First, a lumped-parameter electromechanical coupled dynamic model of this structure is established, and the analytical expressions of the nonlinear restoring force, the equivalent capacitance and the electromechanical coupling coefficient are derived; then the dynamic equations of the system are solved numerically by the Runge-Kutta method, and the frequency-domain response, phase trajectory characteristics and energy harvesting performance under 0.3g~0.6g acceleration excitation are analyzed. The results show that under 0.3g and 0.4g low amplitude excitation the system is in the single-well vibration state and presents softening stiffness characteristics; when the excitation amplitude reaches the threshold of 0.5g, the system triggers bistable cross-well motion, presents softening-hardening composite stiffness characteristics, and the output voltage and working bandwidth are significantly improved; under 0.6g excitation the number of steady-state switchings increases, and the peak voltage and bandwidth are further optimized. This study provides a basis for structural design and dynamic analysis of vibration energy harvesting for typical industrial and traffic scenarios.

Keywords: Bistable, negative Poisson's ratio, electromechanical coupling coefficient, vibration energy harvesting

1. Introduction

With the rapid development of the Internet of Things and distributed sensing technology, the self-powered demand of miniature electronic devices is increasingly urgent. The piezoelectric energy harvesting technology, which captures vibration energy from the environment and converts it into electric energy, has become an important technical scheme to replace the traditional battery power supply due to its simple structure, high energy density and easy integration [1]. Vibration in the actual environment generally has the characteristics of low frequency, low amplitude and variable frequency, while the traditional linear piezoelectric energy harvester only has relatively high output

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near the resonance frequency, its working bandwidth is extremely narrow, it is difficult to adapt to complex and variable actual vibration scenarios, and the application scenarios are seriously limited [2].

To broaden the working bandwidth of the energy harvester, the bistable structure that introduces geometric nonlinearity has received wide attention from scholars at home and abroad. The bistable system relies on the double-well potential energy characteristic, and can realize cross-well motion between two stable states under external excitation, thereby maintaining relatively high energy output over a wider frequency range and effectively improving the energy conversion efficiency under low amplitude vibration [3, 4]. Among various bistable configurations, the buckled beam structure needs no additional magnetic elements, and the geometric nonlinearity produced by axial buckling of the structure is enough to form the bistable characteristic. It has the advantages of simple structure, easy miniaturization and system integration, and has become a research hotspot in the field of vibration energy harvesting [5].

The negative Poisson's ratio (Auxetic) cell structure has the special mechanical behavior of expanding under tension and contracting under compression. Introducing it into the buckled beam energy harvester can regulate the distribution of structural stiffness and the transfer law of strain through configuration design, and further improve the electromechanical coupling efficiency and the load-bearing capacity of the structure [6]. Existing studies mostly carry out dynamic analysis on traditional straight beam bistable energy harvesters, and the research on the bistable triggering mechanism, threshold characteristics and bandwidth evolution law of buckled beams with negative Poisson's ratio configuration is still not systematic enough [7, 8]. The analysis of structural adaptability for different environmental vibration scenarios also needs to be improved [9]. Table 1 lists the frequencies and amplitudes of two common environmental vibrations. It can be seen from the table that the frequency of mechanical vibration is relatively high, while the vibration frequency of buildings is relatively low. And most of the vibration amplitudes are relatively low, and the vibration frequency is variable. Therefore it is necessary to study the excitation threshold and the working bandwidth of bistable vibration.

For this purpose, this paper designs a bistable negative Poisson's ratio (BNPR) piezoelectric vibration energy harvesting model based on a cosine curved beam, establishes the lumped-parameter electromechanical coupled dynamic equations, and adopts numerical simulation to analyze the dynamic behavior and energy output characteristics under different acceleration excitations, so as to make clear the bistable triggering threshold and the working bandwidth, and to provide theoretical support for the engineering application of this kind of structure in actual vibration environments [10].

Table 1. Frequencies and amplitudes of common environmental vibrations

Vibration environment	Frequency (Hz)	Amplitude (m/s2)
Vertical vibration of the workshop floor slab caused by a spinning machine	10	9.89
Ground vibration caused by a vehicle passing over a speed bump	/	4.5

2. Initial model design

The composition of the model designed in this paper is shown in Figure 1. The bistable structure has a double-well potential energy function. When the external excitation is relatively large, the bistable system can carry out cross-well motion between the two wells. The deeper the potential energy well of the potential energy function, the larger the external excitation required. However, in a real

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vibration environment, most of the excitations are relatively small. In order to study the dynamic characteristics of the bistable structure, this paper designs a new type of Bistable Negative Poisson's Ratio structure. The height H is 2mm; the span length L is 235.85mm, the width W is 30mm and the thickness T is 0.1mm.

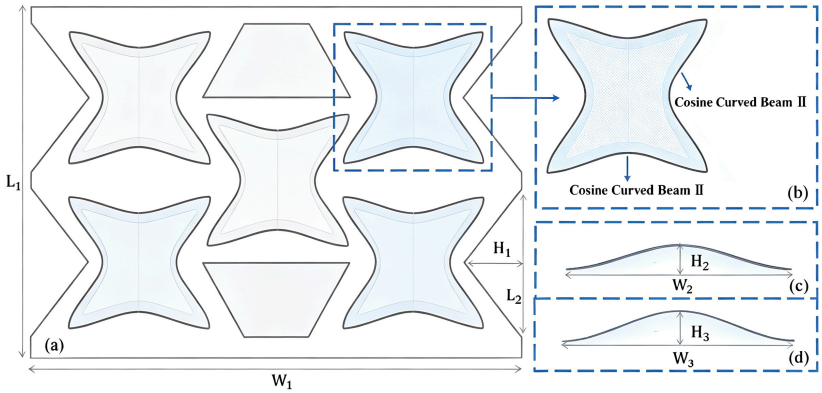


Figure 1. Model design

3. Method introduction

3.1. Equivalent lumped parameter model

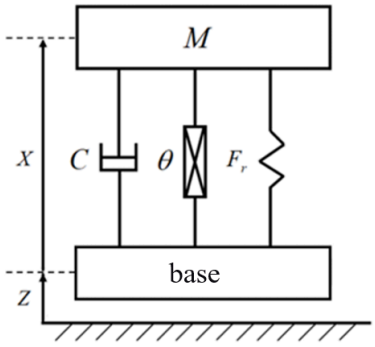


Figure 2. Lumped parameter model of the piezoelectric vibration energy harvesting device

Carrying out lumped parameter modeling according to the structure diagram of the piezoelectric vibration energy harvesting device, as shown in Figure 2. Where: M denotes the equivalent mass of the straight beam or the buckled beam, Z denotes the vibration displacement of the base, X denotes the displacement of the mass block M relative to the base, C denotes the equivalent damping of the system, θ denotes the electromechanical coupling coefficient of the system, F_r denotes the nonlinear restoring force of the system.

3.2. Equivalent dynamic model

According to the lumped parameter model established in Figure 2, the mechanical equation of the energy harvesting system is obtained by Newton's second law:

$$M\ddot{X} + C\dot{X} + F_r + 2\theta V = -M\ddot{Z} \tag{1}$$

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Where M is the equivalent mass of the beam, C is the equivalent damping, $\ddot{Z}$ is the external excitation signal, $\ddot{Z} = Acos(\omega t)$, A is the acceleration amplitude, ω is the output frequency.

When the piezoelectric vibrator vibrates under external excitation, it shows a complex electromechanical coupling process. The piezoelectric sheet outputs charge along with the bending deformation of the beam, and the piezoelectric element is regarded as an equivalent current source connected to an internal capacitance. As shown in Figure 3. Where: R_L is the load resistance, V is the output voltage, C_p is the equivalent capacitance, $2\theta V$ is the electromechanical coupling force.

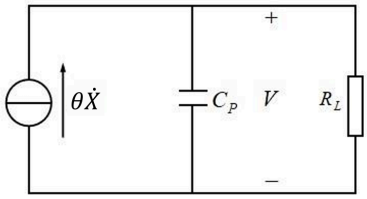


Figure 3. Equivalent circuit model of the piezoelectric vibrator

According to Kirchhoff's current law, the electrical equation of the collecting circuit can be obtained:

$$C_p V + V R_L^{-1} - \theta \dot{X} = 0 \tag{2}$$

Combining the mechanical equation and the electrical equation gives the electromechanical coupled equation. The expressions of the equivalent capacitance C_P and the electromechanical coupling coefficient θ are as follows:

$$C_p = \frac{\varepsilon_{31}\varepsilon_0 W_M l_M}{t_p} \tag{3}$$

$$\theta = e_{31}\Psi_r^{'} W_M \frac{t_M + t_p}{2} \tag{4}$$

Where ε_{31} is the relative dielectric constant, ε_0 is the absolute dielectric constant of vacuum, w_M and l_M are the width and length of the beam respectively, t_p is the thickness of PVDF, e_{31} is the piezoelectric stress coefficient, $\psi_r^{'}$ is the spatial derivative of the mechanical mode shape vector, t_M is the thickness of the buckled beam.

In the above equation F_r is the nonlinear restoring force. In the finite element simulation, the stiffness of the static structures of the straight beam and the buckled beam energy harvesters was analyzed respectively, the force-displacement curves were obtained, the relationship between force and displacement was fitted with a high-order polynomial, and the nonlinear restoring force F_r expressions of the straight beam system and the buckled beam system were obtained.

4. Numerical simulation results

This paper uses the Matlab Ode45 solver to solve the electromechanical coupled dynamic equations by the Runge-Kutta method, and obtains the output voltage and displacement of the BNPR model in the frequency domain. The external excitation signal is set as a linear frequency sweep excitation, and numerical simulation analysis is carried out on the energy harvesting device. To study the

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bandwidth of the energy harvester and the energy harvesting effect, the relevant parameters of the harvester in the coupled equations need to be determined. Through the dielectric constant of the PVDF piezoelectric sheet, the equivalent capacitance is obtained. Through modal analysis of the straight beam and the buckled beam systems, the mode shape vector is obtained, and then the electromechanical coupling coefficient of the system is calculated. The logarithmic decrement method of free vibration is used to obtain the equivalent mass and the damping coefficient. The parameters of the energy harvester are summarized in Table 2.

Table 2. Model parameters of the energy harvester

Parameter	Straight beam	Buckled beam
$M(g)$	4.9	4.9
$C(N/(m/s))$	0.0747	0.0747
$\theta(V/N)$	5.62e-7	3.7e-6
$C_p(nF)$	20.8	20.8
$R_L(k\Omega)$	40	40

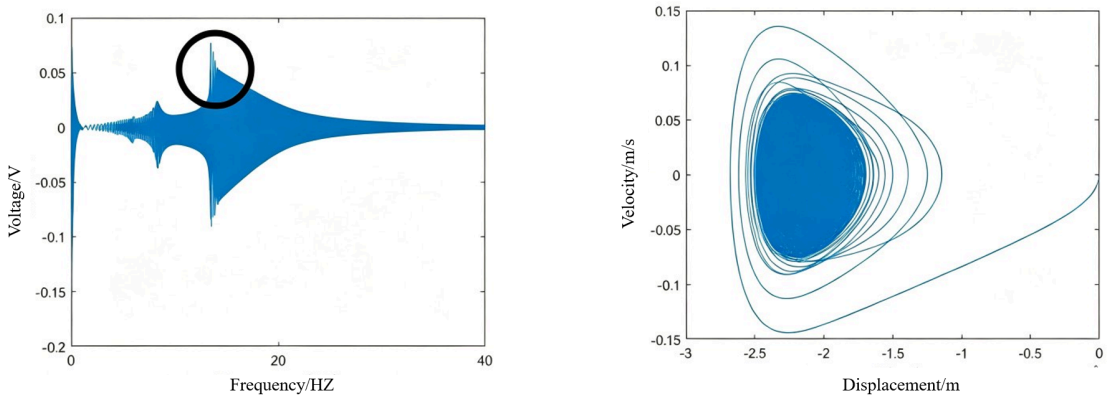


Figure 4. Results of the BNPR1 model under 0.3g acceleration excitation: left) frequency-voltage plot, right) phase portrait

According to the calculated parameters, we carry out numerical simulation analysis of the BNPR model. To study the working bandwidth and output voltage of the energy harvester, this paper tests the open-circuit frequency response of the energy harvester under different acceleration excitations (0.3g, 0.4g, 0.5g),and obtains the frequency-voltage plot and the phase portrait. As can be seen from Figure 4, under 0.3g acceleration excitation, the BNPR model always vibrates around the first stable point (at a displacement of about -0.0022m), and no bistable vibration is generated. In the frequency-voltage plot we can also see that the voltage suddenly increases at the black circle and then decreases continuously, showing the characteristic of softening stiffness. Figure 5 shows the same characteristic as Figure 4, that is, under 0.4g acceleration excitation, the BNPR model does not

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generate bistable vibration, and shows the characteristic of softening stiffness in the frequency-voltage plot.

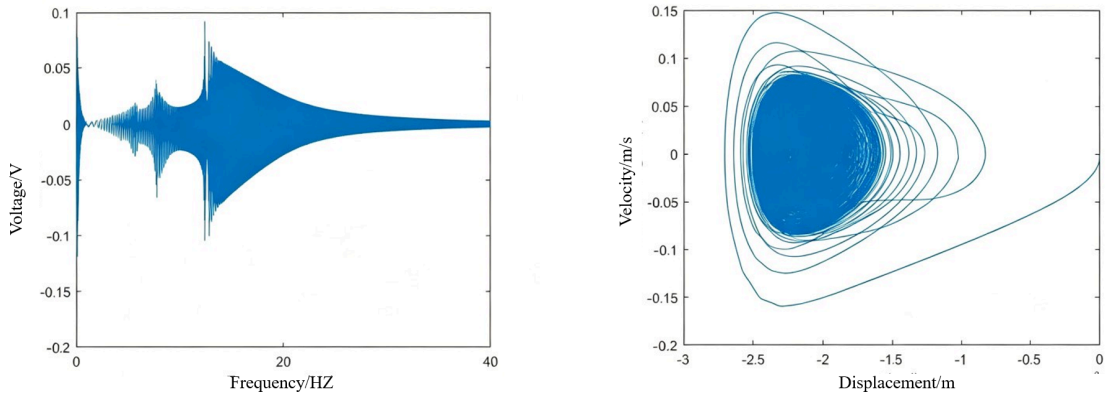


Figure 5. Results of the BNPR1 model under 0.4g acceleration excitation: left) frequency-voltage plot, right) phase portrait

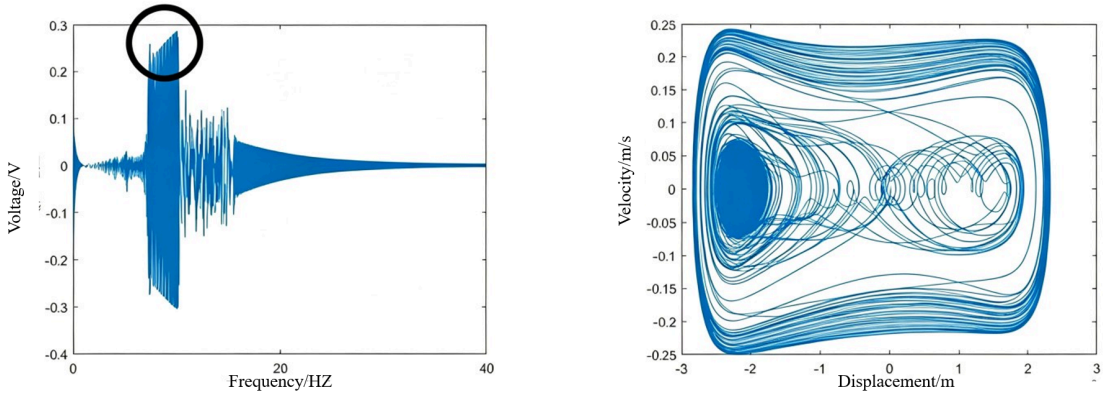


Figure 6. Results of the BNPR1 model under 0.5g acceleration excitation: left) frequency-voltage plot, right) phase portrait

As can be seen from Figure 6, under 0.5g acceleration excitation, the phase portrait rotates around two points (at displacements of about -0.0022m and +0.0015m), indicating that the BNPR model generates bistable vibration. The sudden rise of the voltage shows the softening stiffness characteristic, but the BNPR model generates bistable vibration, and after the softening stiffness there is also a hardening stiffness characteristic. Therefore the voltage keeps rising and then suddenly drops, showing the frequency response of hardening stiffness.

5. Conclusions

This paper designs a bistable negative Poisson's ratio (BNPR) piezoelectric vibration energy harvester based on a cosine curved beam, establishes the lumped-parameter electromechanical coupled dynamic model, and systematically analyzes the dynamic behavior and energy harvesting characteristics under 0.3g~0.5g acceleration excitation through numerical simulation. The following main conclusions are obtained:

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(1) Under low amplitude excitation, the system always vibrates around a single stable point, is in the single-well motion state, the frequency-domain response shows typical softening stiffness characteristics, the output voltage first increases and then decreases as the frequency rises, and the effective working bandwidth is relatively narrow;

(2) When the excitation amplitude reaches 0.5g, the system breaks through the potential energy barrier and triggers bistable cross-well motion, the phase trajectory evolves periodically around two stable points, the frequency-domain response presents softening-hardening composite stiffness characteristics, and the output peak voltage and working bandwidth are significantly improved compared with the single-well state;

(3) Combined with the comparison of typical environmental vibration parameters, it can be seen that this BNPR model can generate stable bistable vibration. According to the data given in Table 1, it can adapt to engineering scenarios such as ground vibration caused by vehicle speed bumps and workshop floor slab vibration caused by spinning machines, providing a new structural scheme for energy harvesting from low frequency and low amplitude environmental vibration. In the following work, structural multi-parameter optimization and prototype experimental verification can be further carried out to improve the practical engineering application value.

References

[1] Liu, Y., Wang, Z., Li, H., et al. (2024). Advances in piezoelectric vibration energy harvesting for industrial IoT applications: A review. *Energy Conversion and Management, 303*, 118215.

[2] Zhang, Q., Chen, L., & Sun, K. (2024). Bandwidth limitation of linear piezoelectric energy harvesters under random environmental vibrations. *Journal of Sound and Vibration, 568*, 117982.

[3] Wang, X., Zhou, S., & Zhang, Z. (2024). Mechanism of bandwidth enhancement in bistable piezoelectric energy harvesters with double-well potential. *Smart Materials and Structures, 33*(5), 055012.

[4] Mao, J., Li, Y., & Ding, W. (2025). High-efficiency bistable energy harvesting via inter-well motion under low-level excitations. *Applied Energy, 372*, 123890.

[5] Chen, B., Liu, X., & Wu, Y. (2024). Static and dynamic characteristics of buckled-beam bistable piezoelectric energy harvesters. *International Journal of Mechanical Sciences, 269*, 109078.

[6] Xu, M., Zhao, L., & Tian, X. (2025). Auxetic structure enhanced piezoelectric energy harvesters: Design and performance optimization. *Composite Structures, 341*, 118562.

[7] Guo, R., Feng, H., & Jiang, C. (2024). Nonlinear dynamics of auxetic buckling structures for energy harvesting: A review. *Mechanical Systems and Signal Processing, 212*, 111289.

[8] Lin, D., Xiao, Y., & Huang, W. (2025). Threshold characteristics of snap-through motion in bistable buckled beams with different configurations. *Sensors and Actuators A: Physical, 379*, 115097.

[9] Zhou, K., Wang, H., & Liu, S. (2026). Performance matching of bistable energy harvesters with typical environmental vibration scenarios. *Energy, 335*, 132541.

[10] Tang, L., Zhu, X., & Wang, Q. (2024). Numerical analysis of electromechanical coupled dynamics for piezoelectric energy harvesting systems. *IEEE Transactions on Industrial Electronics, 71*(10), 10234–10243.